On the nonlinear state of the fluctuation dynamo

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Outline

- Dynamo in a non-helical flow
- Description of magnetic structures
  - Fourier spectrum
  - Minkowski functionals
- Nonlinear state of the fluctuation dynamo
  - Morphology of magnetic structures
  - Volume filling factor of magnetic field

Dynamo in a non-helical flow

Weakly compressible isothermal flow,
electrically conducting fluid,
random driving force:

$$\frac{D\ln\rho}{Dt} = -\nabla\cdot\vec{u}\,,$$

$$\rho\frac{D\vec{u}}{Dt} = -c_{\rm s}^2\rho\nabla\ln\rho + \vec{J}\times\vec{B} + \mu(\nabla^2\vec{u} + \tfrac{1}{3}\nabla\nabla\cdot\vec{u}) + \rho\vec{f}\,,$$

$$\frac{\partial\vec{A}}{\partial t} = \vec{u}\times\vec{B} - \eta\mu_0\vec{J}\,.$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{u}\cdot\nabla\,, \quad \vec{B} = \nabla\times\vec{A}\,, \quad \vec{J} = \nabla\times\vec{B}/\mu_0\,.$$

- Weak compressibility, $|f| \le 0.09$, $\nu = \mu/\rho = \text{const.}$

- $\vec{f}$: δ-correlated in time, single scale = ⅓(box size).

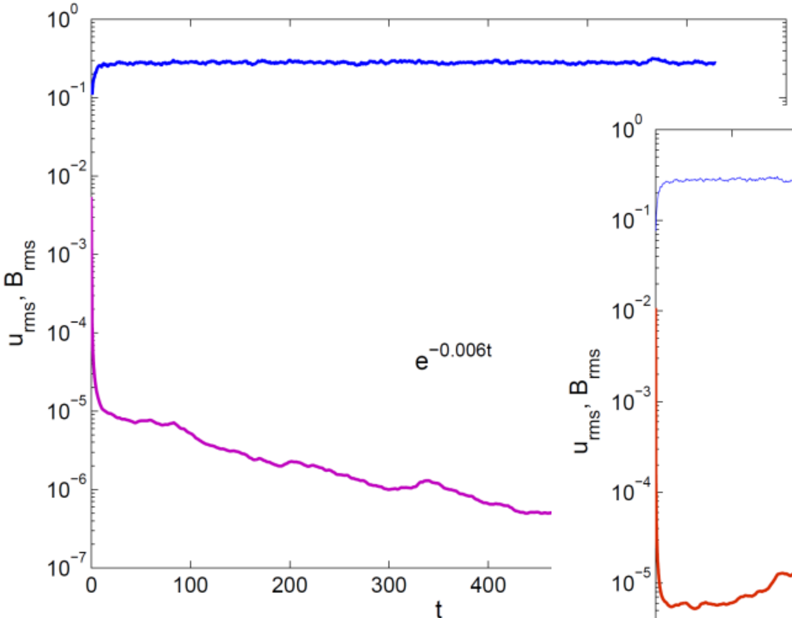
- Initial conditions: $\ln \rho = 0$, $\vec{u} = 0$,
  $\vec{A}$: smoothed Gaussian random field, δ-correlated in space.

- Periodic box, size L, 256^3 grid, the Pencil Code.
  Units: $c_s = 2\pi/L = \langle\rho\rangle = \mu_0 = 1$.

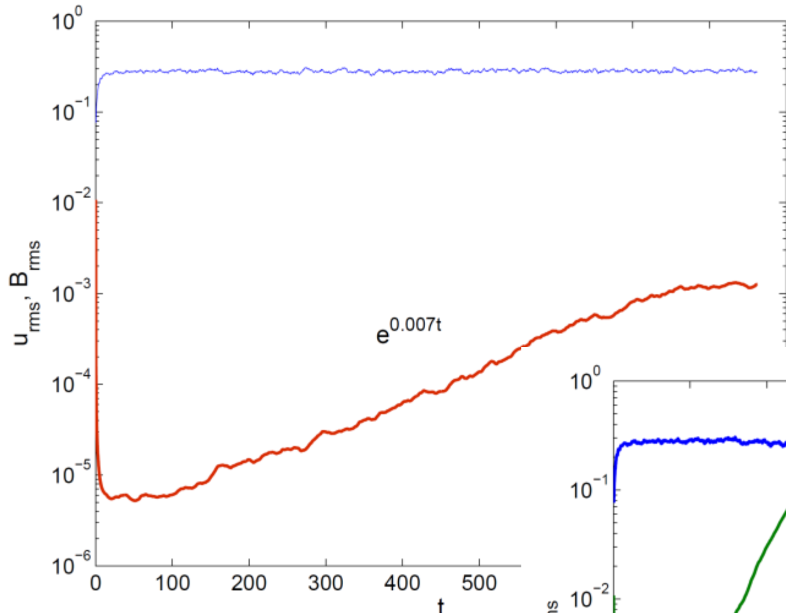
- $\mathrm{Re} \approx 20$ fixed, a range of R_m values.

The fluctuation dynamo: u_{rms} & B_{rms} versus time

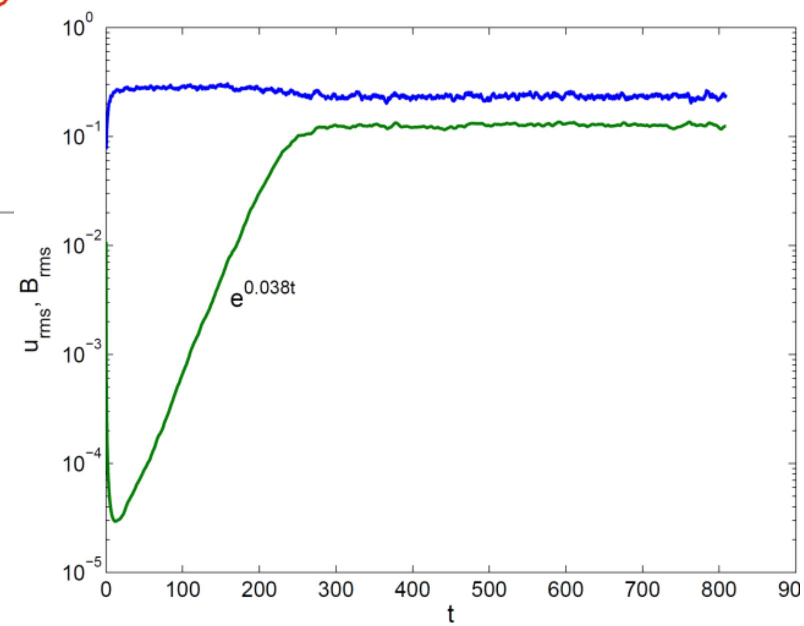
$R_m = 25$



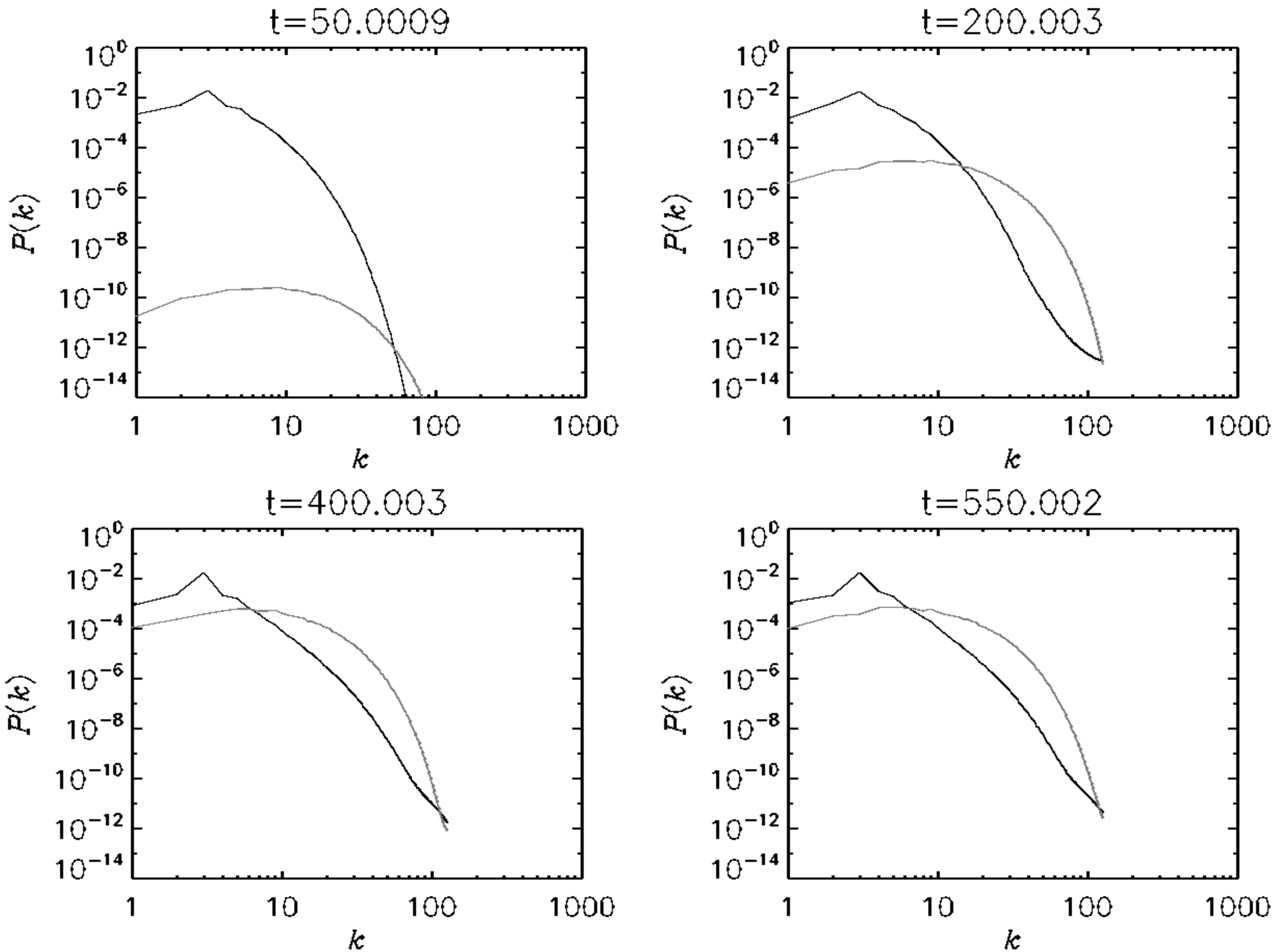
$R_m = 36$



$R_m = 133$

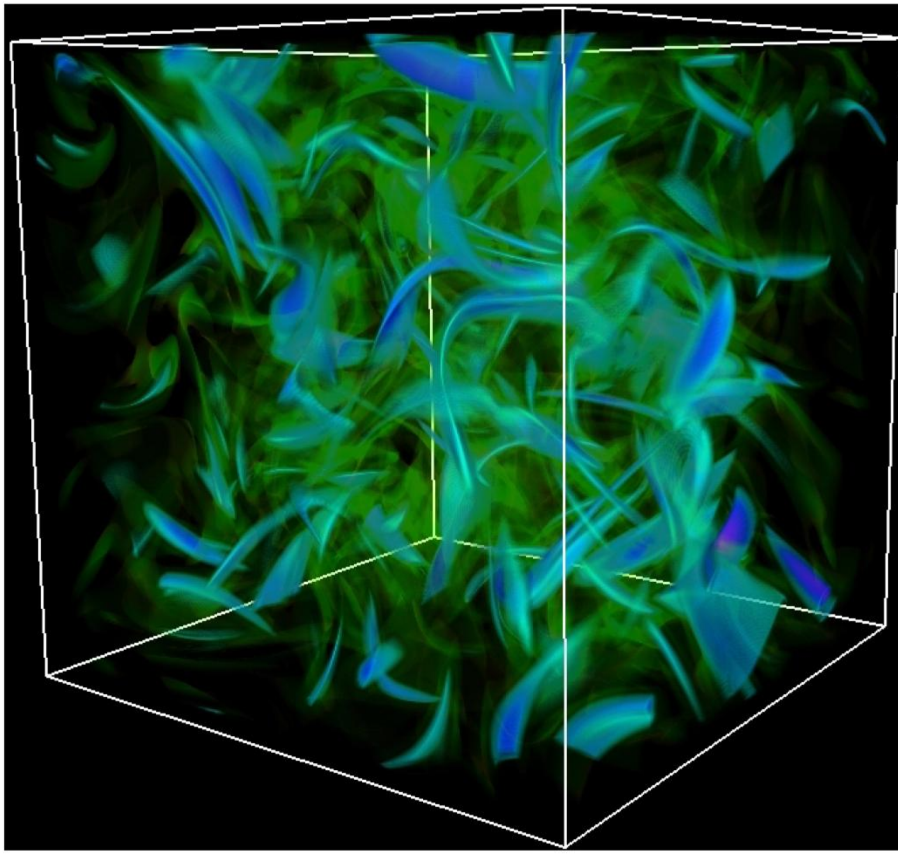


Evolving kinetic & magnetic energy spectra, $R_{\mathrm{m}} \approx 133$

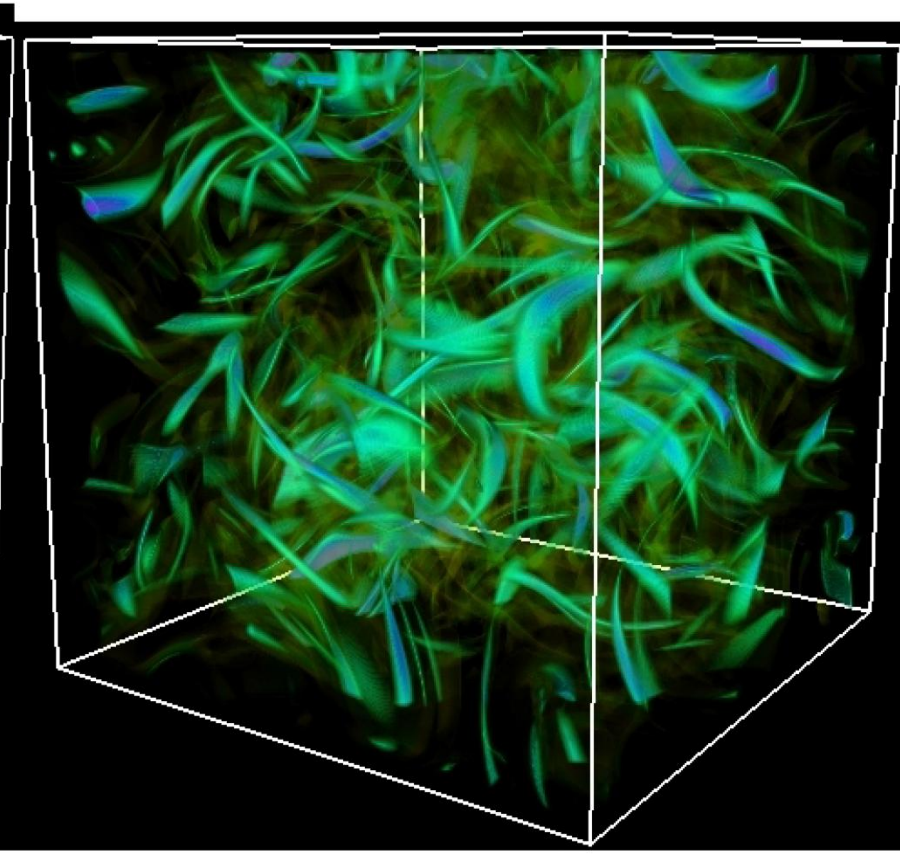


Isosurfaces $B = \text{const}$, kinematic dynamo: intermittent filaments/ribbons

$R_\mathrm{m} \approx 133$



$R_\mathrm{m} \approx 196$

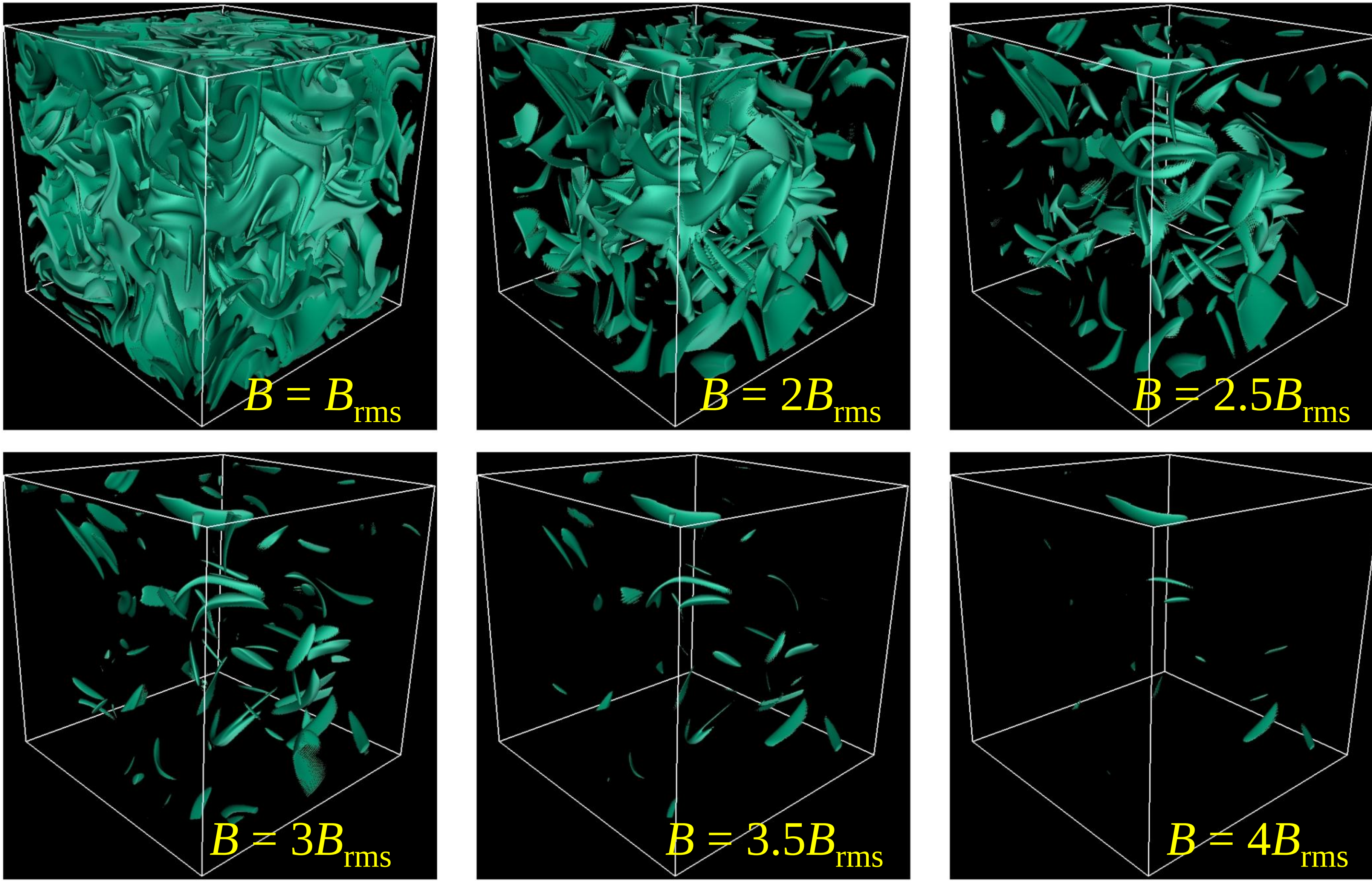


Green: $B = 2.5\, B_\mathrm{rms}$ Blue: $B = 3\, B_\mathrm{rms}$

Power spectra or second-order moments miss the most important features of magnetic structures produced by the dynamo,

and hence provide little evidence for the physical nature of the dynamo action and its saturation.

Need for a quantitative morphological analysis.



Which of these is the right/most informative picture?

Minkowski functionals

Morphology of structures in 3D is completely characterised by four Minkowski functionals (Hadwiger's theorem, 1957).

$$V_0 = \int\int\int dV$$ Volume

$$V_1 = \frac{1}{6}\int\int dS$$ Surface area

$$V_2 = \frac{1}{6\pi}\int\int(\kappa_1 + \kappa_2)dS$$ Integral mean curvature

$$V_3 = \frac{1}{4\pi}\int\int \cdot_1 \cdot_2 \, dS$$ Euler characteristic

$\kappa_1, \ \kappa_2 =$ principal curvatures

Minkowski functionals (per unit volume) on a grid (Adler 1981)

- $n_0 =$ number of grid points within the structure,
- $n_1 =$ number of complete edges,
- $n_2 =$ number of faces within the structure,
- $n_3 =$ total number of grid cubes,
- $N =$ total number of grid points in the domain.

$$V_0 = n_3, \qquad V_1 = \frac{2(n_2 - 3n_3)}{9N}$$

$$V_2 = \frac{2(n_1 - 2n_2 + 3n_3)}{9N^2}, \qquad V_3 = \frac{n_0 - n_1 + n_2 - n_3}{N^3}$$

Shapefinders

$$V_0 = \iiint dV$$

$$V_1 = \frac{1}{6}\iint dS$$

$$V_2 = \frac{1}{6\pi}\iint (\kappa_1 + \kappa_2) dS$$

$$V_3 = \frac{1}{4\pi}\iint \kappa_1 \kappa_2 dS$$

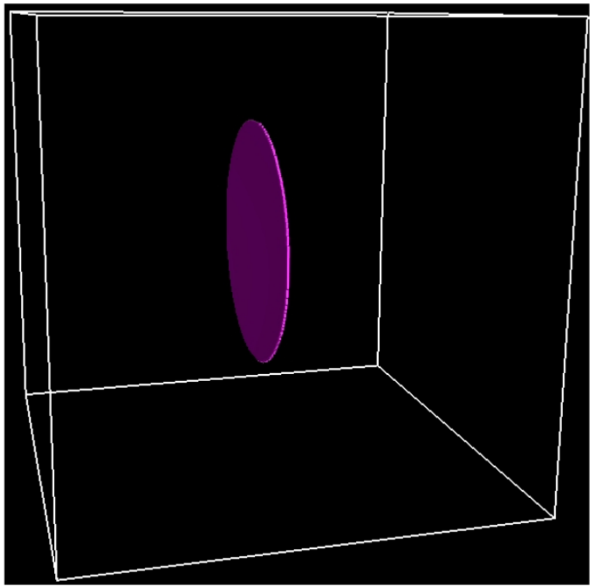
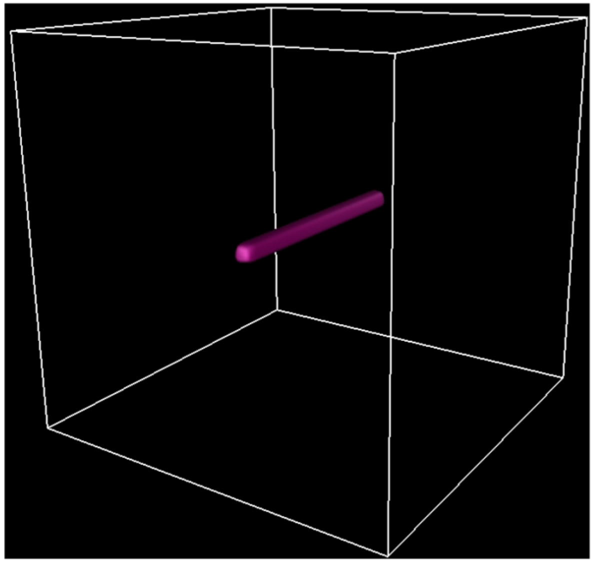
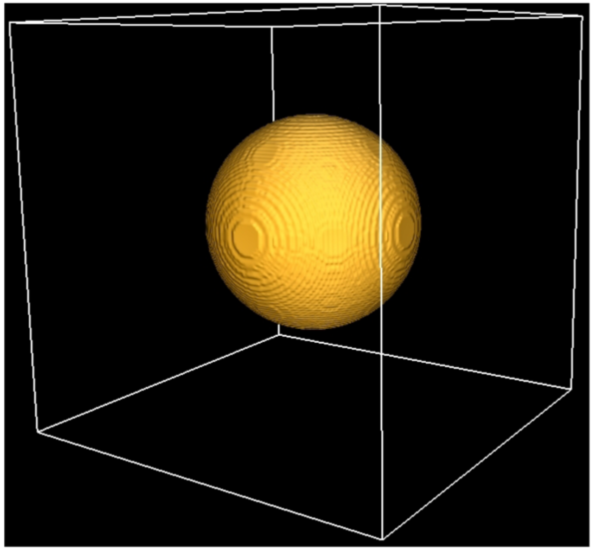
Thickness, **W**idth, **L**ength

$$T = \frac{V_0}{2V_1}, \quad W = \frac{2V_1}{\pi V_2}, \quad L = \frac{3V_2}{4V_3}$$

Planarity and **F**ilamentarity

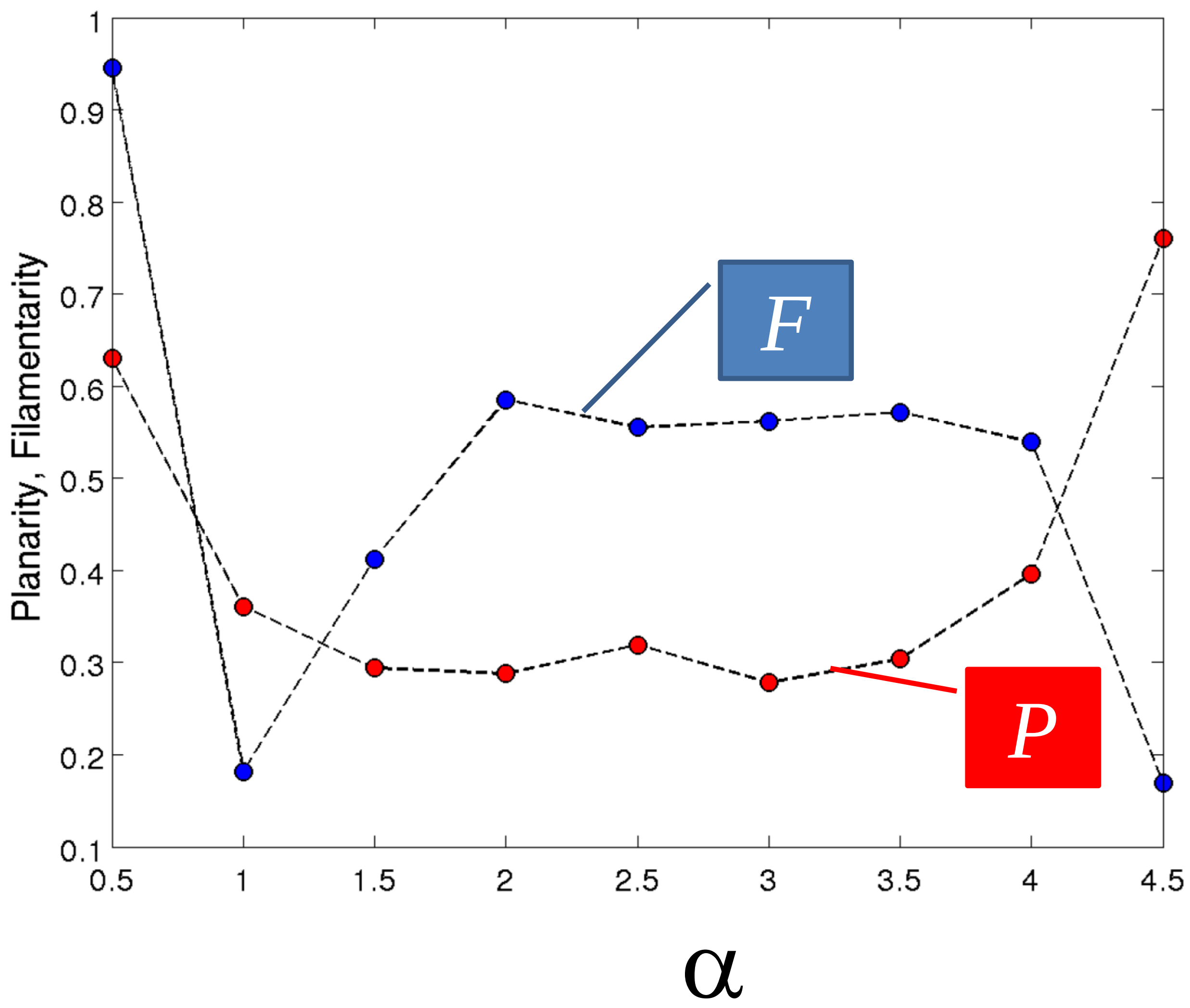
$$P = \frac{W - T}{W + T}, \quad F = \frac{L - W}{L + W}$$

- Filament: $P = 0$, $F = 1$;
- Pancake: $P = 1$, $F = 0$;
- Sphere: $P = F = 0$

(P, F) = (1.0,0.14)   (0.05, 0.69)   (0.016, 0.025)

Working range: $B = \alpha B_{\mathrm{rms}}$, $2 < \alpha < 4$

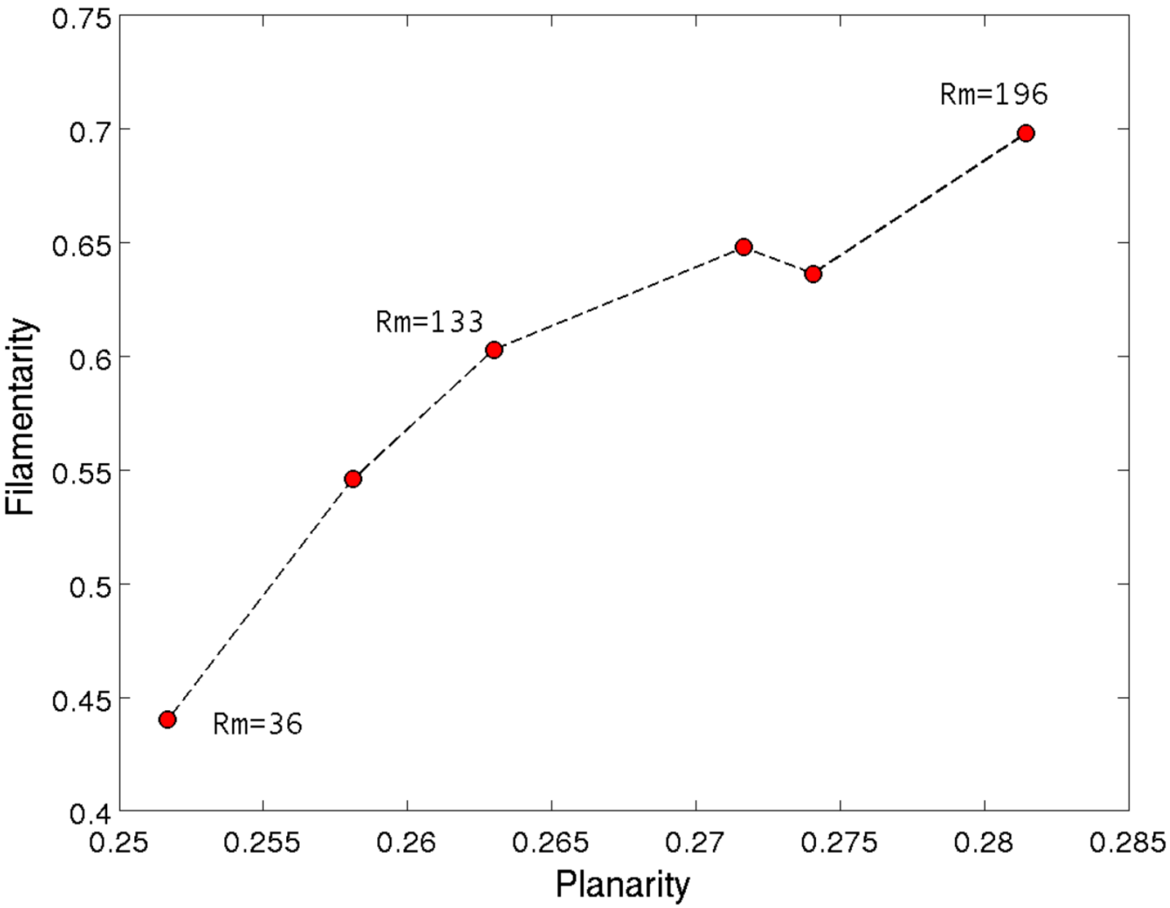


$R_{\mathrm{m}} = 133$

Results: R_m scaling & evolution

- P and F of the isosurfaces $B = \alpha B_{rms}$,
- f = fractional volume where $B \geq \alpha B_{rms}$,
- $2 \leq \alpha \leq 4$,
- averaged over 50 realizations.

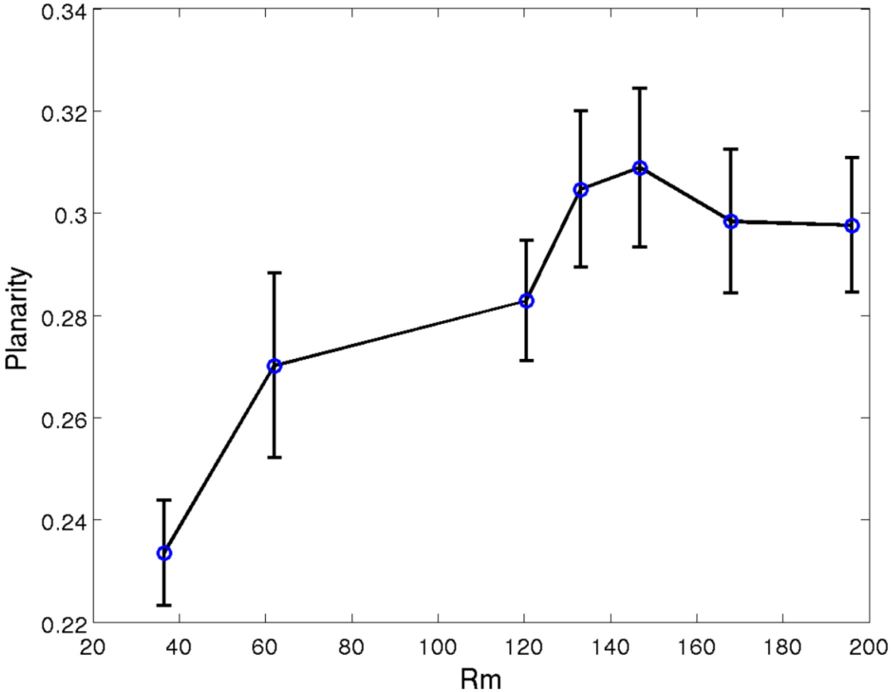
(P, F)-plane, kinematic state, varying R_m

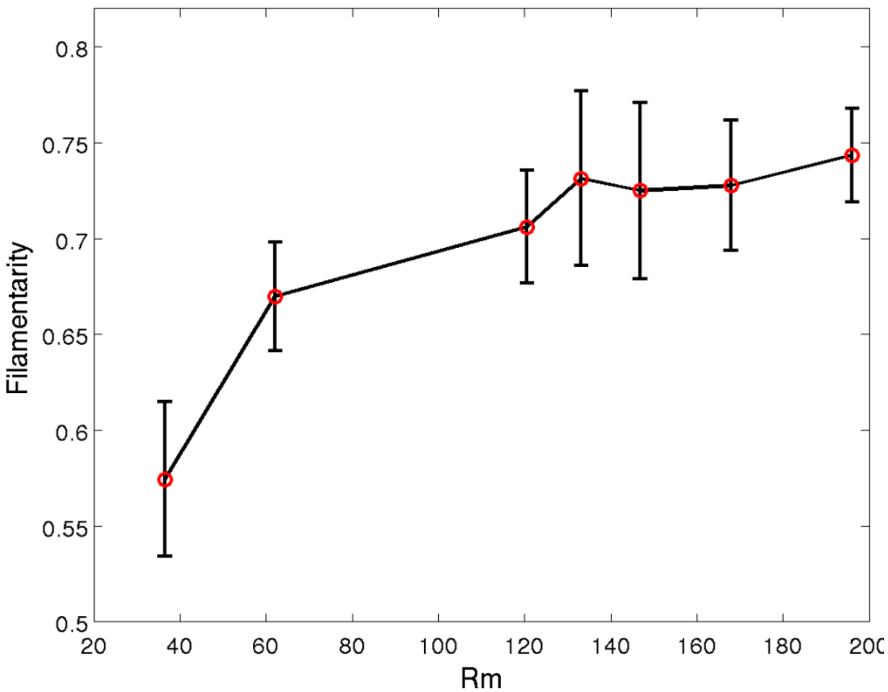


F/P grows with R_m: magnetic filaments at $R_m \gg 1$;

Consistent with Wilkin et al. (PRL, 2007)

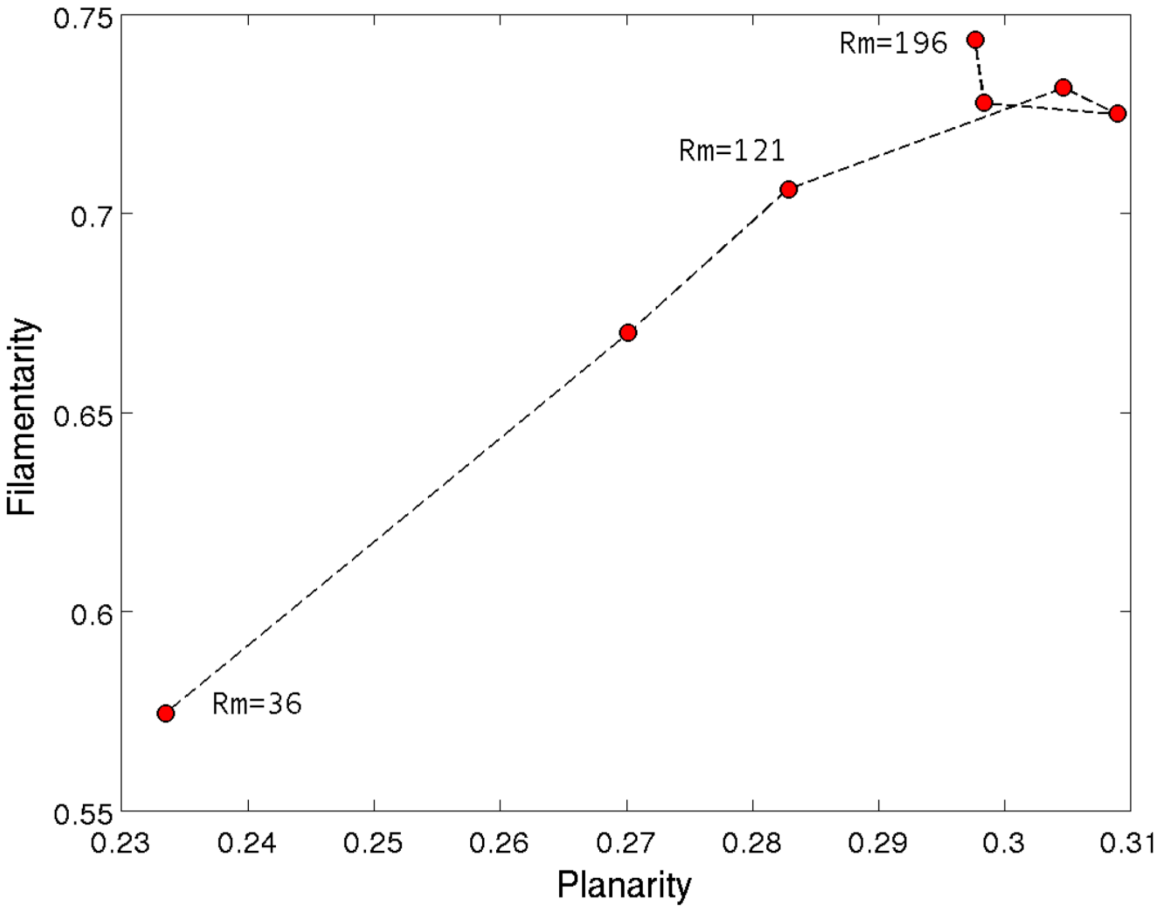
P and F in the saturated state





- ❑ $P \to$ const, $F \to$ const as R_m increases;
- ❑ asymptotic regime at $R_\mathrm{m} > 150$ (???)

(P, F)-plane, saturated state, varying R_m



$R_m \gg 1$:

- ❑ $F, P \to$ const (?)
- ❑ Magnetic filaments (rather than ribbons)

$(P, \; F) =$

(a) (0.096, 0.81);

(b) (0.66, 0.23);

(c) (0.66, 0.12);

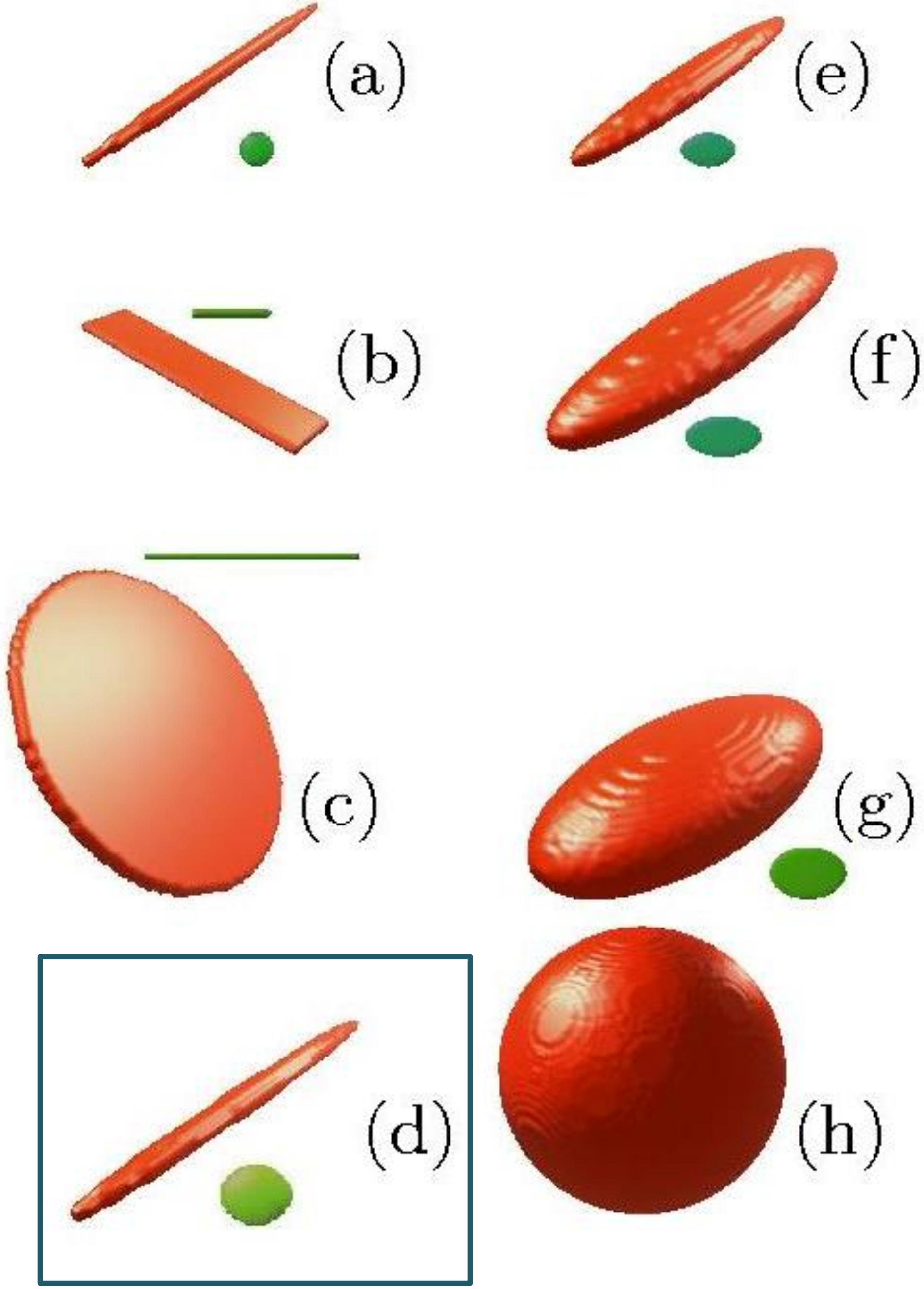
(d) (0.25, 0.66);

(e) (0.18, 0.43);

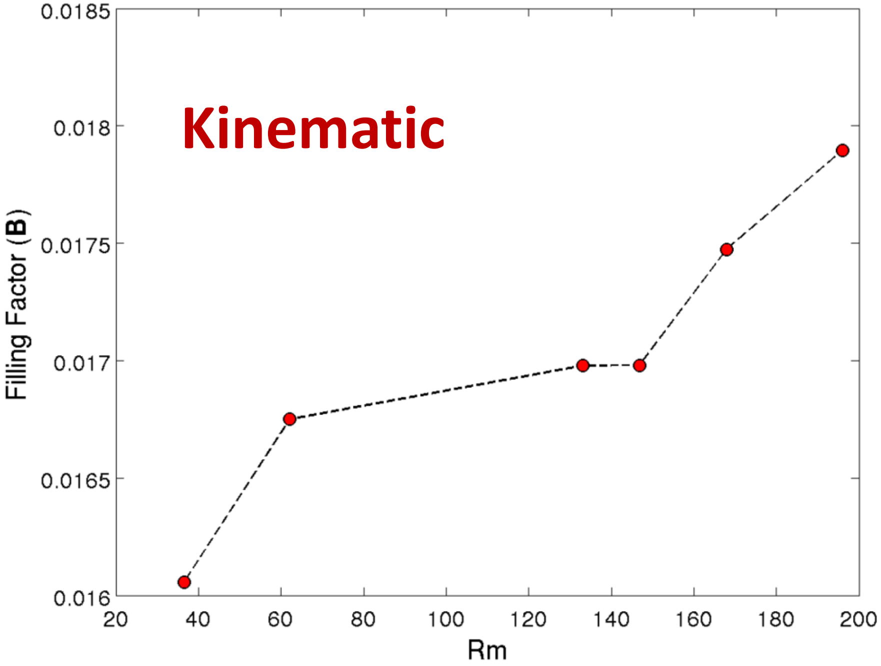
(f) (0.14, 0.23);

(g) (0.087, 0.073);

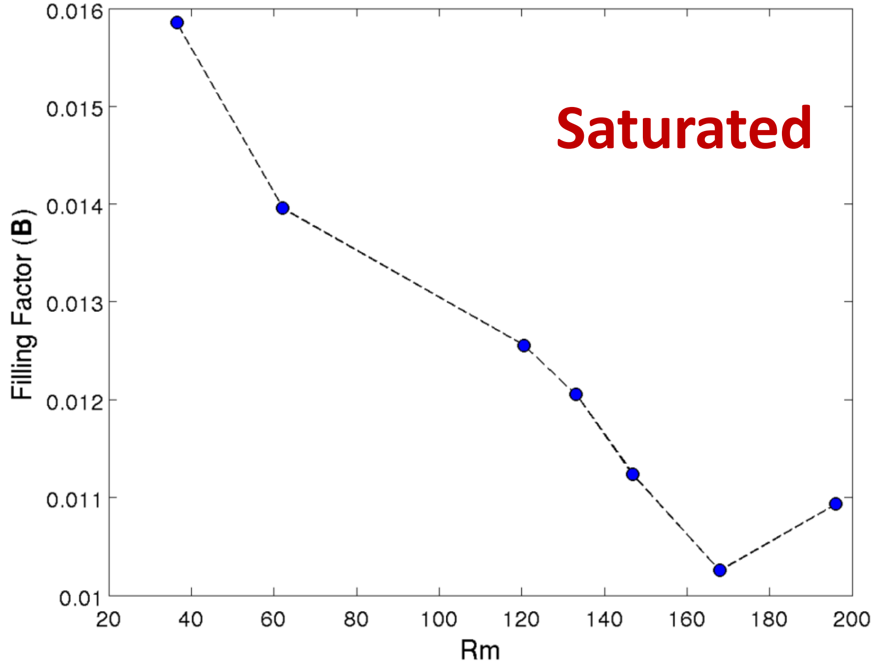
(h) (0.0036, -0.0047).



Fractional volume of magnetic structures vs R_{m}

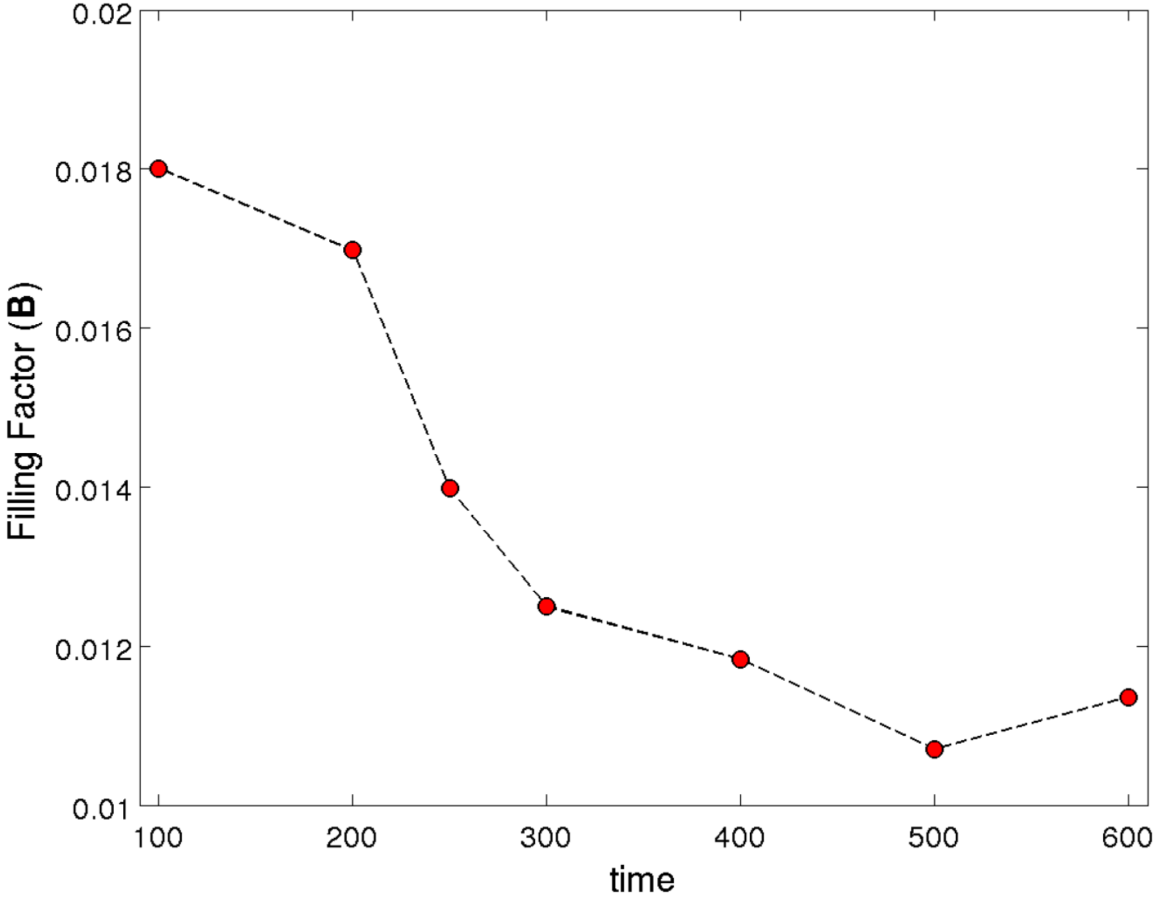


Slightly growing or constant



Slightly decreasing or constant

Time variation of the fractional volume: $R_m = 133$, saturation at $t \approx 250$



Decreasing at the kinematic stage (intermittency), constant in the nonlinear state