REFLECTIONS ON MAGNETOCONVECTION

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Thanks to

MIKE PROCTOR

Dan Moore
Andrew Soward
Neal Hurlburt
Steve Tobias
Paul Bushby
Dave Galloway
David Hughes
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Jon Dawes
Steve Houghton

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(L.H.M. Rouppe van der Voort)

Outline

- Linear Theory
- Numerics (2D Boussinesq)
- Nonlinear Dynamics
- Compressible Magnetoconvection
- Small-scale Dynamos
- Back to Sunspots

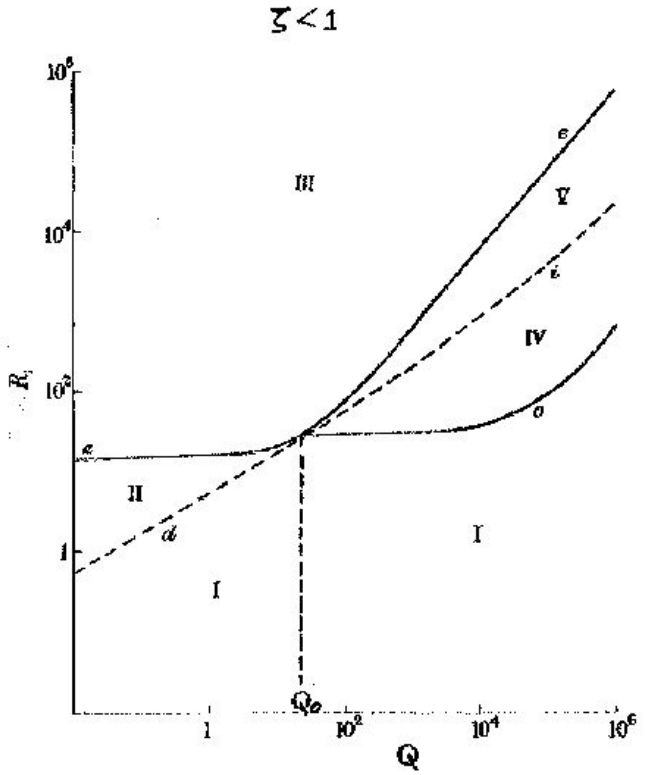
Linear Theory (1950-1965)

- Stationary and oscillatory instabilities
- Bifurcations in *QR*-plane
- Importance of zeta
- Eigenvalues in complex *s*-plane

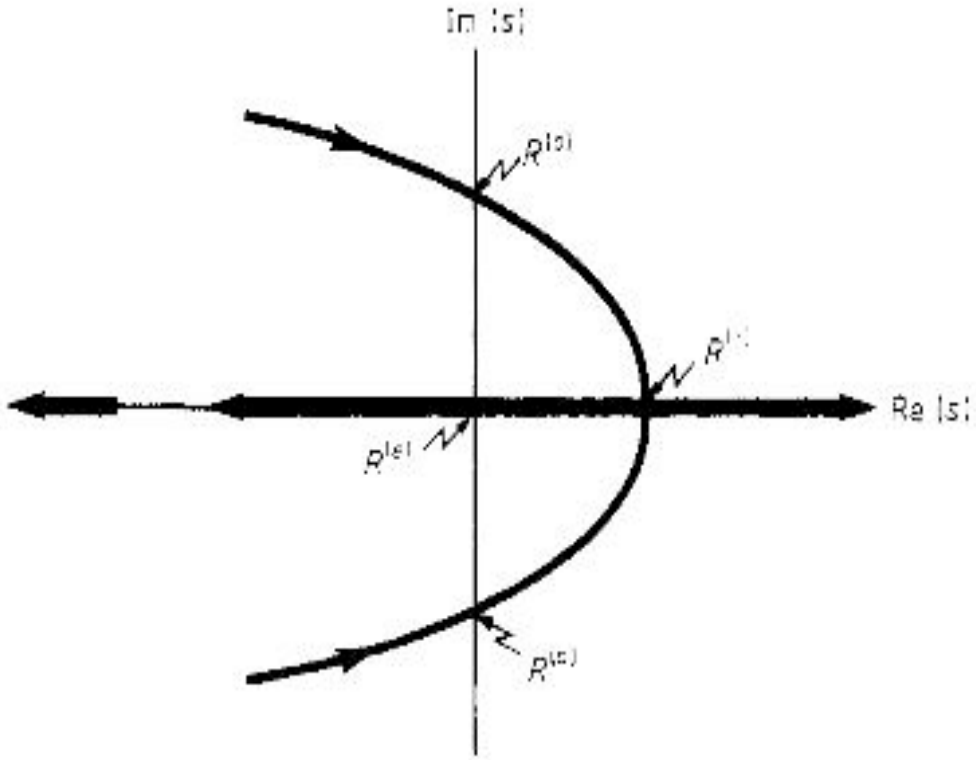
The Rayleigh number R and the Chandrasekhar number Q are given by

$$R = \frac{g\alpha_T \Delta T d^3}{\kappa\nu} \quad \text{and} \quad Q = \frac{B_0^2 d^2}{\mu_o \rho_0 \eta \nu},$$

while the Prandtl number $\sigma = \nu/\kappa$, and the diffusivity ratio $\zeta = \eta/\kappa$.

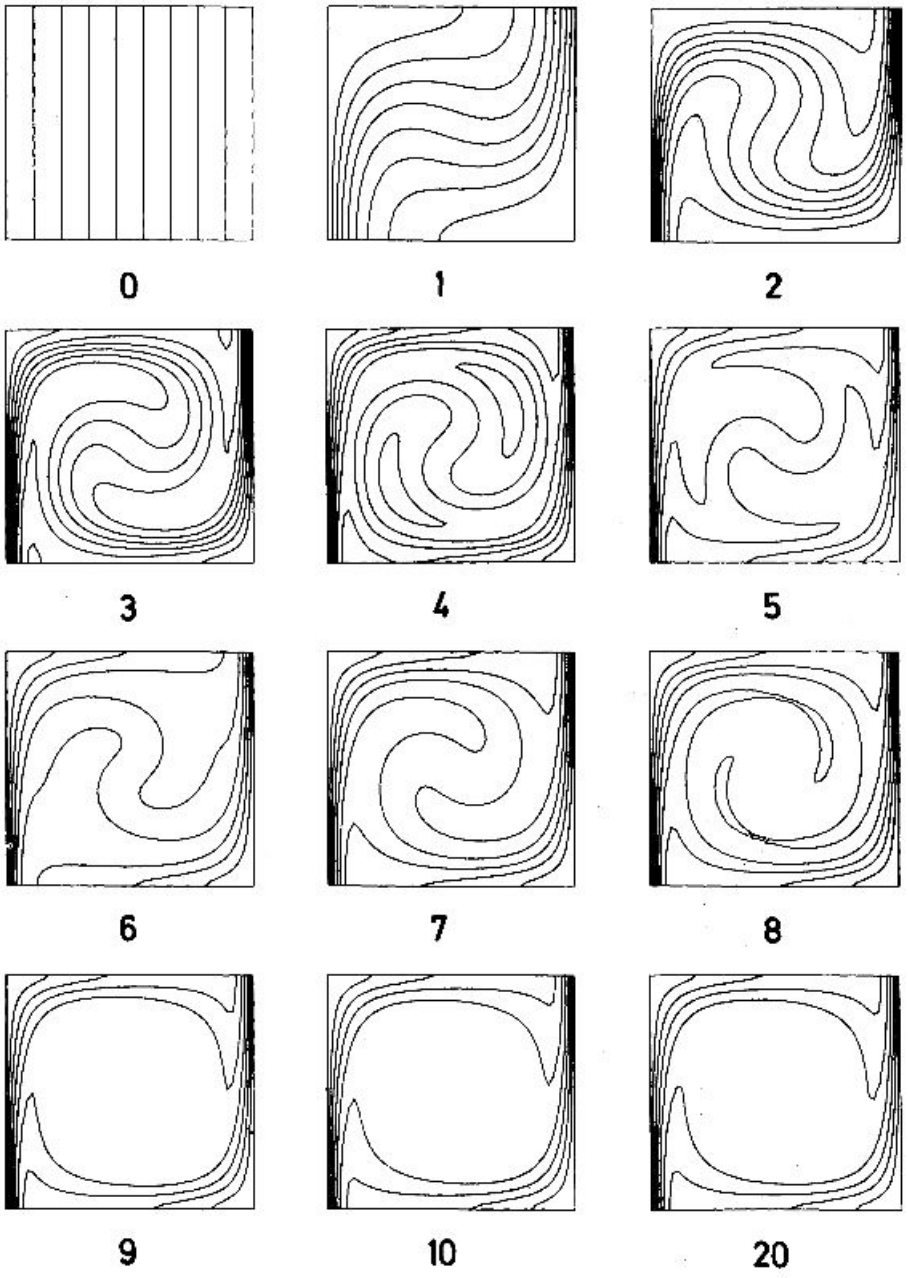


Eigenvalues in the complex *s*-plane



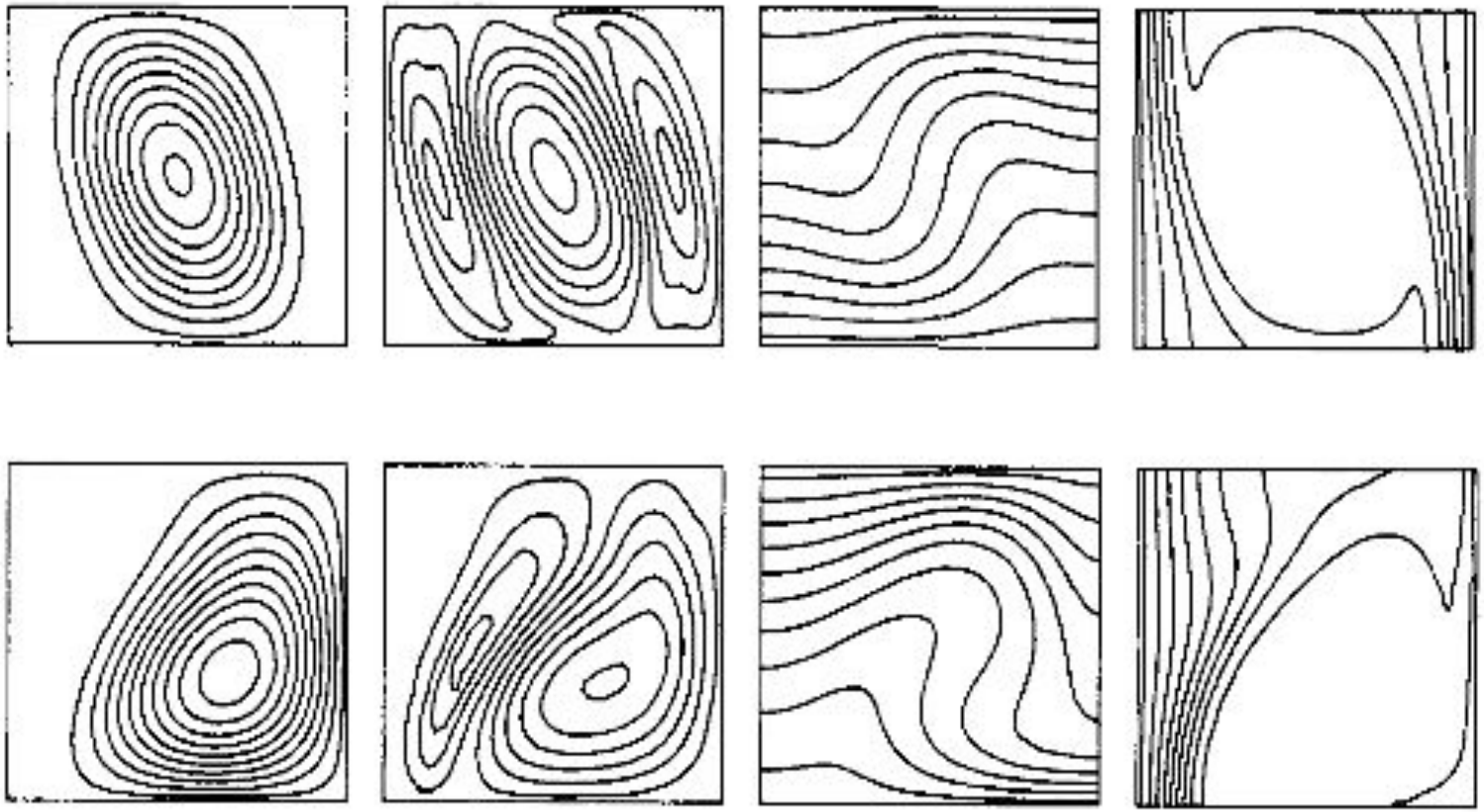
Numerics (1963-2002)

- 2D kinematic flux expulsion
- Idealized 2D nonlinear convection
- Cartesian and cylindrical models
- Dynamical flux concentration
- Bifurcation structure



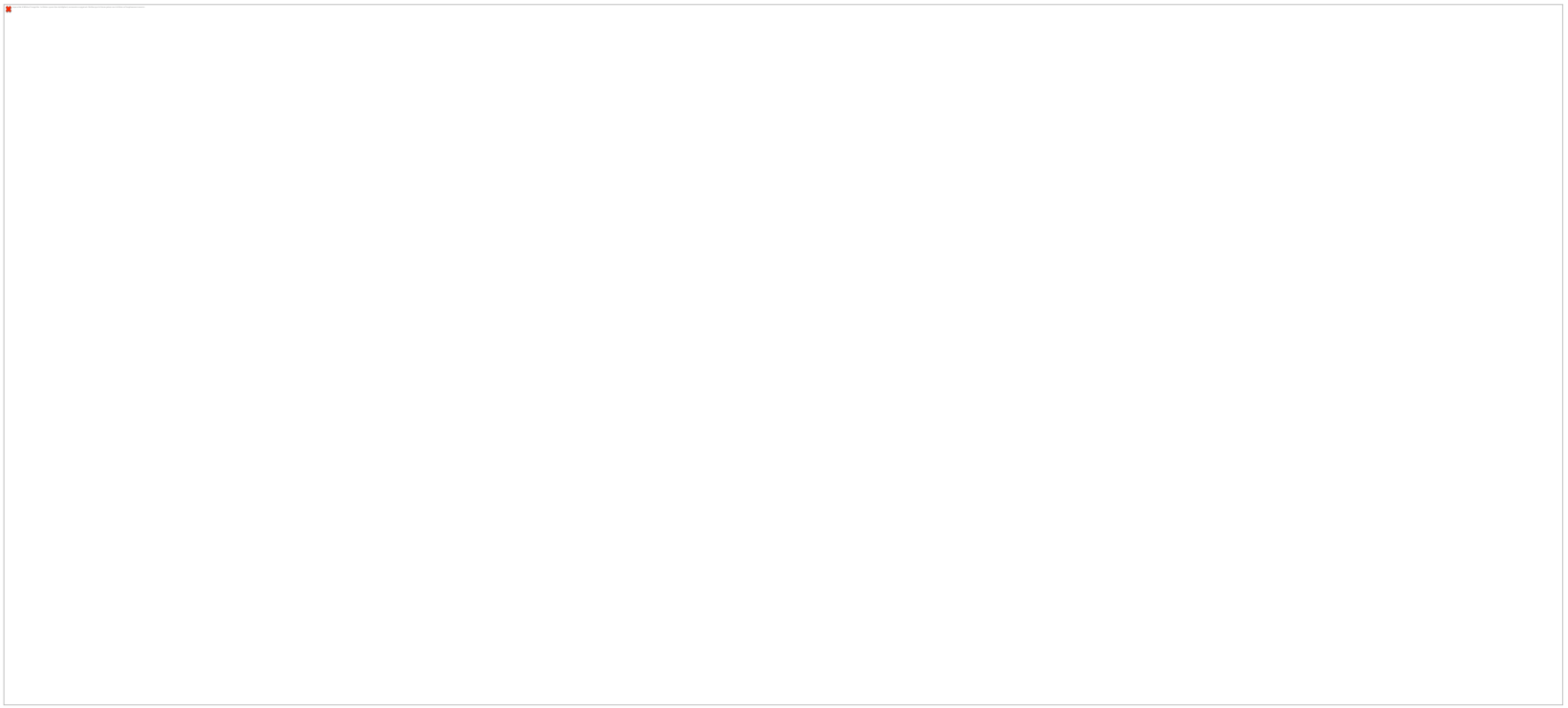
(Galloway & W. 1981)

Separation of motion from the field

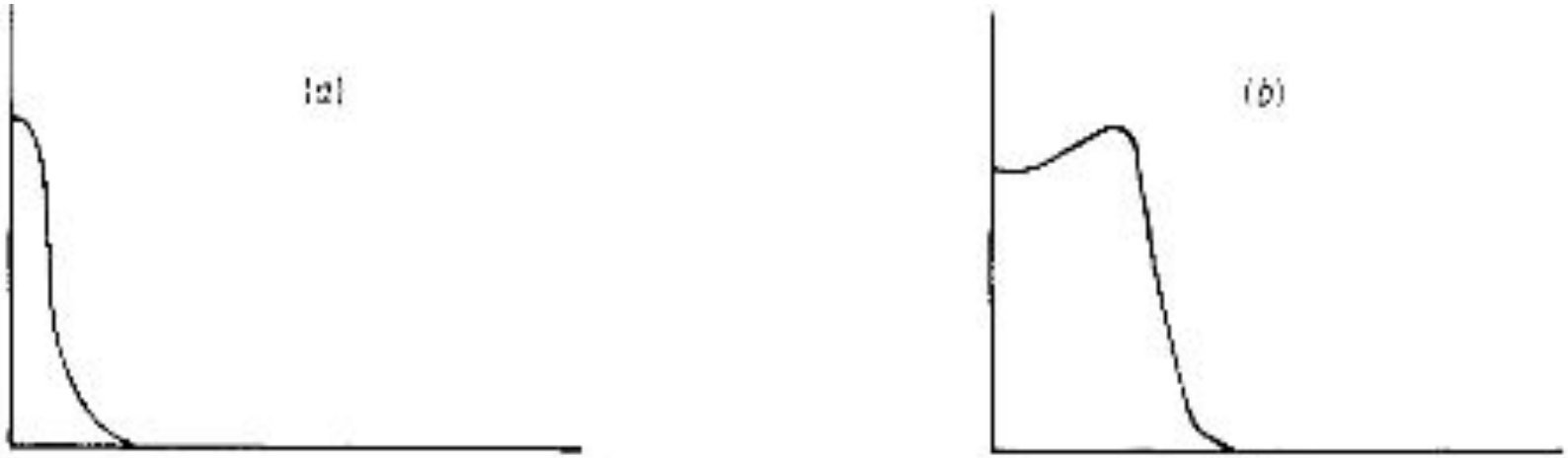


(W. 1981)

Cylindrical geometry



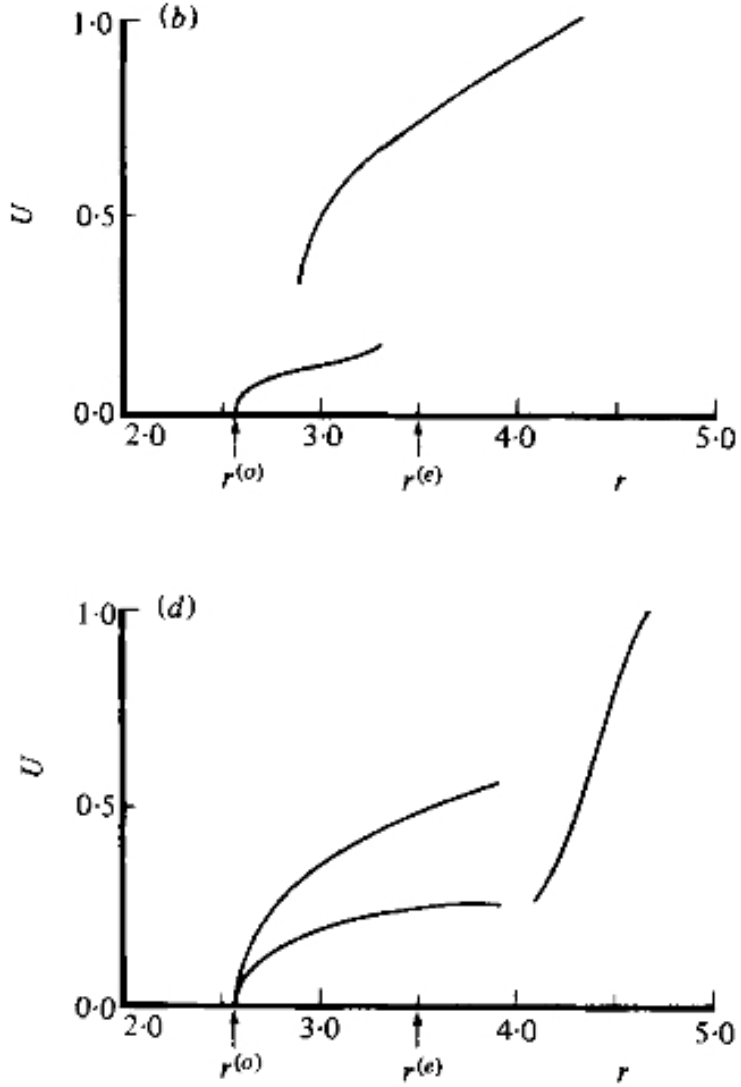
(Galloway, Proctor & W. 1978)



$$B^2_{\max} \sim \frac{B^2_{\rm eq}}{\ln R_m}\left(\frac{\nu}{\eta}\right) .$$

(Galloway & Proctor 1979)

Subcritical and supercritical convection

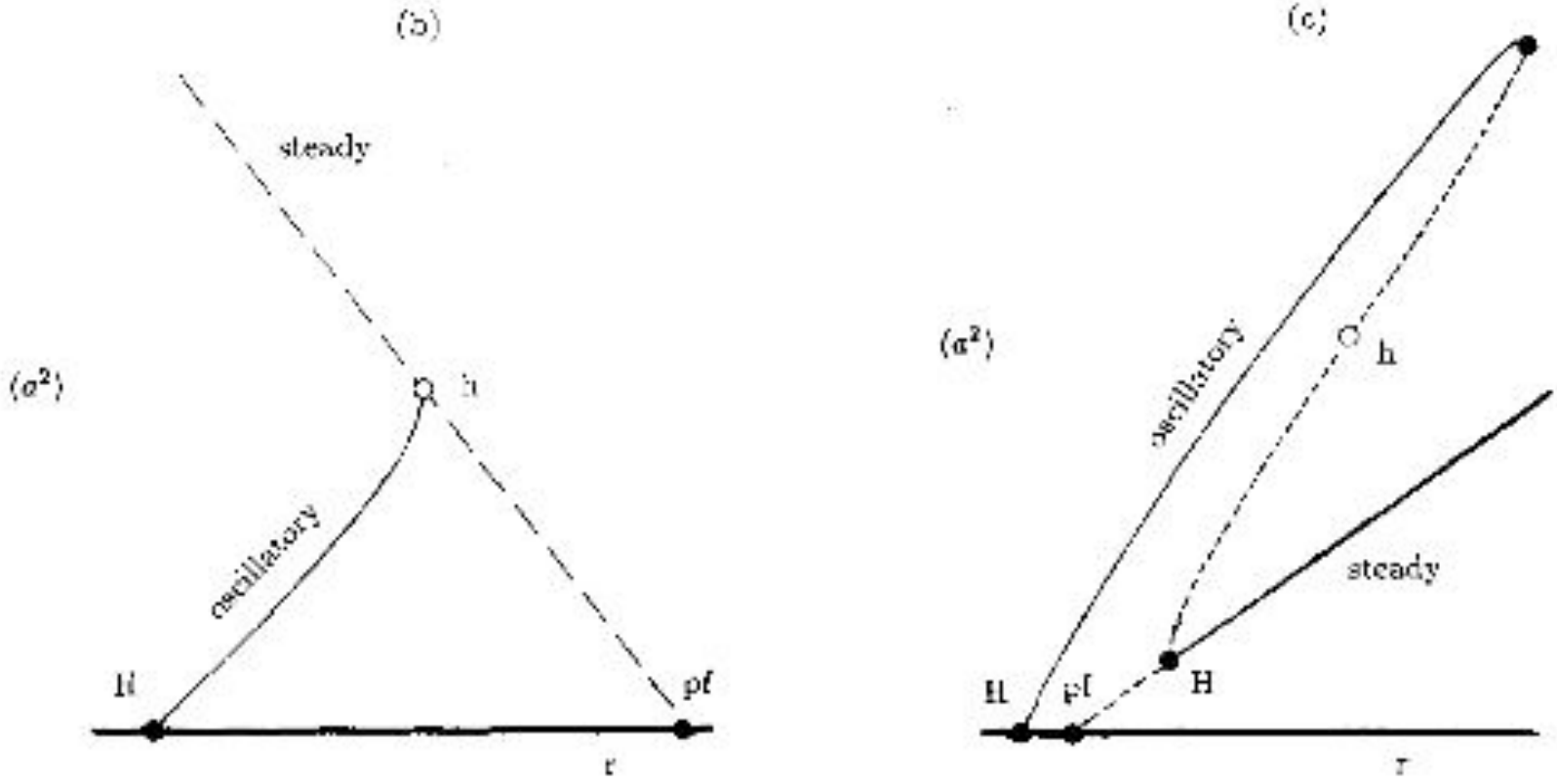


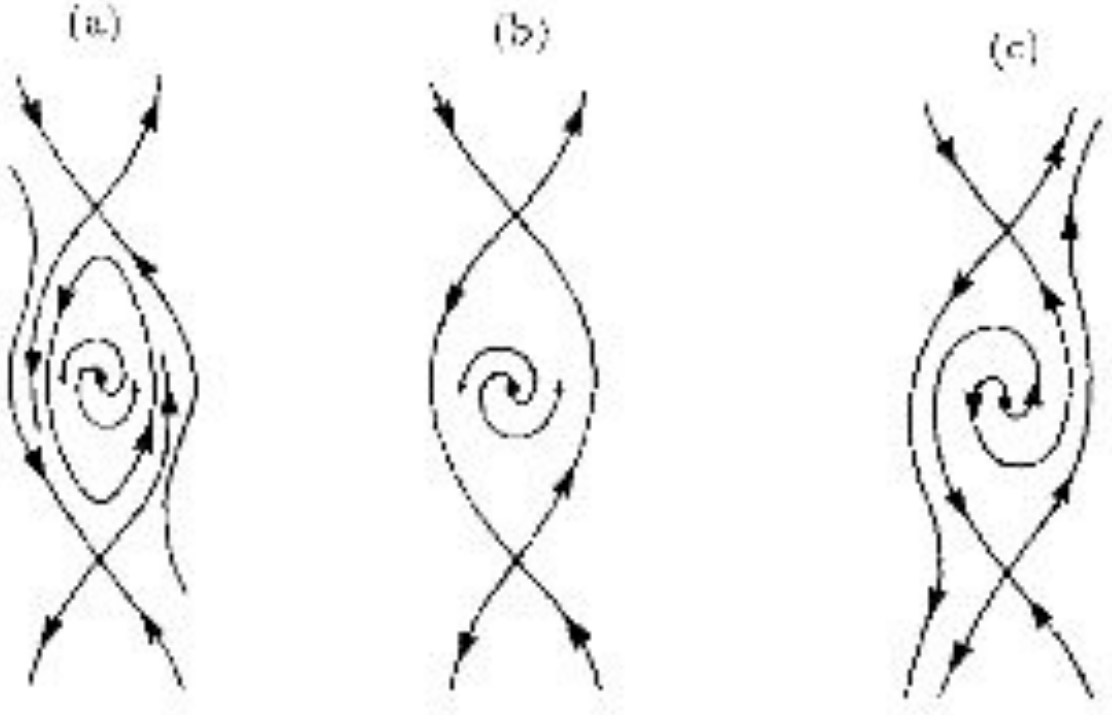
Nonlinear Dynamics (1978-2000)

- Takens-Bogdanov bifurcation
- Fifth-order model
- Shilnikov effect and chaos
- Third-order models and chaos
- Chaos in thermosolutal convection
- Symmetry-breaking

The normal form equation can be written as

$$a'' - (\beta + Ba^2)a' - (\alpha + Aa^2)a = 0\,.$$



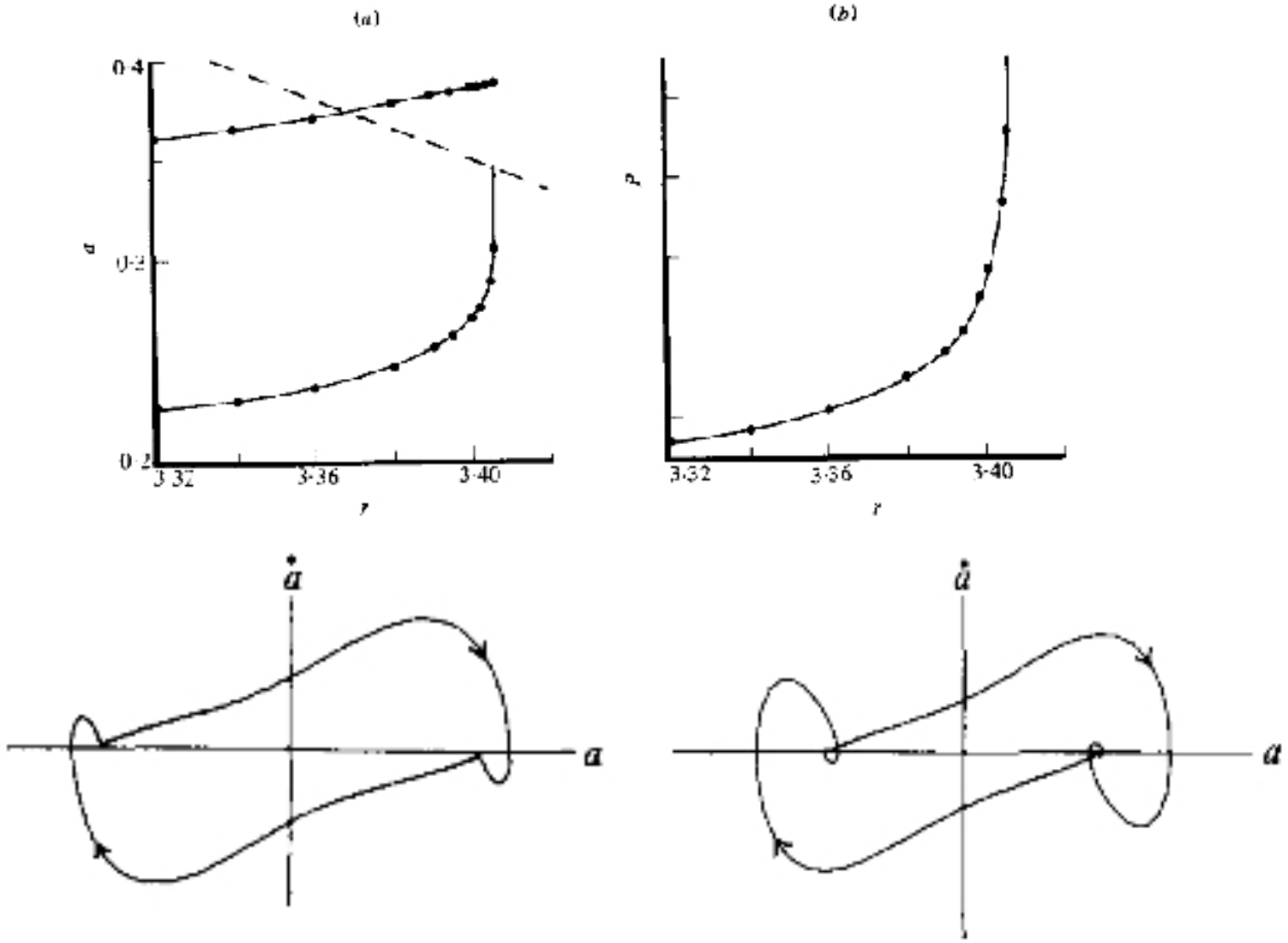


(Knobloch & Proctor 1981)

(Knobloch & Proctor 1981)

Fifth-order model system

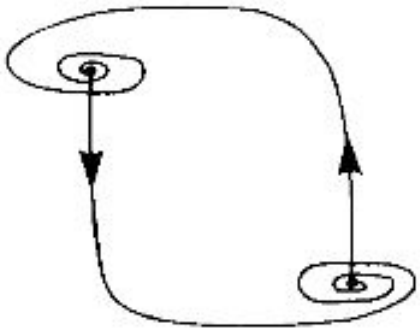
$$
\begin{aligned}
\dot{a} &= \sigma[-a + rb - \zeta q d\{1 - (3 - \varpi)e\}] \,, \\
\dot{b} &= -b + a(1 - c) \,, \\
\dot{c} &= \varpi(-c + ab) \,, \\
\dot{d} &= [-\zeta d + a(1 - e)] \,, \\
\dot{e} &= -(4 - \varpi)\zeta e + \varpi a d \,.
\end{aligned}
$$

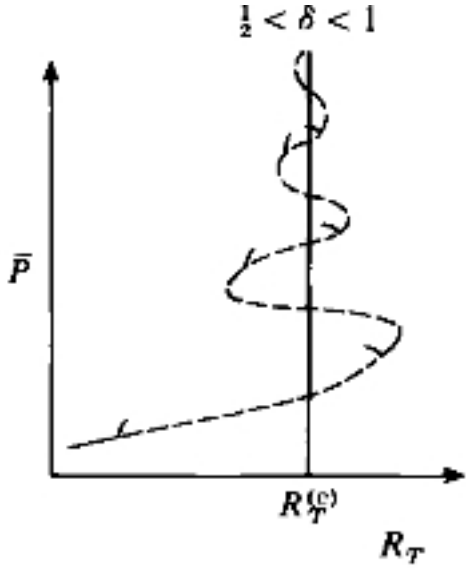


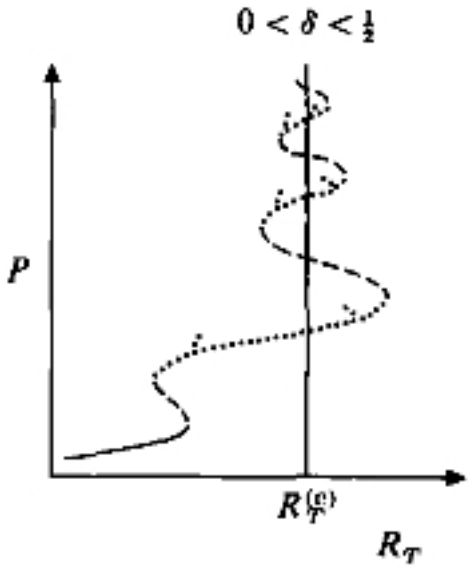
(Knobloch & W. 1983)

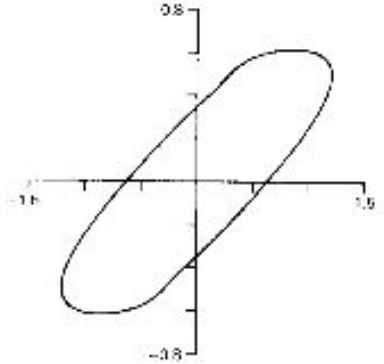
Shilnikov mechanism

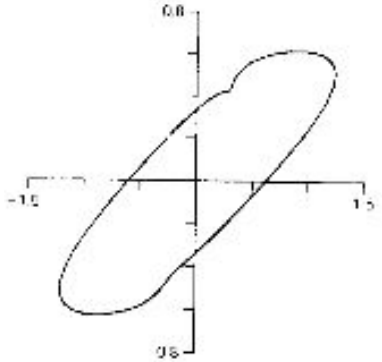
There are three eigenvalues at the saddle-focus:
a and $b \pm i\omega$ such that $a > 0 > b$.
Define the ratio $\delta = |b|/a$.

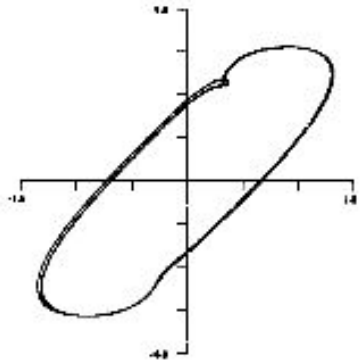


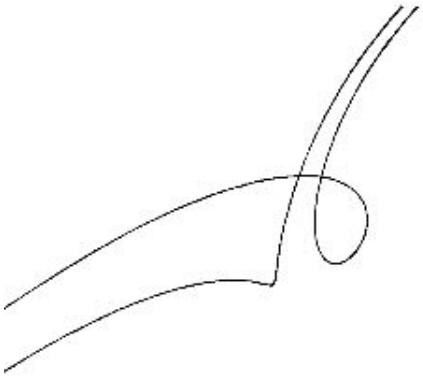


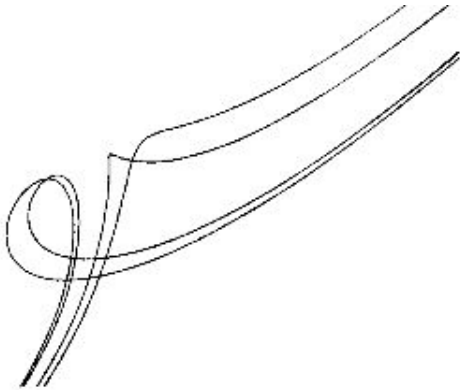


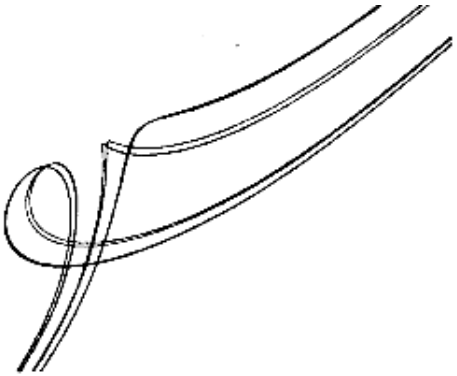


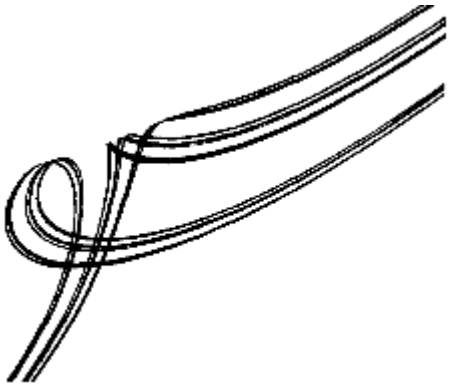


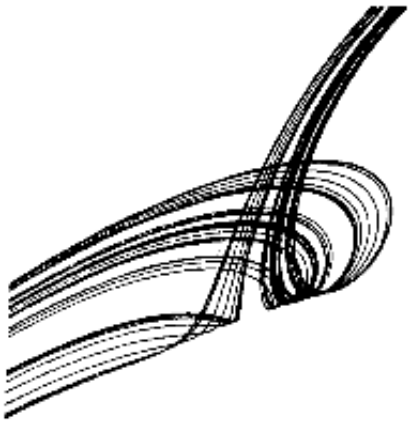






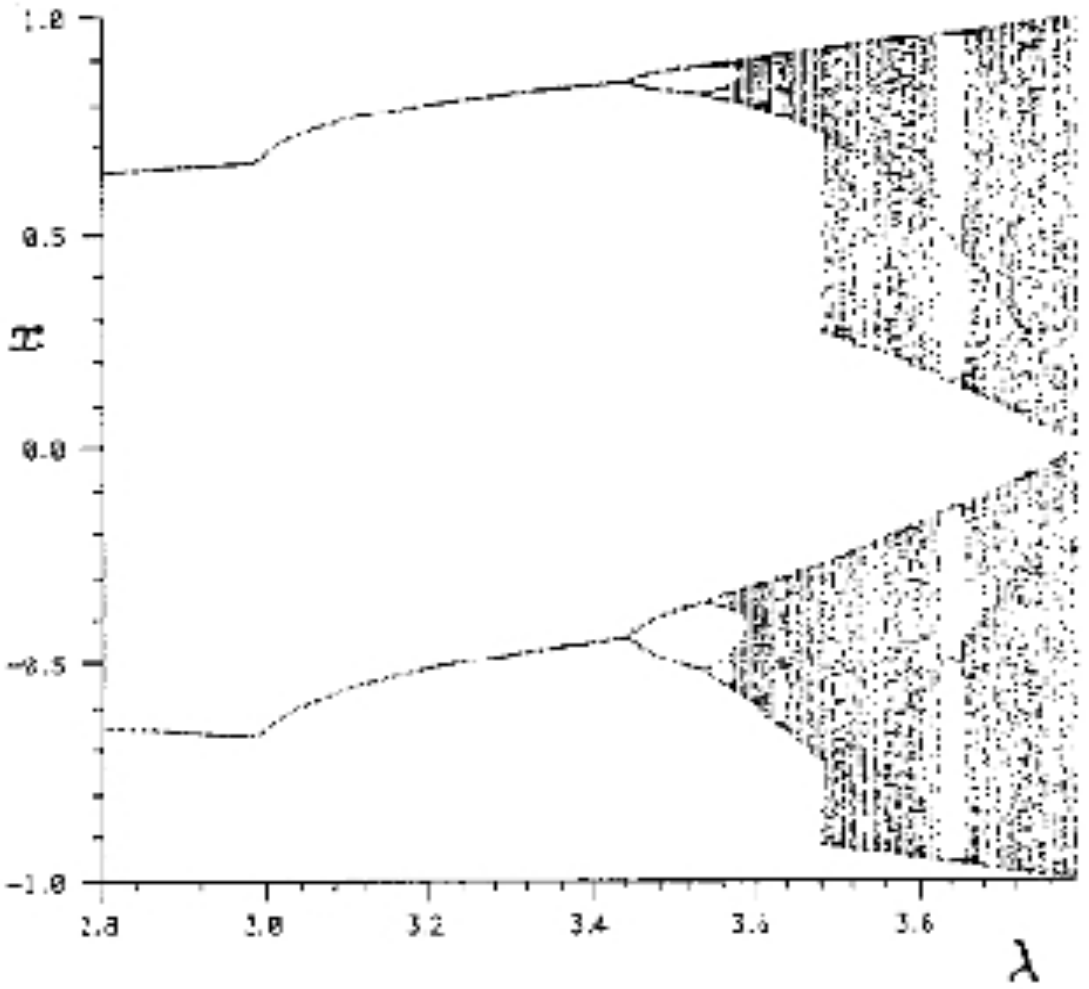






Logistic map

$$x_{n+1} = -\lambda x_n (1 - x_n) \, .$$



Impossible d'afficher l'image liée. Le fichier a peut-être été déplacé, renommé ou supprimé. Vérifiez que la liaison pointe vers le fichier et l'emplacement corrects.

(Knobloch & W. 1983)

(A. J. Bernoff)

Nonlinear Dynamics (1978-2000)

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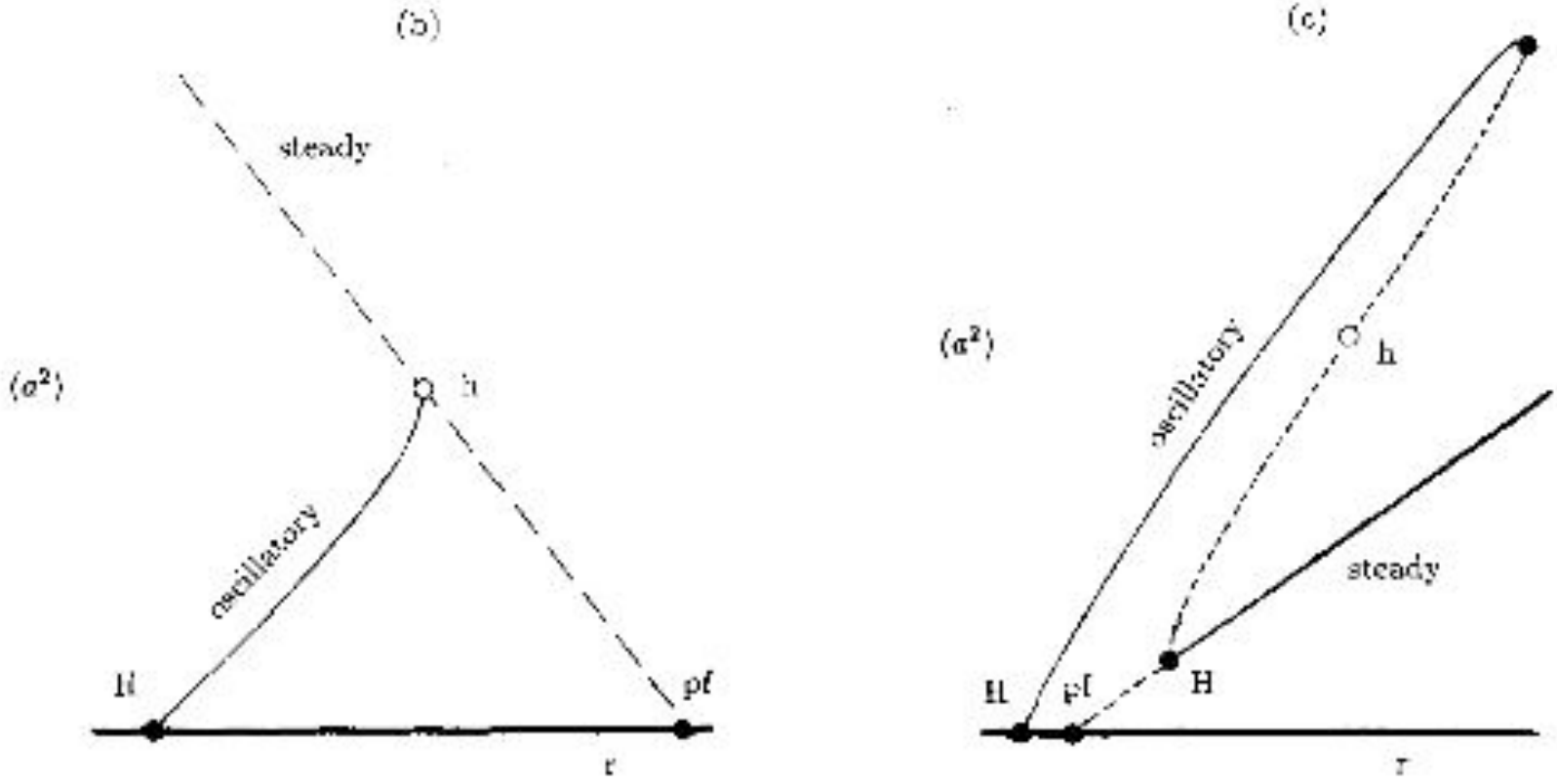
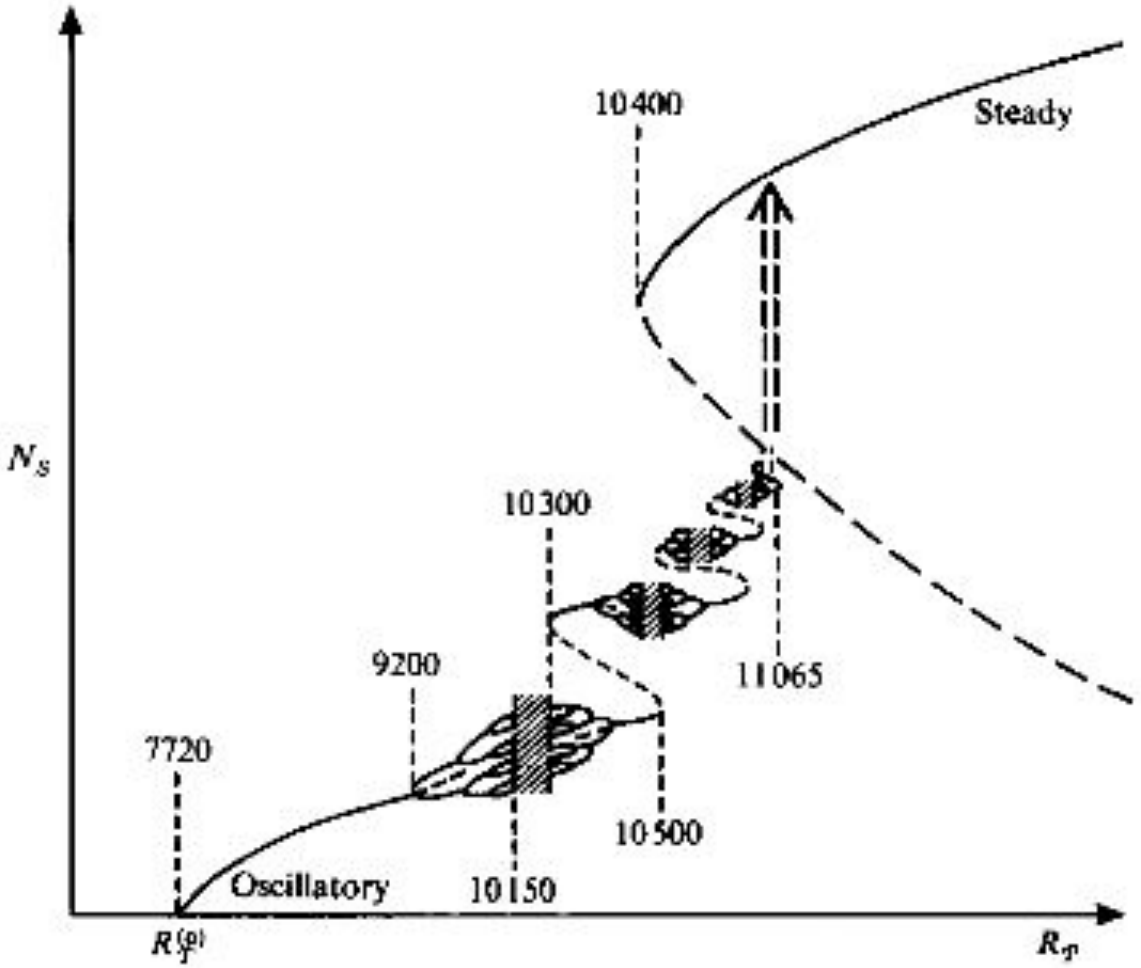
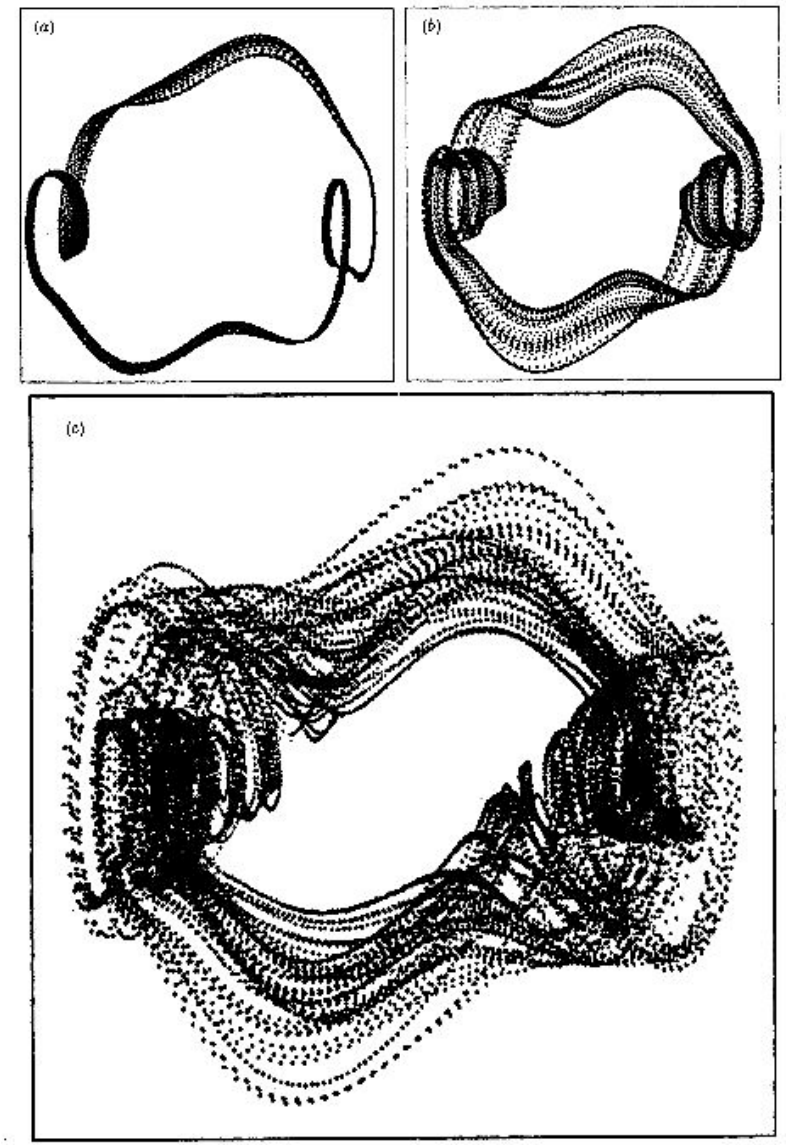


Thermosolutal convection



(Knobloch, Moore, Toomre & W. 1986)



(W. 1987)

Compressible magnetoconvection

- 2D (1985-1994)

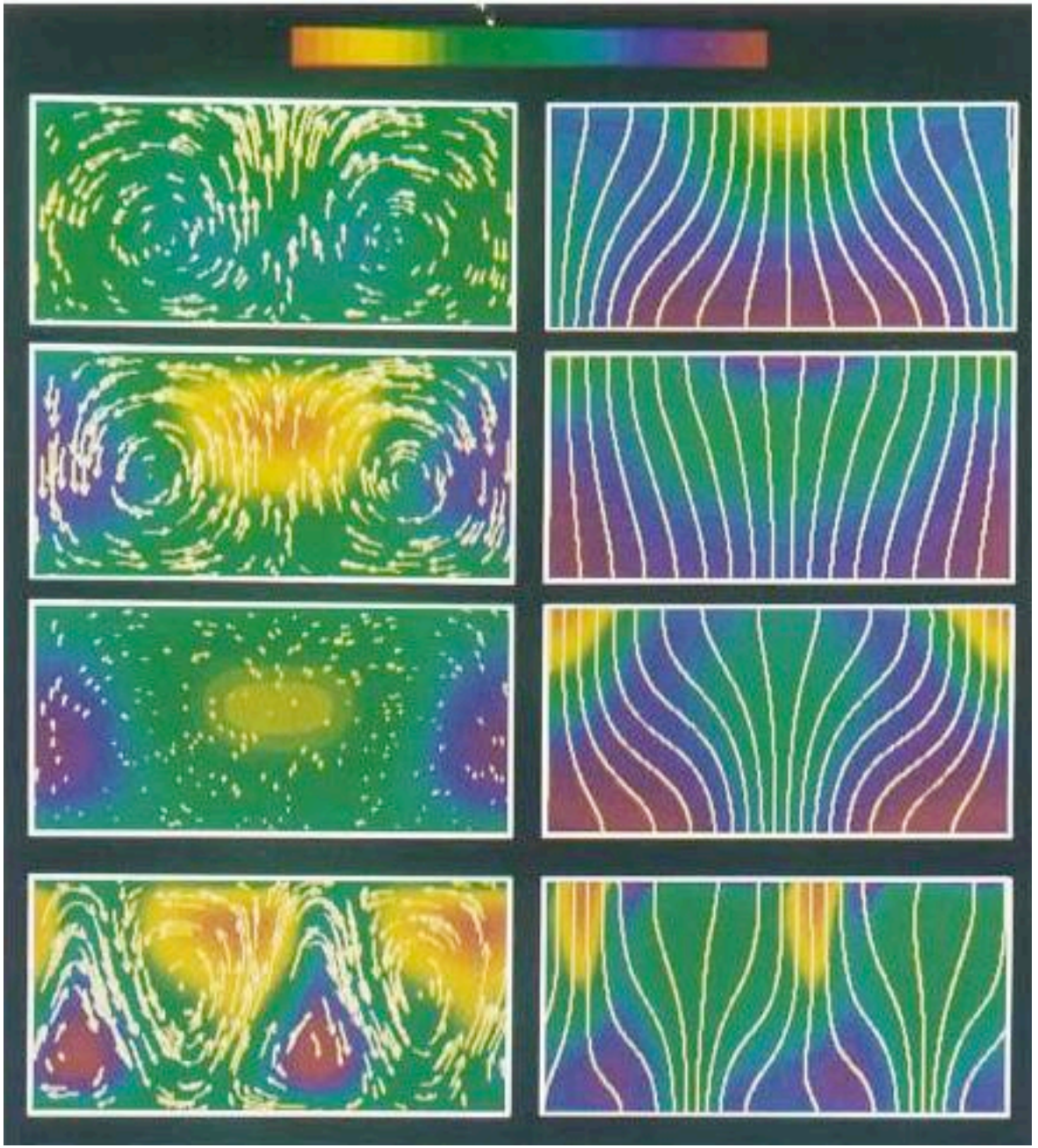
  Shallow layers: SW, TW and PW

  Deep layers: spatially modulated oscillations
- 3D (1990-2008)

  Planforms and pattern selection

  Increasing *R* for fixed *Q*: flux separation

  and intense small-scale fields



(Hurlburt, Proctor, W. & Brownjohn 1989)

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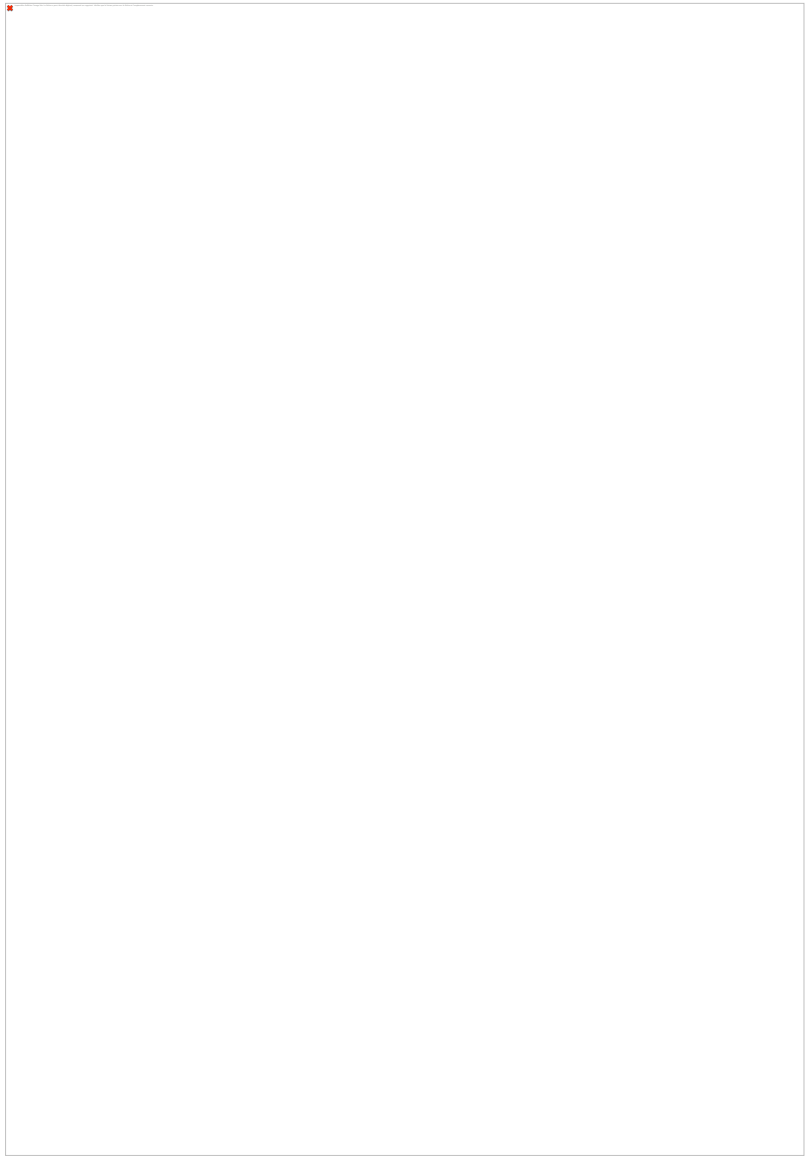
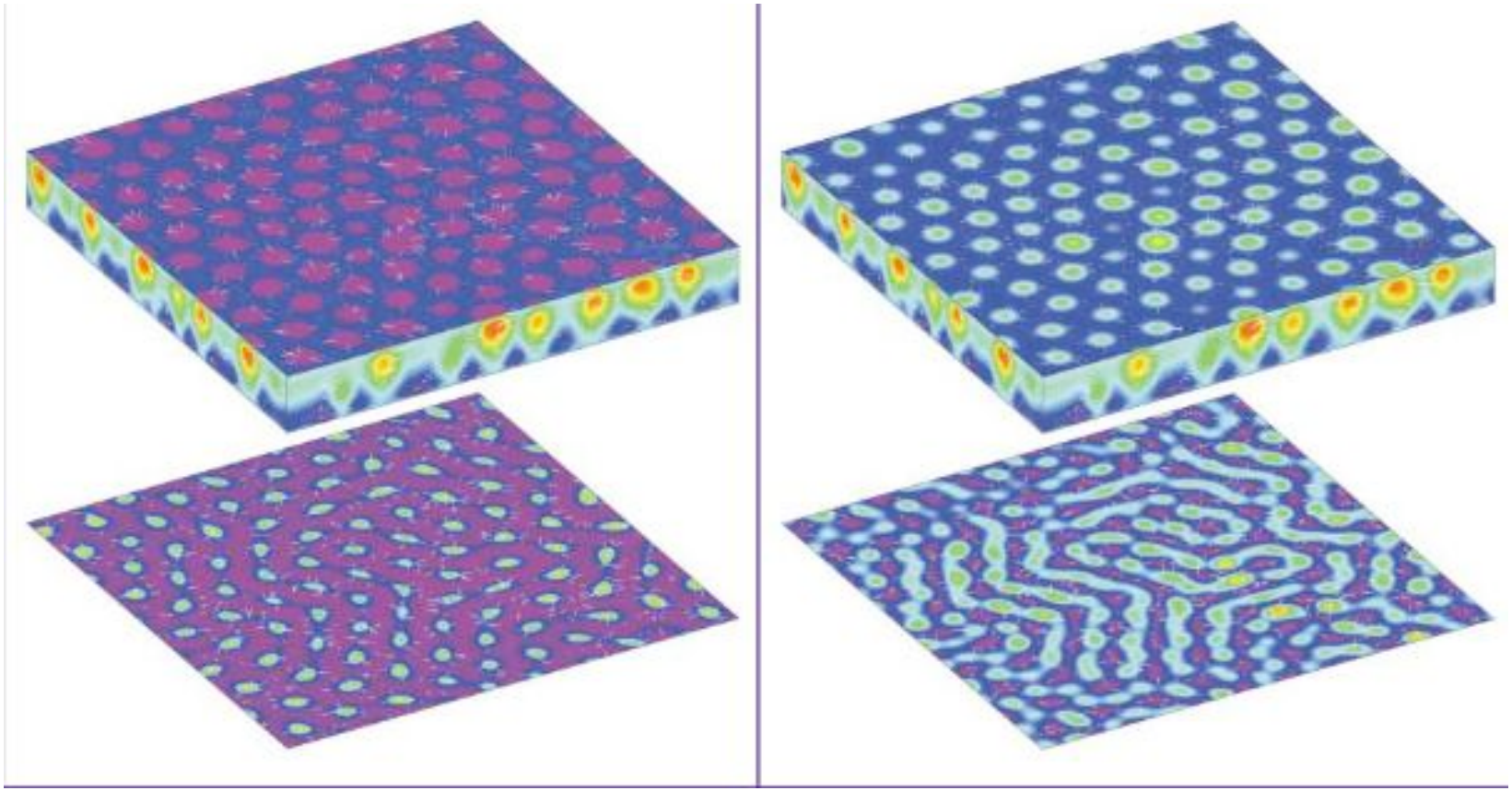
(Proctor & W. 1993)



(Hurlburt, Matthews & Rucklidge 2000)

3D compressible



(W. 2001)

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(W. 2001)

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(W. 2001)

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(W. 2001)

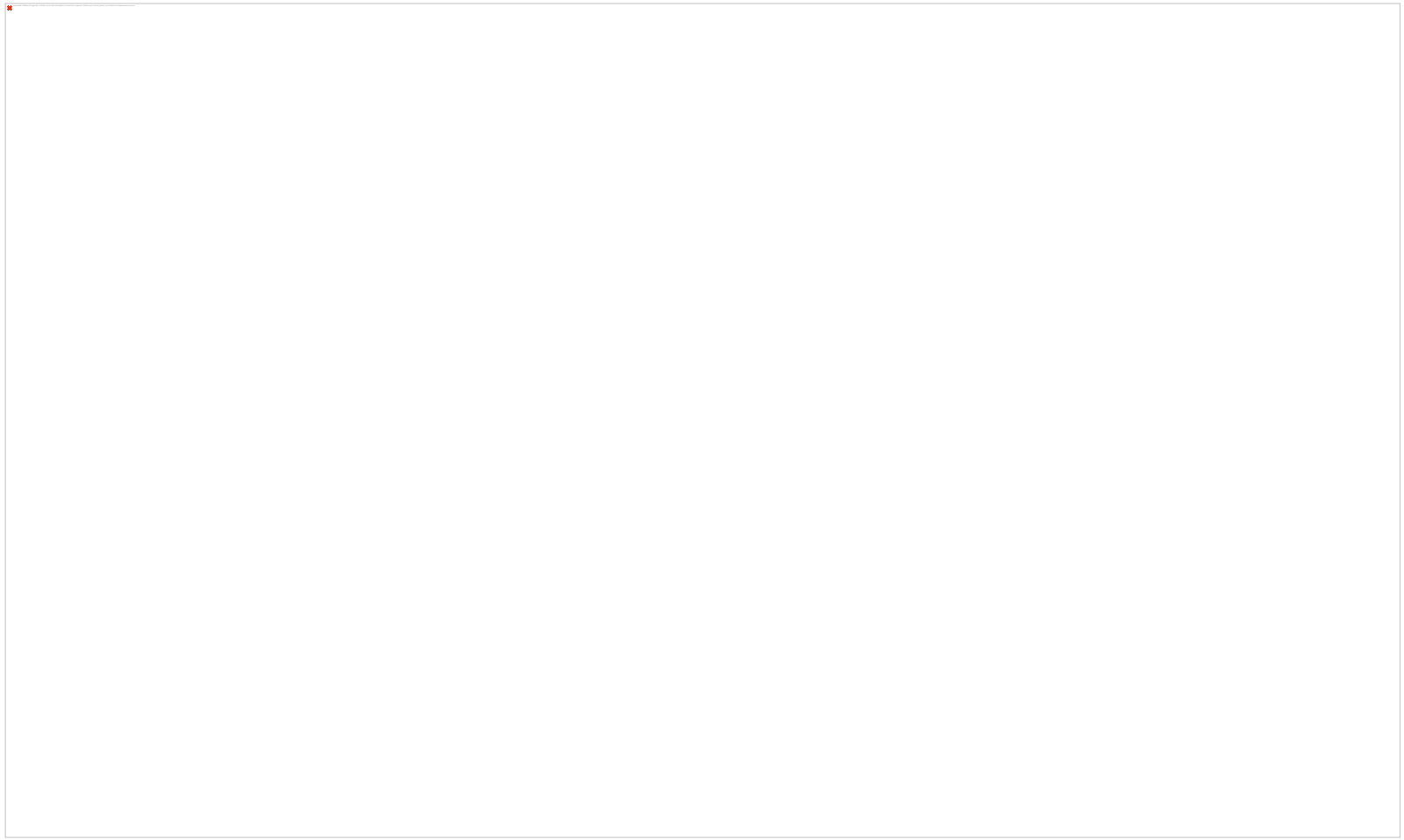
(Bushby, Houghton, Proctor & W. 2008)

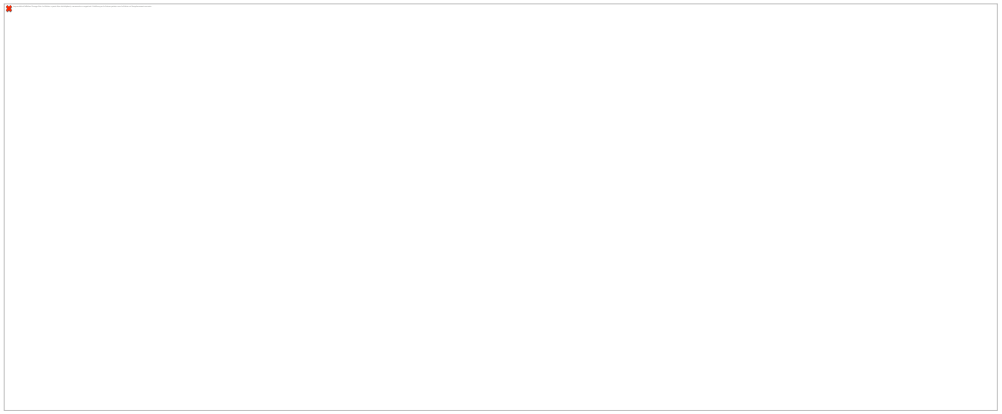
Small-scale dynamos

- 3D Boussinesq (1999-2003)
  Transition from magnetoconvection to small-scale dynamo action
- Subcritical Hopf bifurcation: isolated states and convectons

{Cattaneo, Emonet & W. 2003)

(Cattaneo, Emonet & W. 2003)





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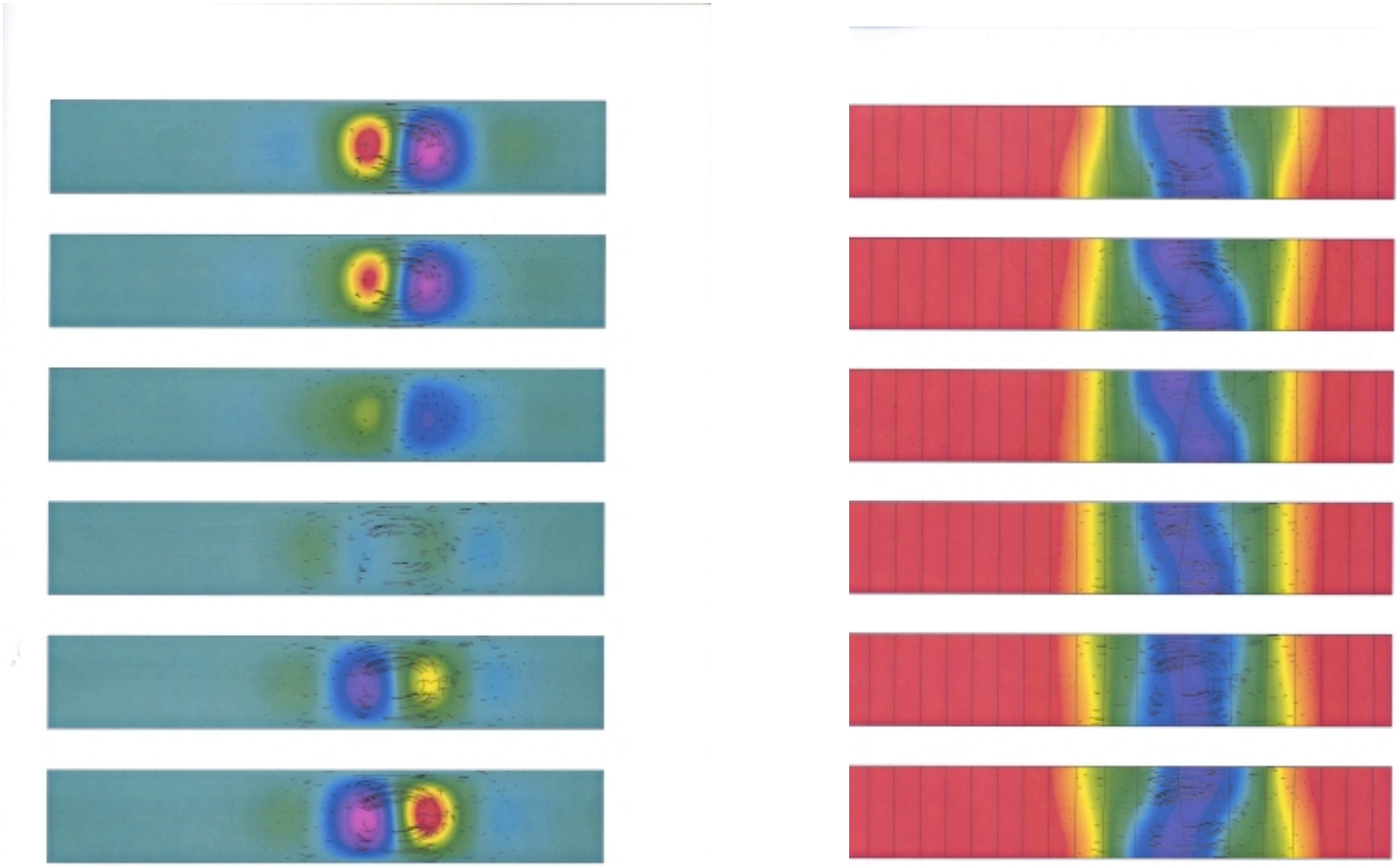
3D convecton

(Blanchflower 1999)



(Blanchflower 1999)

Back to sunspots

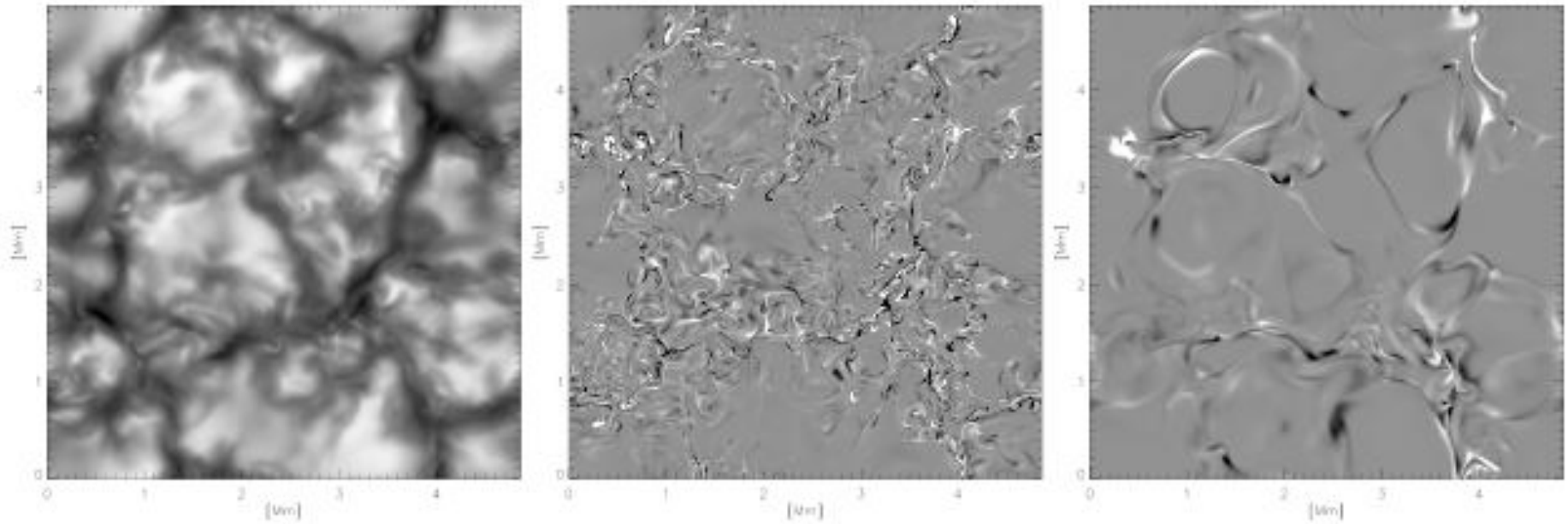
- *Realistic* modelling: multiple constituents, ionization and radiative transfer
- Photospheric magnetoconvection
- Sunspot models

  Umbral dots; inner penumbra;

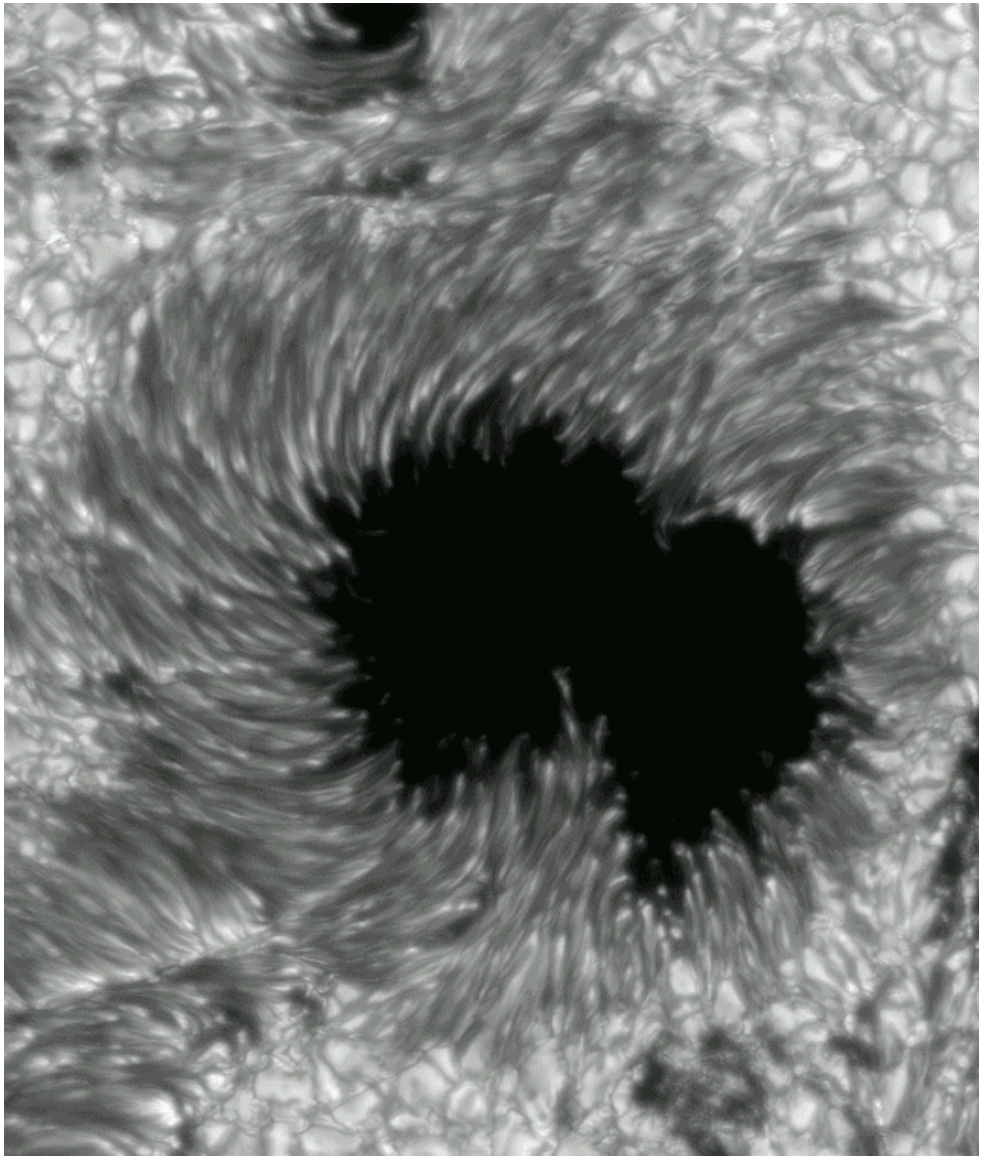
  outer penumbra and flux pumping

(Berger et al. 2004)

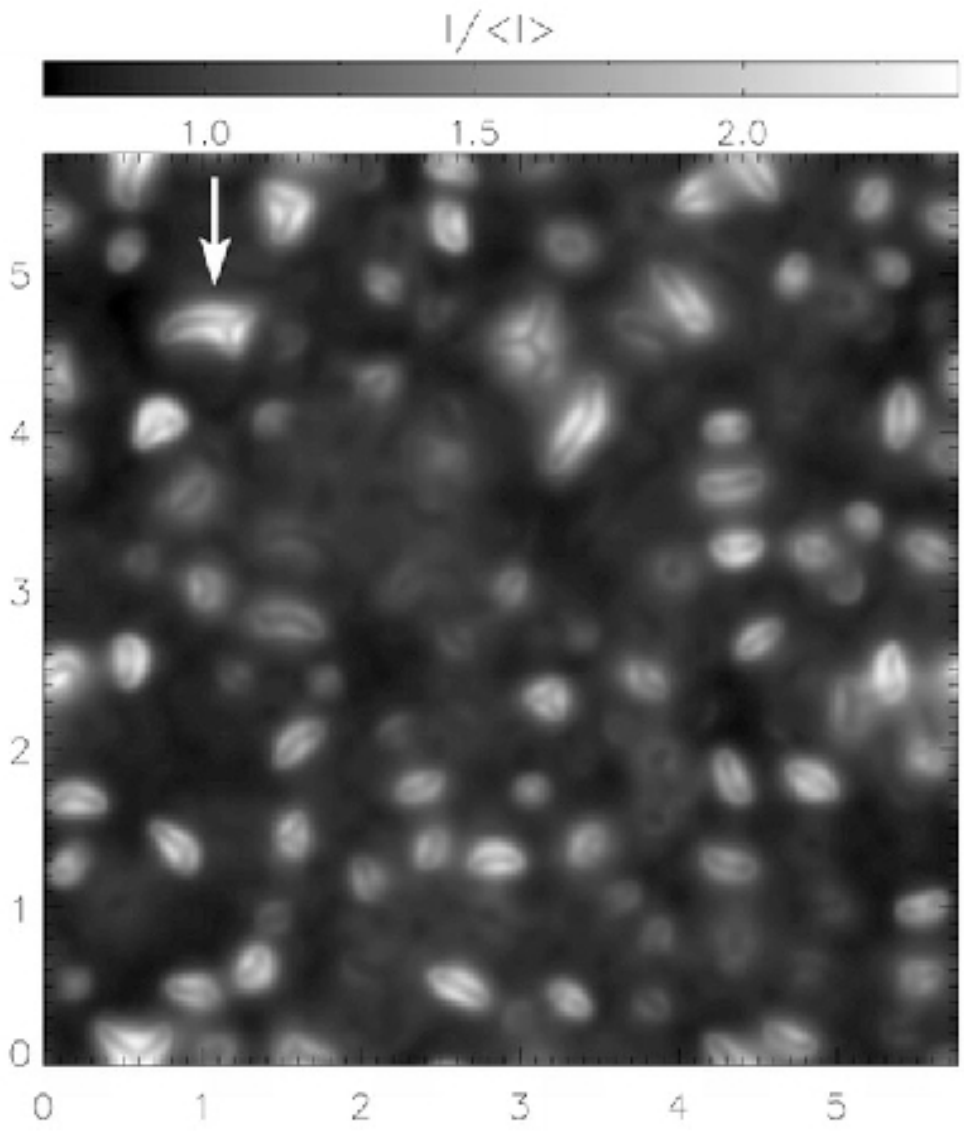
(Voegler et al. 2005)



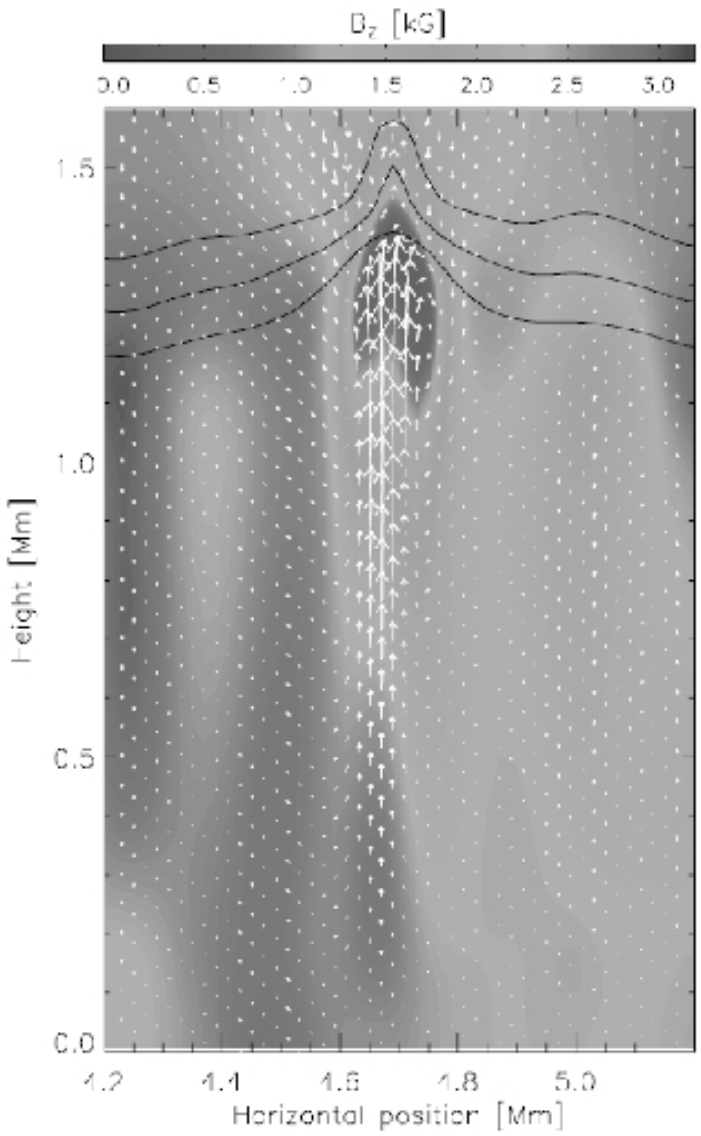
(Voegler & Schuessler 2007)



(G. Scharmer)



(Schuessler & Voegler 2006)



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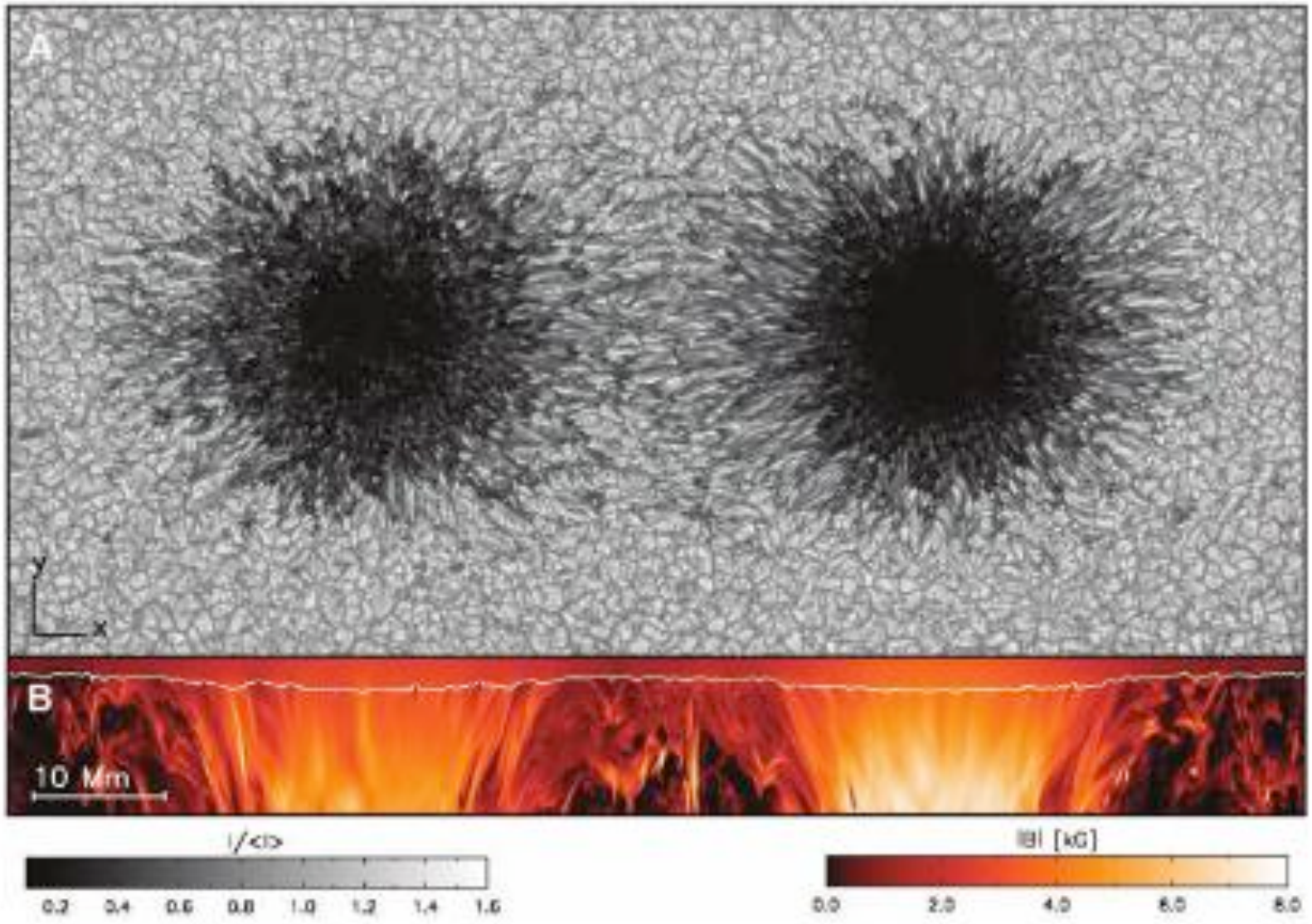
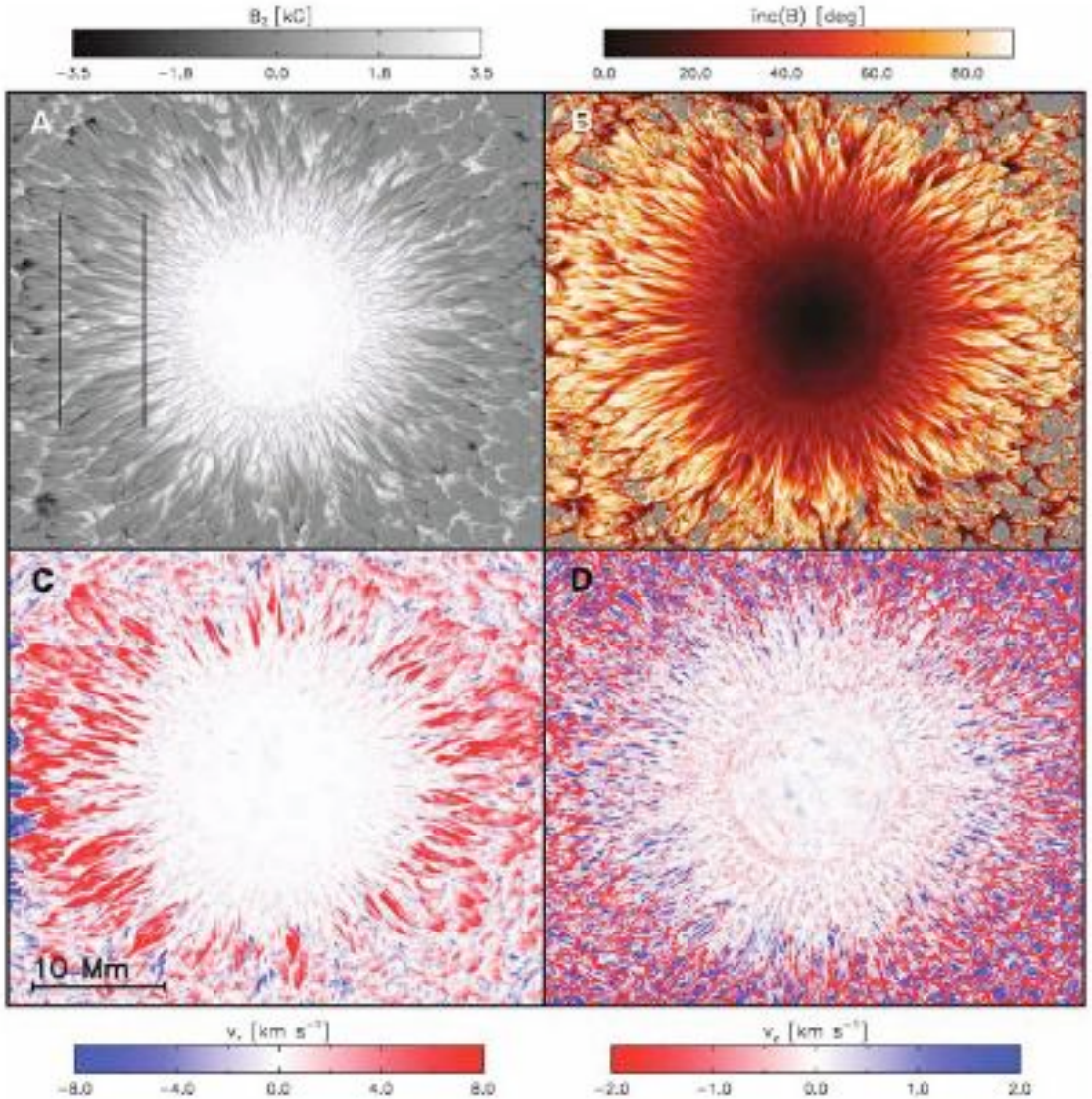


(Rempel, Schuessler, Cameron & Knoelker 2009)



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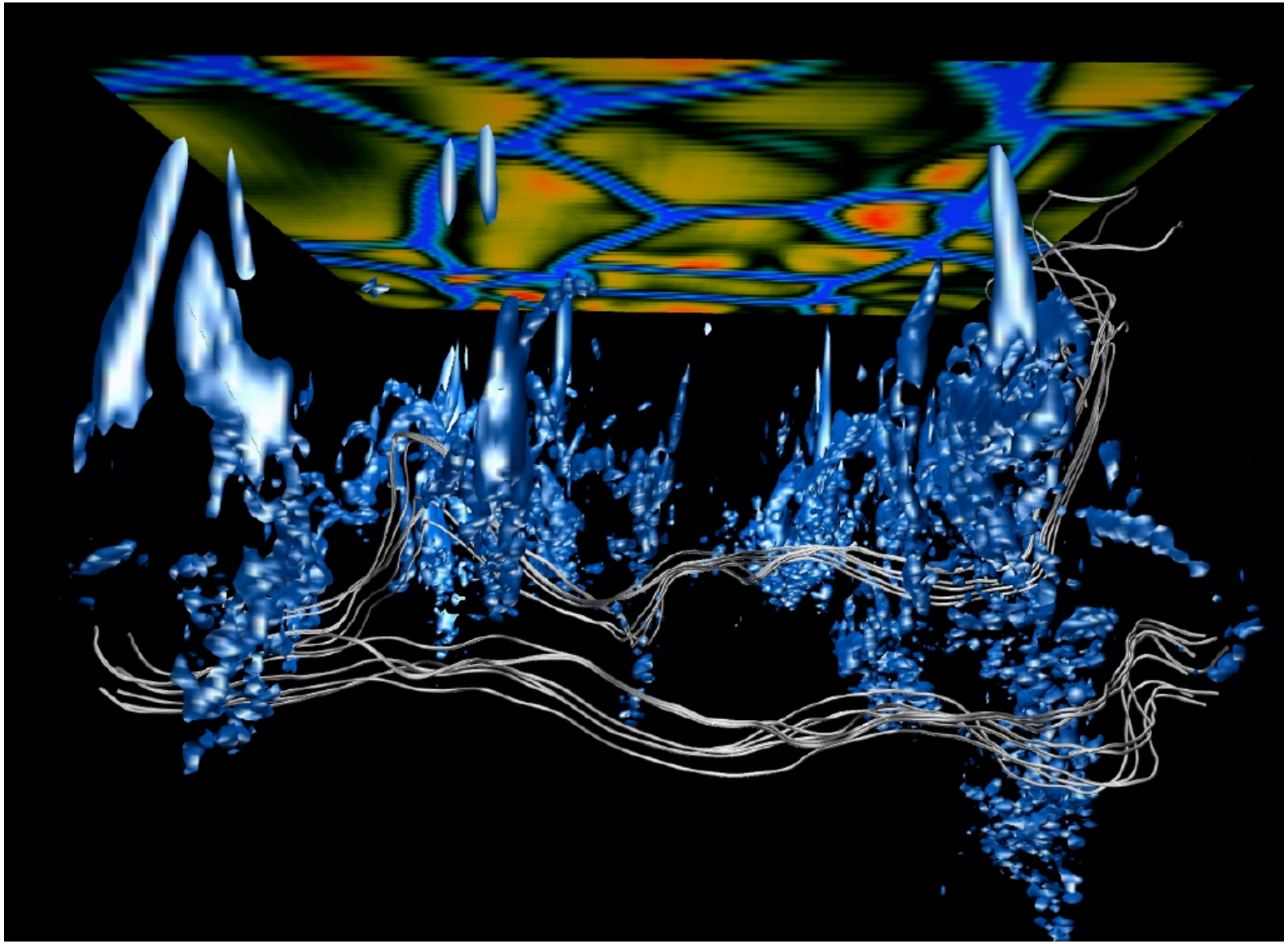

(Thomas, W., Tobias & Brummell 2002}

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