Magnetic flux emergence as an origin of the solar cycle variability

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Mike's birthday

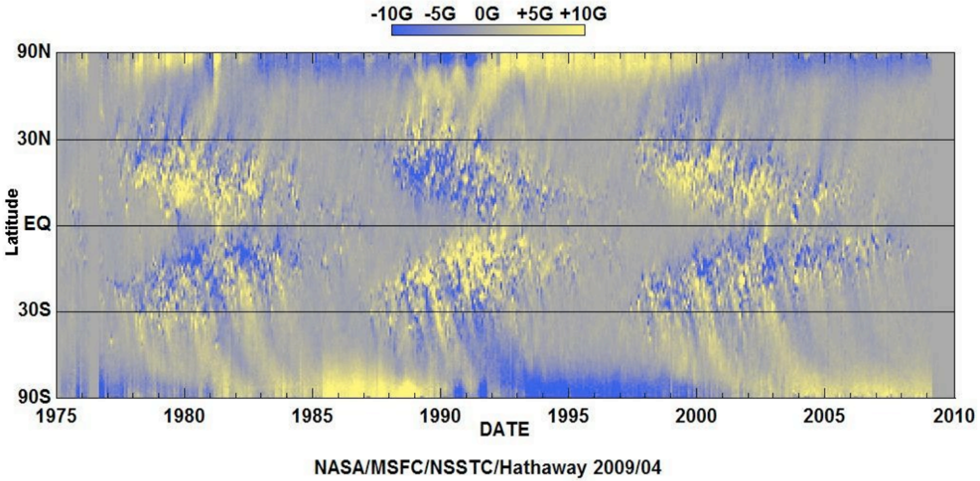
In collaboration with Mike Proctor and Geoffroy Lesur

UNIVERSITY OF CAMBRIDGE

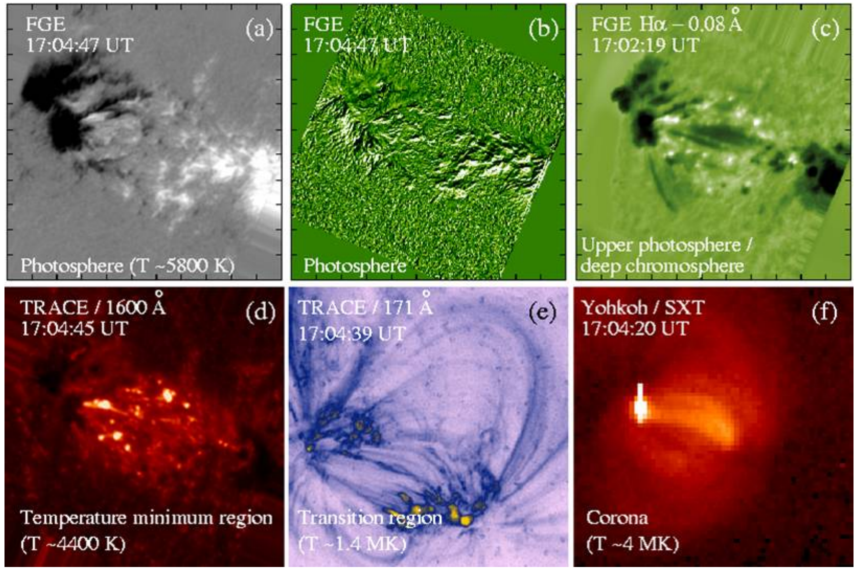
Astrophysical Fluid Dynamics and Nonlinear Dynamics

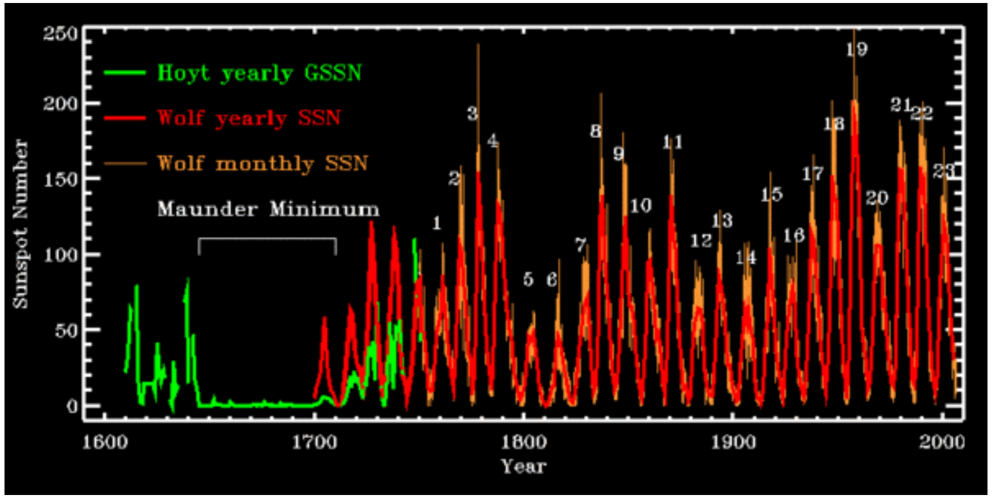
The 22-yr sunspot cycle

Butterfly diagram (observed radial field) (D. Hathaway)



Flux emergence observed at different wavelengths (Georgoulis et al. 2004, Schmieder et al. 2004)

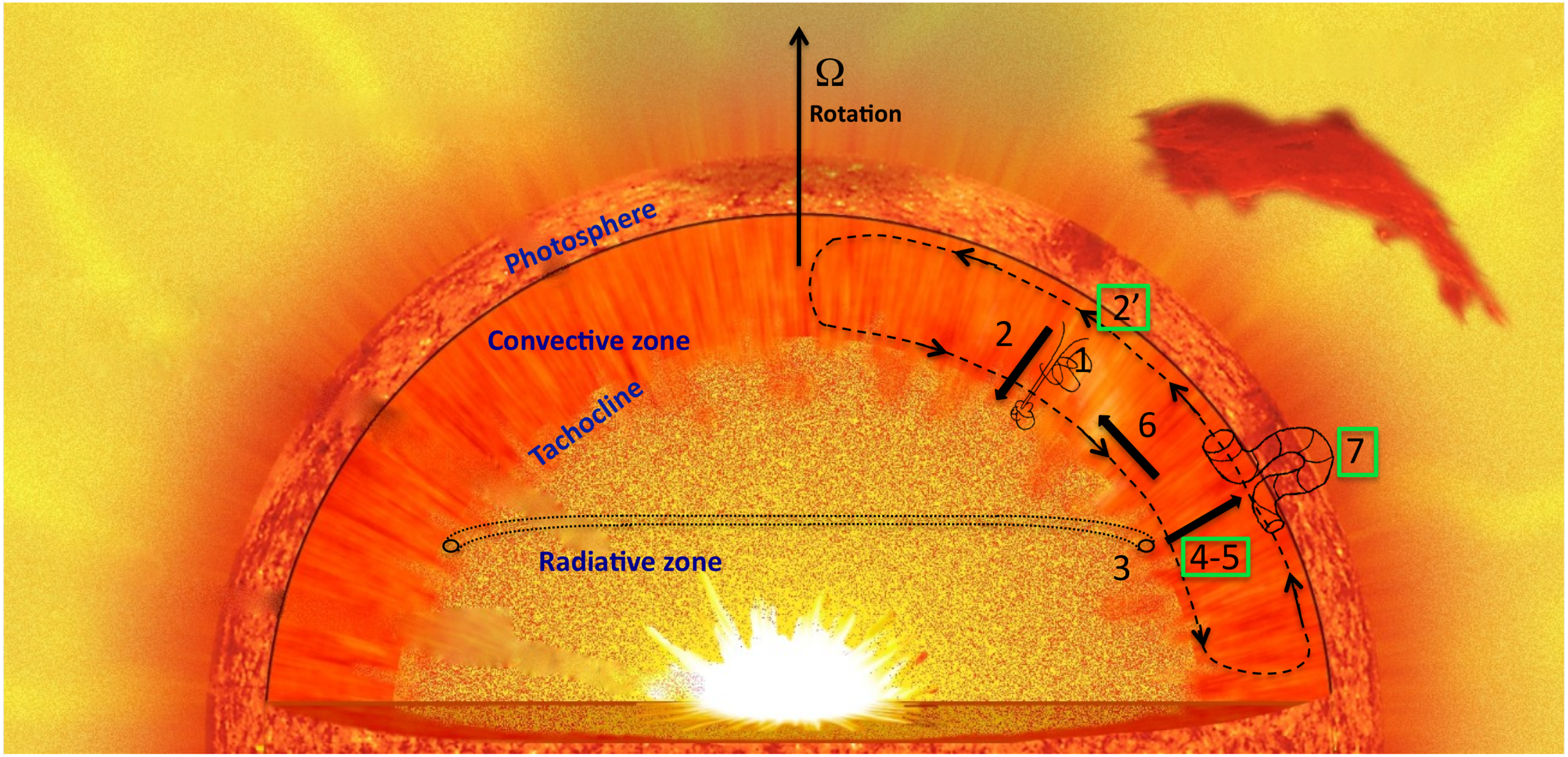




Varying sunspot number (Hoyt & Schatten 1998)

Schematic theoretical view of the solar cycle

Adapted from NB and SB



1: magnetic field generation, self-induction
2: pumping of mag. field
or
2': **transport by meridional flow**
3: stretching of field lines through Ω effect

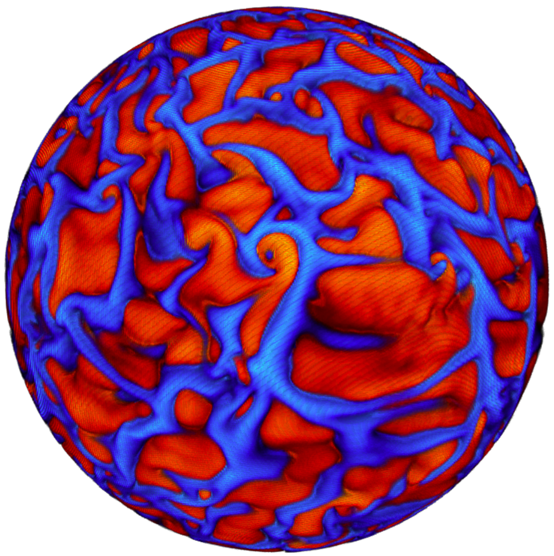
4: Parker instability
5: **emergence+rotation**
6: recycling through α−effect or
7: **emergence of twisted bipolar structures at the surface**

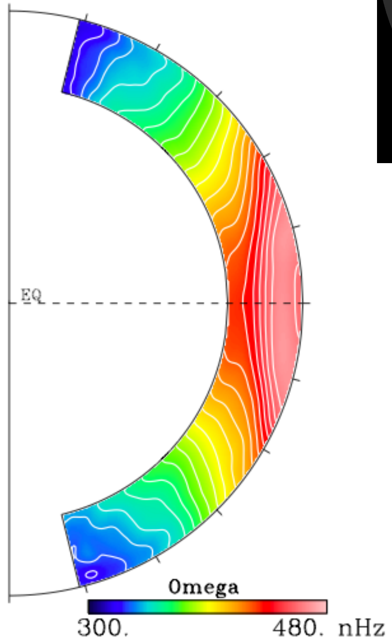
Flux tube rise in a spherical convective shell

η=1.13 10^{12} cm^2 s^{-1}
ν=1.13 10^{12} cm^2 s^{-1}
κ=4.53 10^{12} cm^2 s^{-1}

at mid-CZ,
vary as $1/\bar{\rho}^{1/3}$

Re=120, Pr=0.25, Pm=1





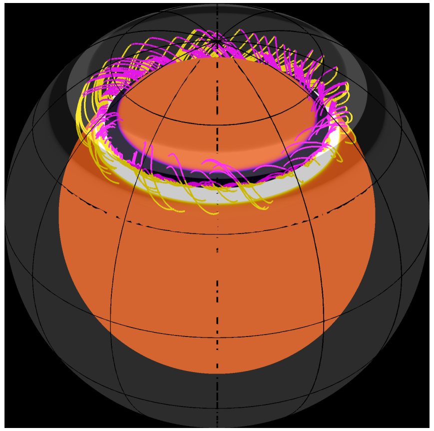
Radial velocity

(volume rendering obtained with SDvision)

Differential rotation

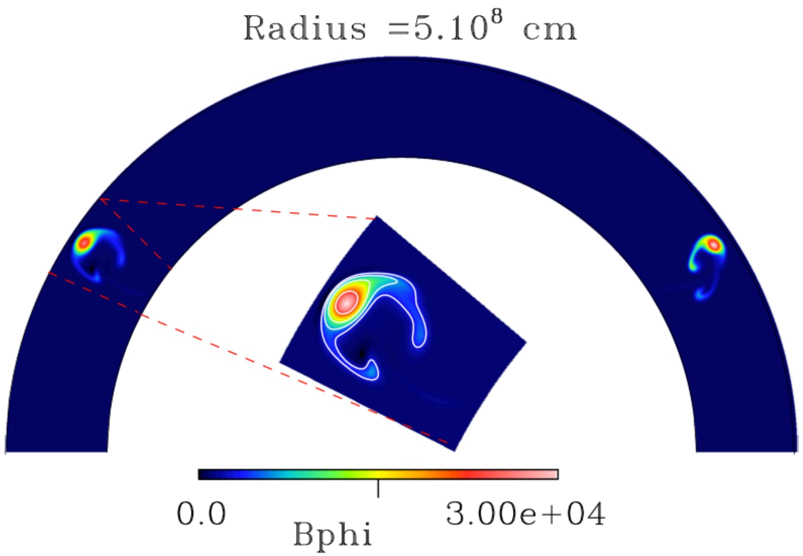
Miesch, Brun & Toomre 2006

$$S(r_{bot},\theta) = a_2\, Y_2^0 + a_4\, Y_4^0$$



✓ Beq is approximately 6 x 10^4 G

✓ A twisted tube is introduced in pressure and entropy equilibrium

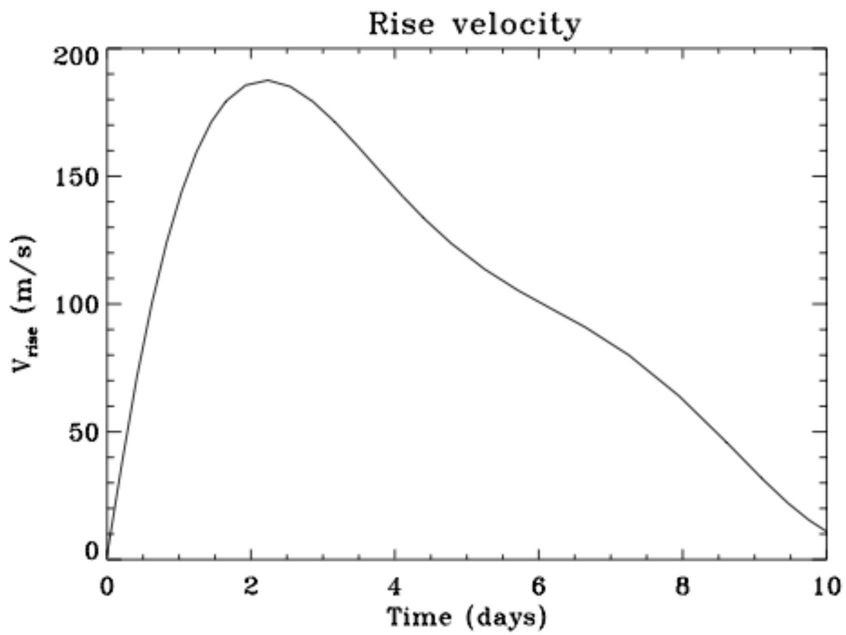


Jouve & Brun, 2009, ApJ

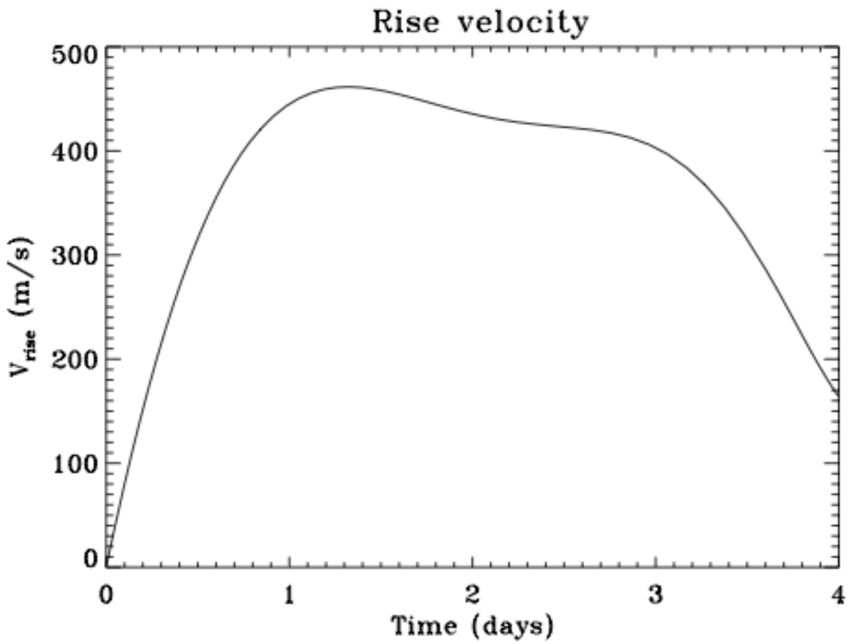
Flux tube rise in a spherical convective shell

Influence of the initial magnetic intensity

B_0=5 Beq



B_0=10 Beq



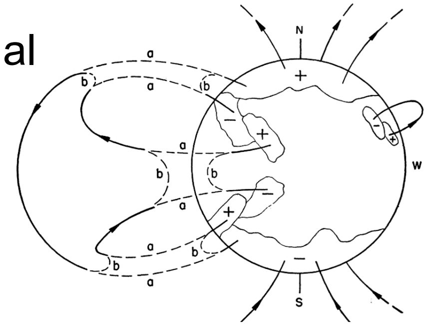
The tube acceleration in the first phase is due to the buoyancy force which is directly linked to the density deficit in the tube.

Thus we have in the acceleration phase: $\boxed{acceleration\ \alpha\ \ \Delta\rho\ \ \ \alpha\ \ B_0^2}$

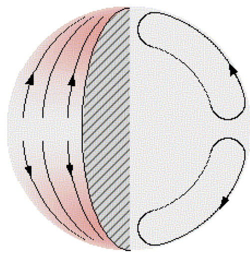
The Babcock-Leighton flux-transport model

(Babcock 1961, Leighton 1969, Wang & Sheeley 1991)

✓ **Source** of poloidal field linked to the rise of toroidal flux concentrations



✓ **Transport** by meridional circulation within the convection zone

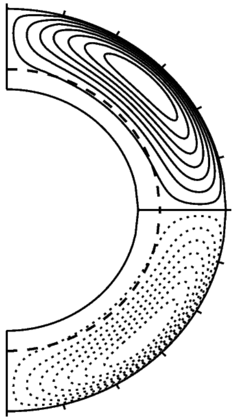


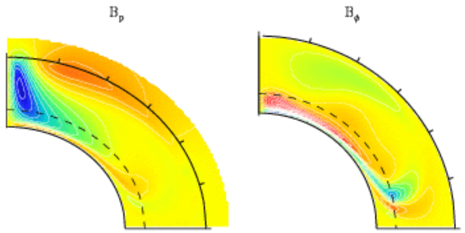
✓ **2 coupled PDEs:**

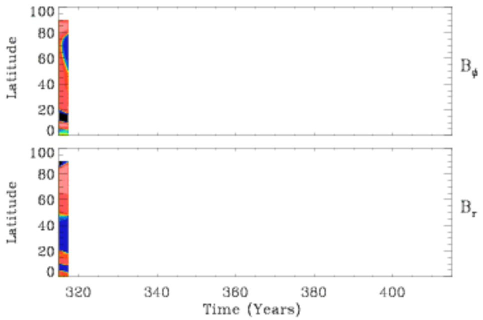
$$\frac{\partial A_\phi}{\partial t} = \frac{\eta}{\eta_t}\left(\nabla^2 - \frac{1}{\varpi^2}\right)A_\phi - R_e \frac{\mathbf{u}_p}{\varpi}\cdot\nabla(\varpi A_\phi) + \cancel{C_\alpha \alpha B_\phi} + \boxed{C_s S(r,\theta,B_\phi)}$$

$$\frac{\partial B_\phi}{\partial t} = \frac{\eta}{\eta_t}\left(\nabla^2 - \frac{1}{\varpi^2}\right)B_\phi + \frac{1}{\varpi}\frac{\partial(\varpi B_\phi)}{\partial r}\frac{\partial(\eta/\eta_t)}{\partial r} - R_e \varpi \mathbf{u}_p \cdot \nabla\left(\frac{B_\phi}{\varpi}\right) - R_e B_\phi \nabla\cdot\mathbf{u}_p + C_\Omega \varpi(\nabla\times(\varpi A_\phi \hat{\mathbf{e}}_\phi))\cdot\nabla\Omega$$

Standard model: single-celled meridional circulation







Cyclic field

Butterfly diagram close to observations

Parameters:

v_0=6.4 m.s^{-1}
η_t=5x10^{10}cm^2.s^{-1}
s_0=20 cm.s^{-1}
Ω_{eq}=460 nHz

The Babcock-Leighton source term:
incorporating results of 3D simulations

Standard

$$S(r,\theta,B_\phi) = \underbrace{\frac{1}{4}\left[1+\mathrm{erf}\left(\frac{r-r_2}{d_2}\right)\right]\left[1-\mathrm{erf}\left(\frac{r-R_\odot}{d_2}\right)\right]}_{\text{Confinement at the surface}} \times \underbrace{\left[1+\left(\frac{B_\phi(r_c,\theta,t)}{B_0}\right)^2\right]^{-1}}_{\text{Quenching}} \underbrace{\cos\theta\sin\theta}_{\text{« Ad hoc » latitudinal dependence}} \underbrace{B_\phi(r_c,\theta,t)}_{\text{Toroidal field at the base of the CZ}}$$

Modified

$$S(r,\theta,B_\phi) = \frac{1}{2}\left[1+\mathrm{erf}\left(\frac{r-r_2}{d_2}\right)\right]\left[1-\mathrm{erf}\left(\frac{r-R_\odot}{d_2}\right)\right] \times \left[1+\boxed{\left(\frac{B_\varphi(r_c,\theta,t-\tau(B_\varphi))}{B_0}\right)^2}\right]^{-1} \cos\theta\sin\theta\,\boxed{B_\varphi(r_c,\theta,t-\tau(B_\varphi))}$$

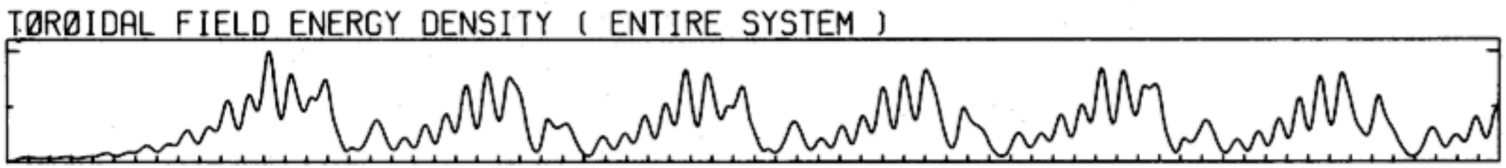
with $\boxed{\tau(B_\varphi) \propto \frac{1}{B_\varphi^2}}$

We get state-dependent delayed equations

Jouve, Proctor & Lesur, 2010, A&A

Previous work on delays and dynamo models

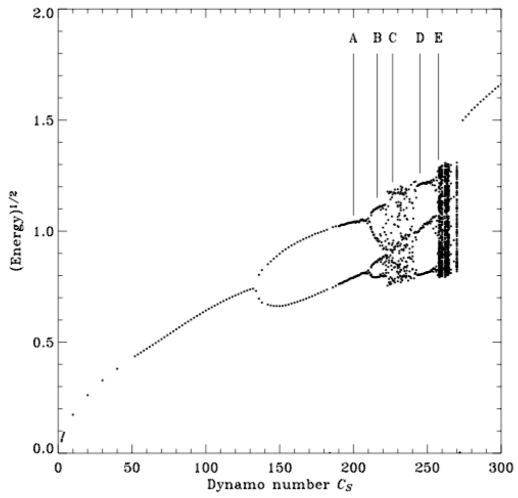
- Yoshimura 78 introduces delays in both equations and gets long-term modulation (but large delays compared to the cycle period: 30-yr delay to get a 80-yr modulation)



- Wilmot-Smith et al 05 consider delays in a BL model but very simple and only delay due to MC

$$\frac{dB_\phi(t)}{dt} = \frac{\omega}{L} A(t - T_0) - \frac{B_\phi(t)}{\tau},$$

$$\frac{dA(t)}{dt} = \alpha_0 f\left(B_\phi(t - T_1)\right) B_\phi(t - T_1) - \frac{A(t)}{\tau}.$$



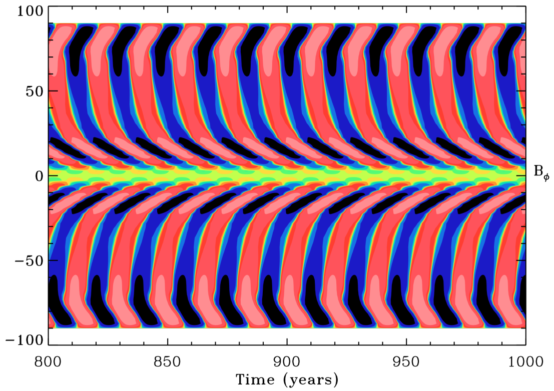
- Charbonneau et al 05 increased S_0 and found a series of bifurcations leading to chaotic behaviour. They argue it's linked to the time delay introduced by the meridional flow.

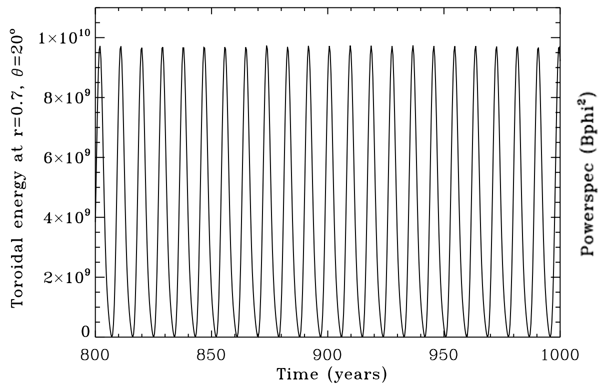
What about considering the time delay due to the buoyant rise of magnetic flux?

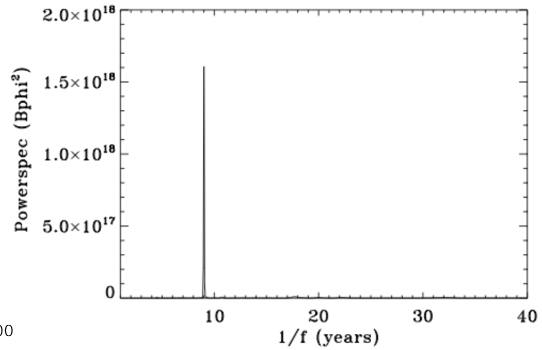
Modulation of the cycle?

Butterfly diagram | Magnetic energy | Powerspectrum

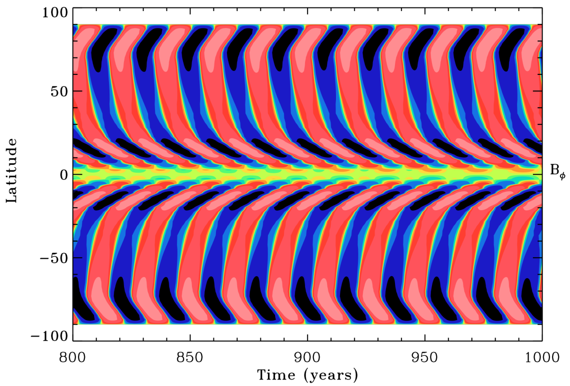
Standard (no delay)

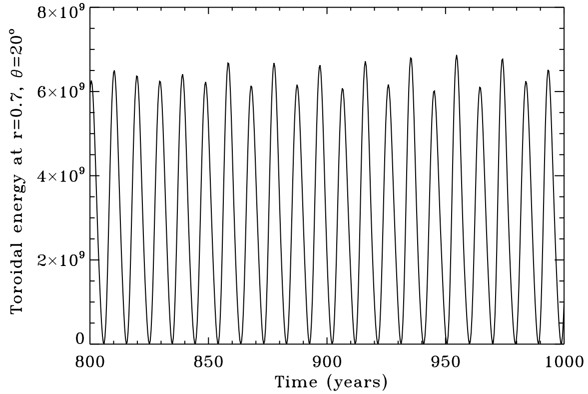


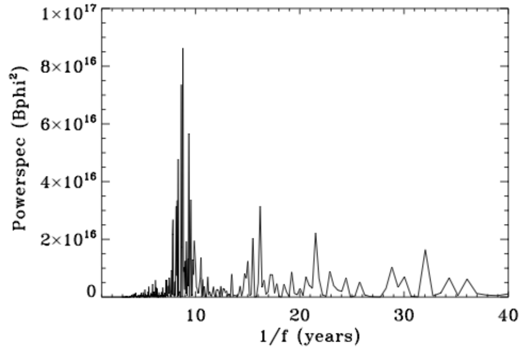




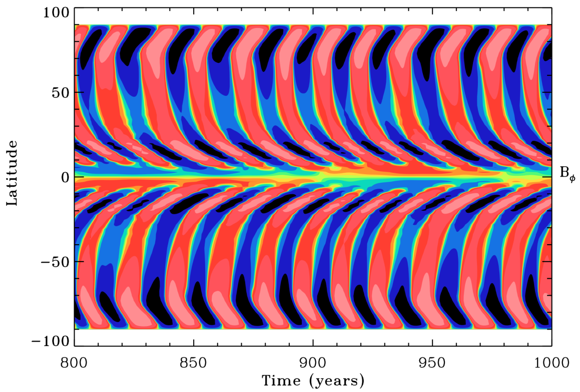
14 days on 10^3G fields

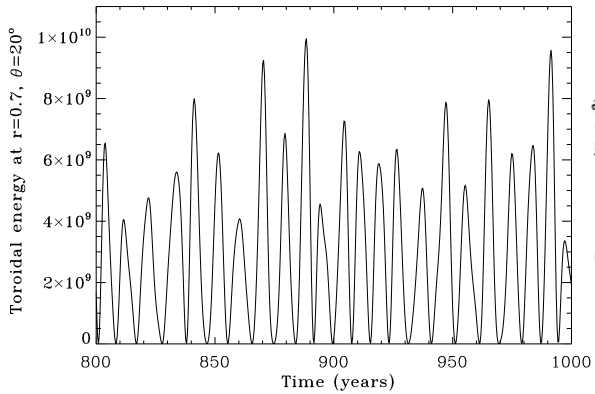


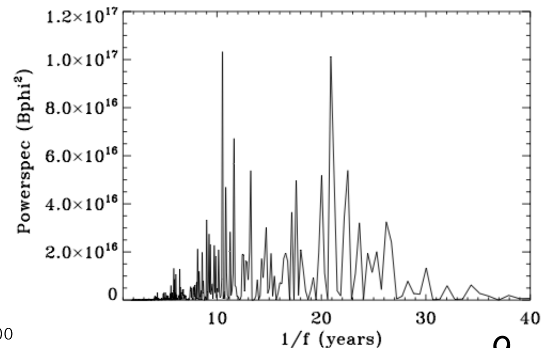




14 days on $5x10^4$G fields







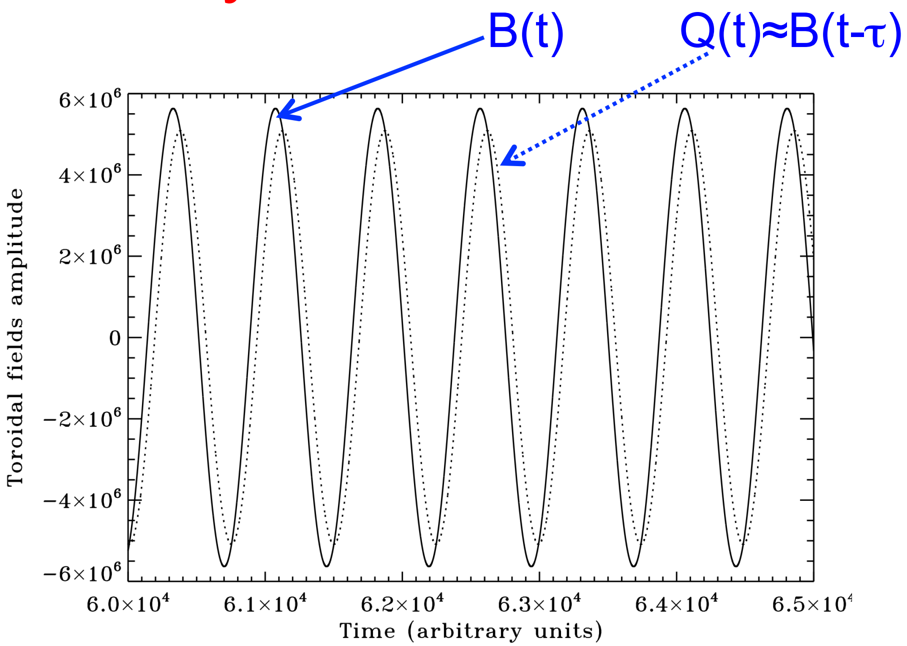
A reduced model of the rise of flux tubes without using explicit delays

We consider the following set of equations, Q is here delayed by τ compared to B:

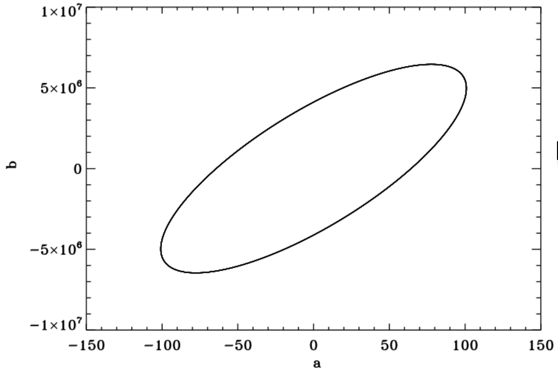
$$\frac{dA_t}{dt} + ikvA_t = \frac{S\,Q_t}{1+\lambda|Q_t|^2} - \eta k^2 A_t$$

$$\frac{dB_t}{dt} + ikvB_t = ik\Omega A_t - \eta k^2 B_t$$

$$\frac{dQ_t}{dt} = \frac{1}{\tau}(B_t - Q_t) \quad \text{with} \quad \tau = \frac{\tau_0}{1+|B_t|^2}$$

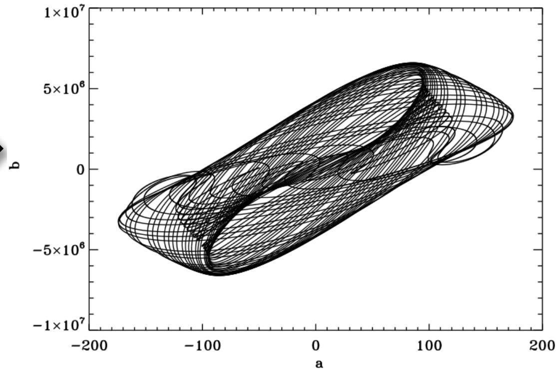


τ_0=0.01



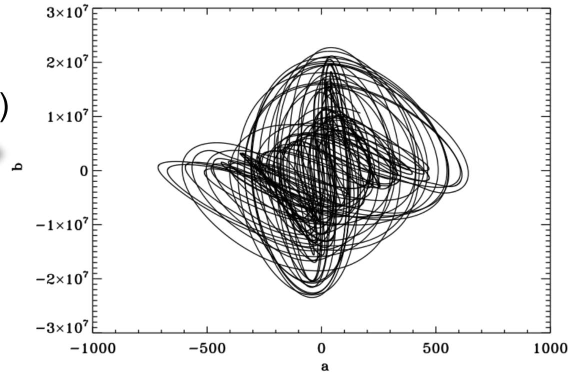
Bifur-
cation

τ_0=0.04



Bifur-
cation(s)

τ_0=0.5



Stability of the harmonic solution

Harmonic solution

$$A_t(t) = A_0 \exp(i\omega t)$$

$$B_t(t) = B_0 \exp(i\omega t)$$

$$Q_t(t) = Q_0 \exp(i\omega t)$$

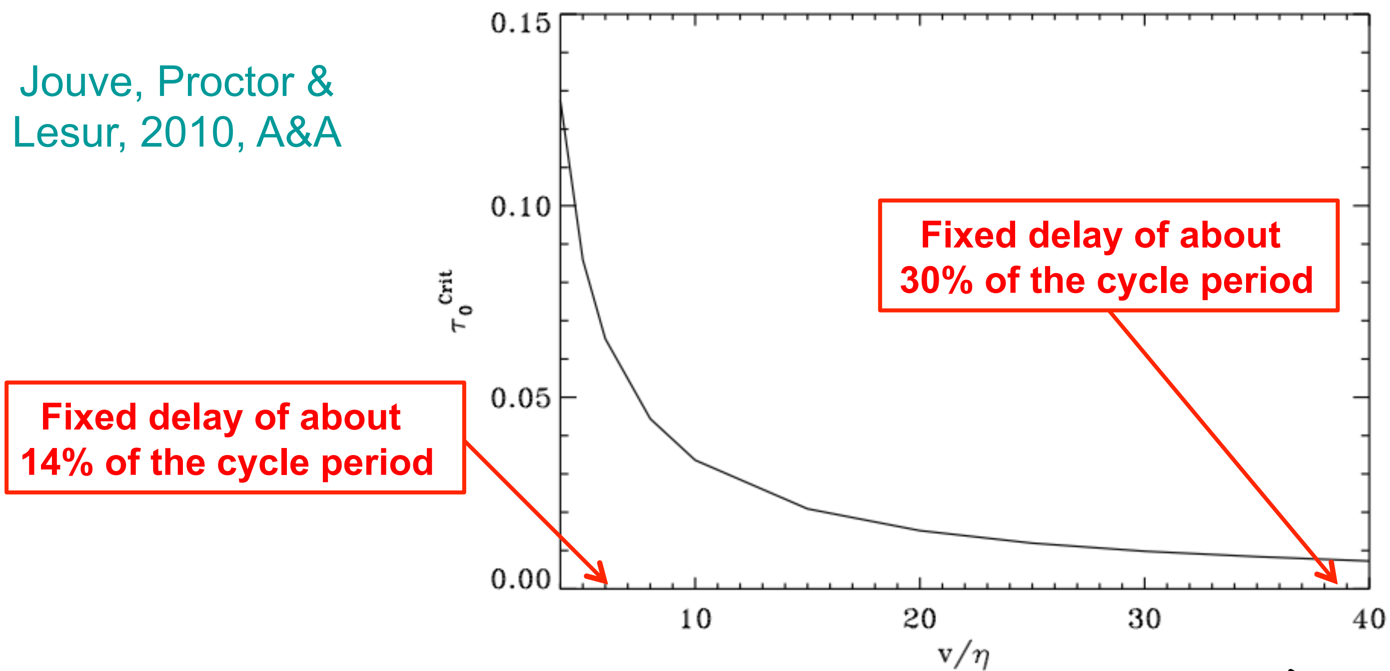
Perturbations with complex growth rate p

$$\tilde{A} = A_t\,(1 + \alpha_1\, e^{pt} + \alpha_2^\star\, e^{p^\star t})$$

$$\tilde{B} = B_t\,(1 + \beta_1\, e^{pt} + \beta_2^\star\, e^{p^\star t})$$

$$\tilde{Q} = Q_t\,(1 + \gamma_1\, e^{pt} + \gamma_2^\star\, e^{p^\star t})$$

Jouve, Proctor & Lesur, 2010, A&A



More and more advection-dominated

Influence of the meridional flow speed on the loss of stability (Hopf bifurcation)

Further reduction of the equations

By noticing the existence of a symmetry in the equations (phase invariance), we can further reduce our system from 6 to 5 degrees of freedom (Jones et al, 1984)

$$A_t = \rho\, y\, e^{i\theta}$$

$$B_t = \rho\, e^{i\theta}$$

$$Q_t = \rho\, z\, e^{i\theta}$$

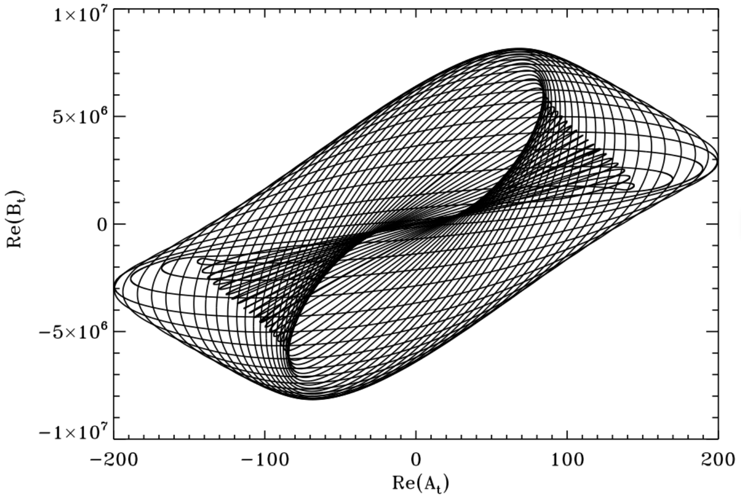
with ρ, θ real and y, z complex

$$\dot{\rho} = -\Omega\rho\,\Im(y) - \eta\rho$$

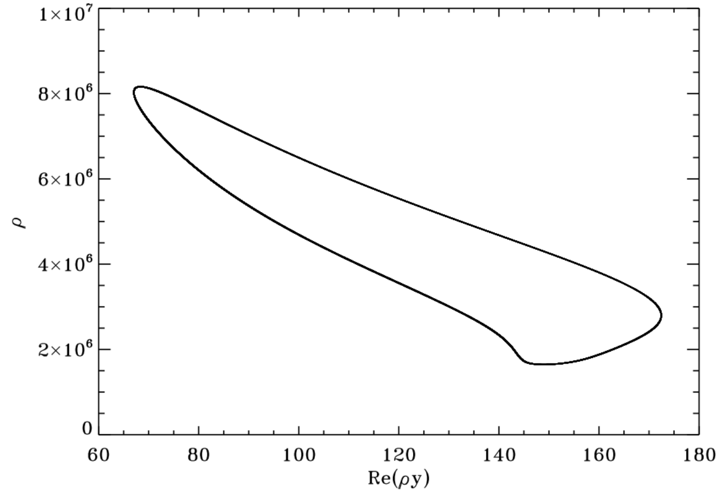
$$\dot{y} = \frac{S\,z}{1+\lambda\rho^2\,|z|^2} - i\,\Omega\,y^2 \qquad \text{with} \qquad \tau = \frac{\tau_0}{1+\rho^2}$$

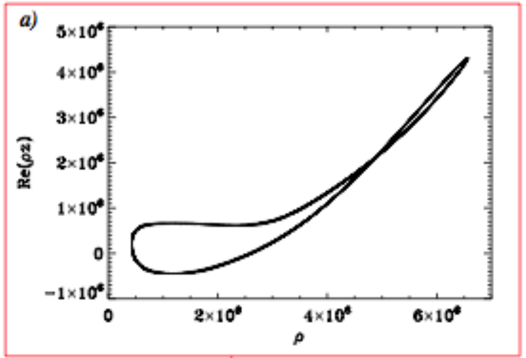
$$\dot{z} = \frac{1-z}{\tau} - (i\,\Omega\,y - i\,v - \eta)\,z$$

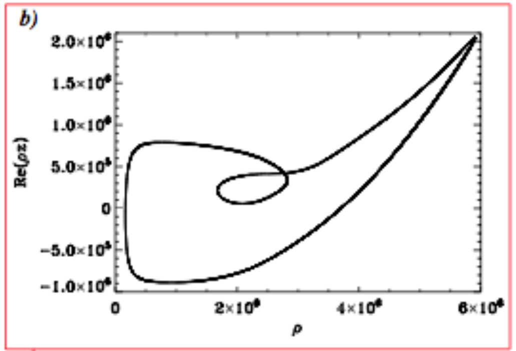
6th order model



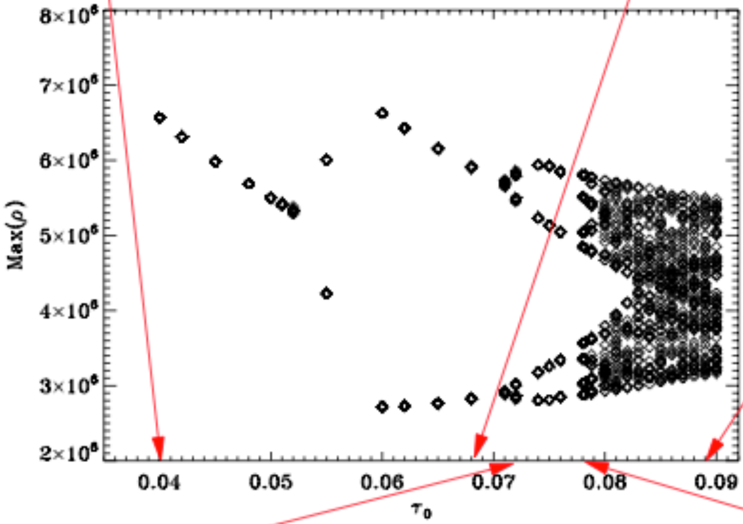
5th order model

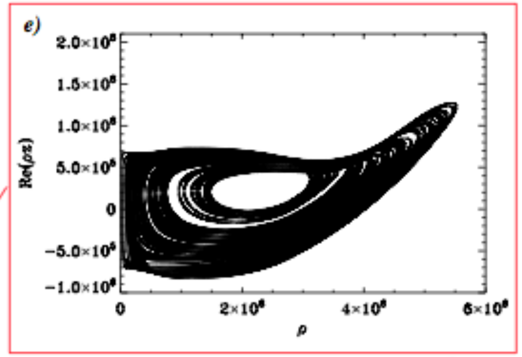




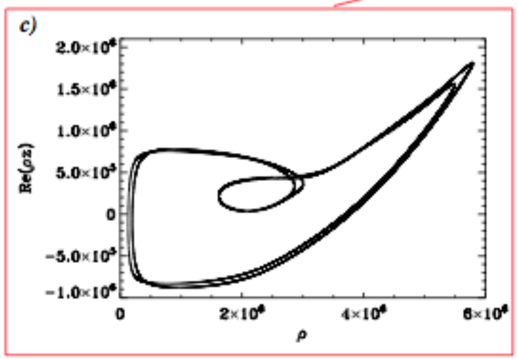


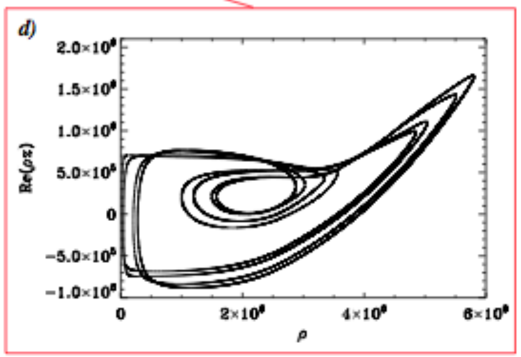
From the harmonic to the aperiodic solution…

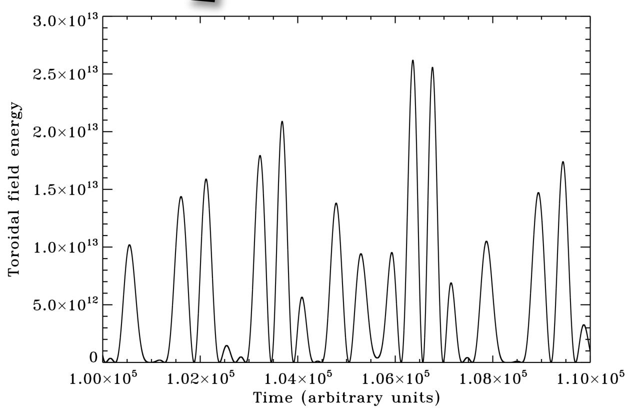




Aperiodic modulation of the cycle







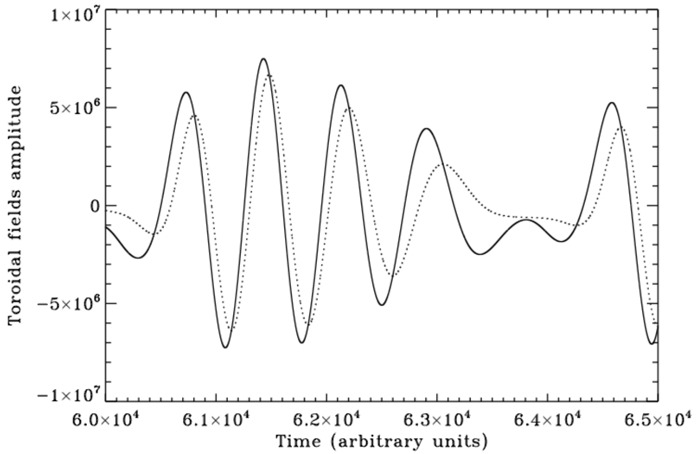
Conclusions on 2D models

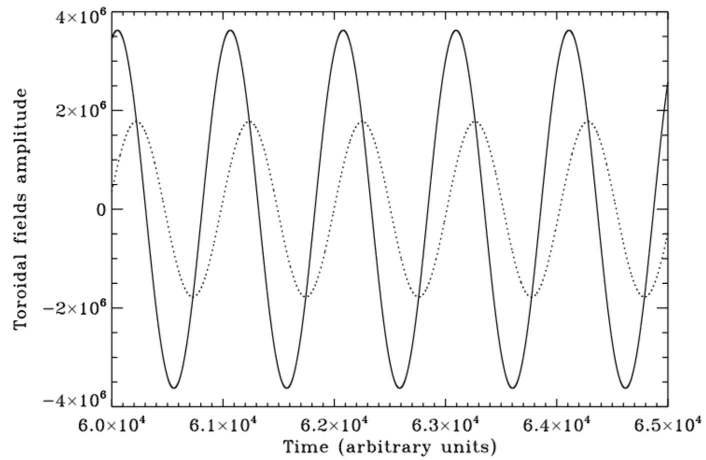
Reintroducing results from **3D calculations in simple mean-field dynamo models** helps to get **some insights** on which process has which effect on the global magnetic cycle.

Varying delay

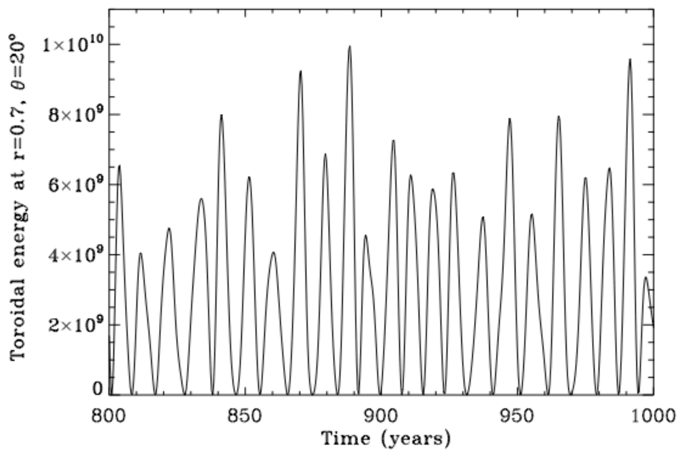
Fixed delay

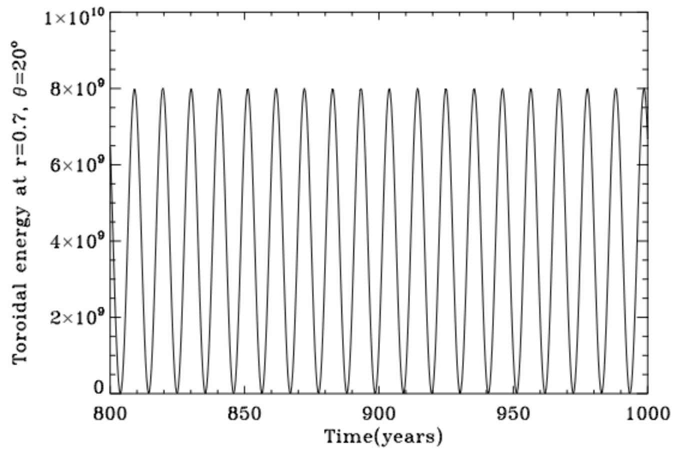
Reduced model





Full 2D model





The magnetic field-dependent rise time of flux concentrations are shown to be a potential source of long-term modulation

Flux tube rise from the BCZ to the surface

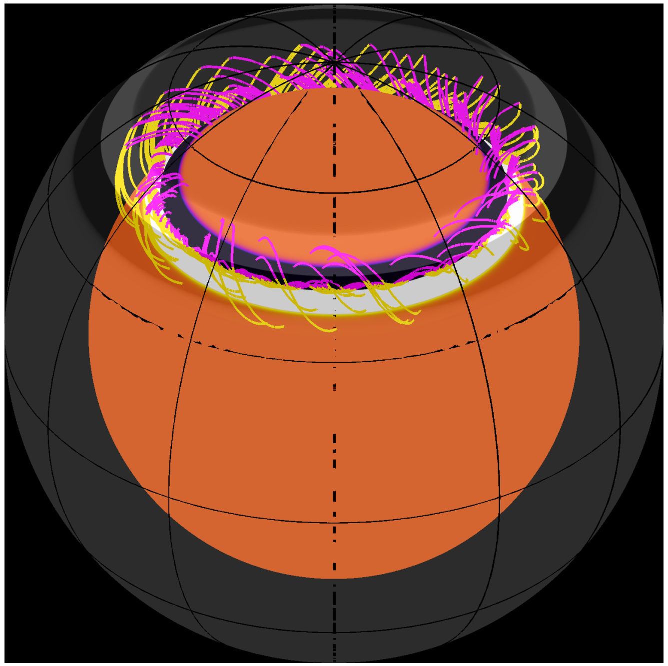
Previous calculations:

Thin flux tubes: all the variables only vary along the tube axis (Spruit,1981)

2D simulations: cartesian or spherical (e.g. Emonet & Moreno-Insertis 1998)

3D simulations: with convection in cartesian geometry (Cline 2003, Fan et al 2003)
without convection in spherical geometry (Fan 2008)

→ **First 3D simulations in a convective rotating spherical shell (ASH code)**



✓ Beq is approximately $\boxed{6 \times 10^4 \text{ G}}$

✓ For the twisted case, the twist is set to a value above the 2D-threshold:

$$\boxed{\sin\psi = 0.5 > \sin\psi_{\min} \approx 0.42}$$

✓ Pressure and entropy equilibrium, density deficit in the tube (magnetic buoyancy)

$$\frac{\rho_{in}}{\rho_{ext}} = \left(1 - \frac{B^2}{8\pi P^g_{ext}}\right)^{1/\gamma}$$

Further reduction of the equations

By noticing the existence of a symmetry in the equations (phase invariance), we can further reduce our system from 6 to 5 degrees of freedom (Jones et al, 1984)

$$A_t = \rho\, y\, e^{i\theta}$$

$$B_t = \rho\, e^{i\theta}$$

$$Q_t = \rho\, z\, e^{i\theta}$$

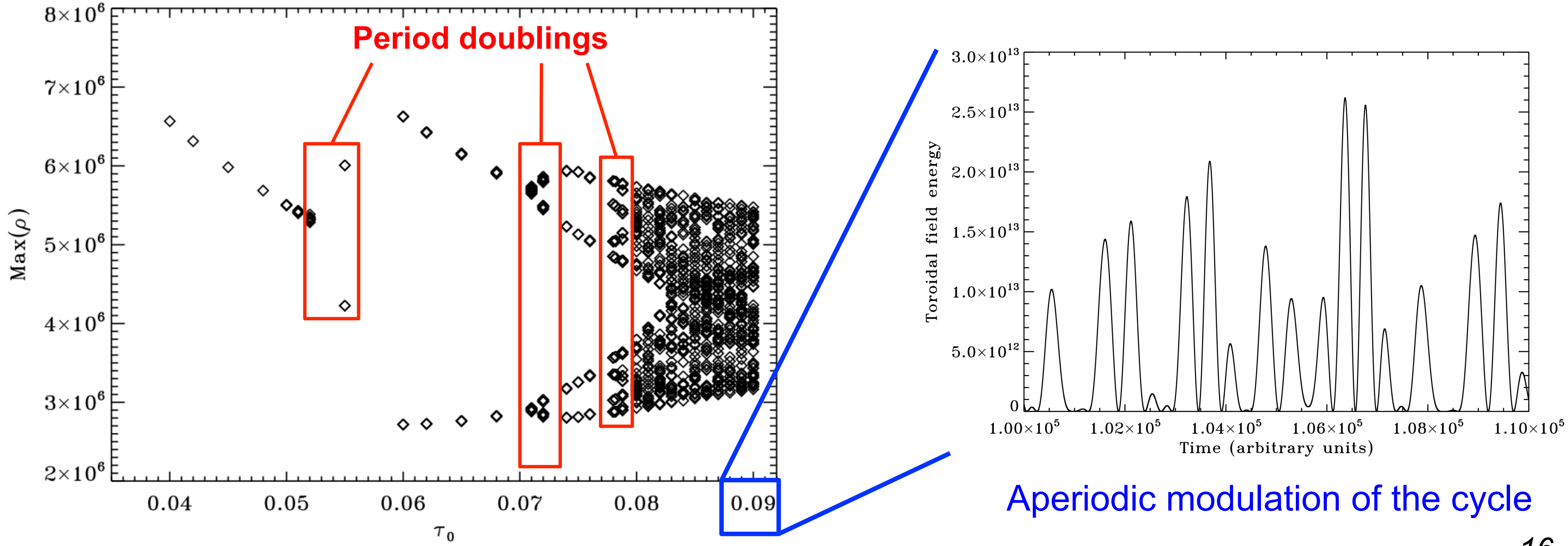
with ρ, θ real and y, z complex

$$\dot{\rho} = -\Omega \rho\, \Im(y) - \eta \rho$$

$$\dot{y} = \frac{S\, z}{1 + \lambda \rho^2 |z|^2} - i\,\Omega\, y^2 \qquad \text{with} \qquad \tau = \frac{\tau_0}{1 + \rho^2}$$

$$\dot{z} = \frac{1 - z}{\tau} - (i\,\Omega\, y - i\, v - \eta)\, z$$

The route to aperiodic solutions (seen in the 2D model) is then very clear…



Aperiodic modulation of the cycle