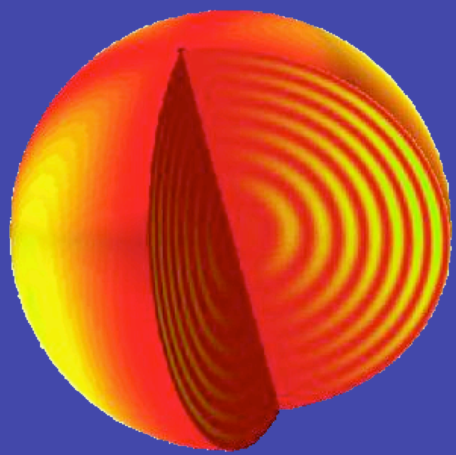
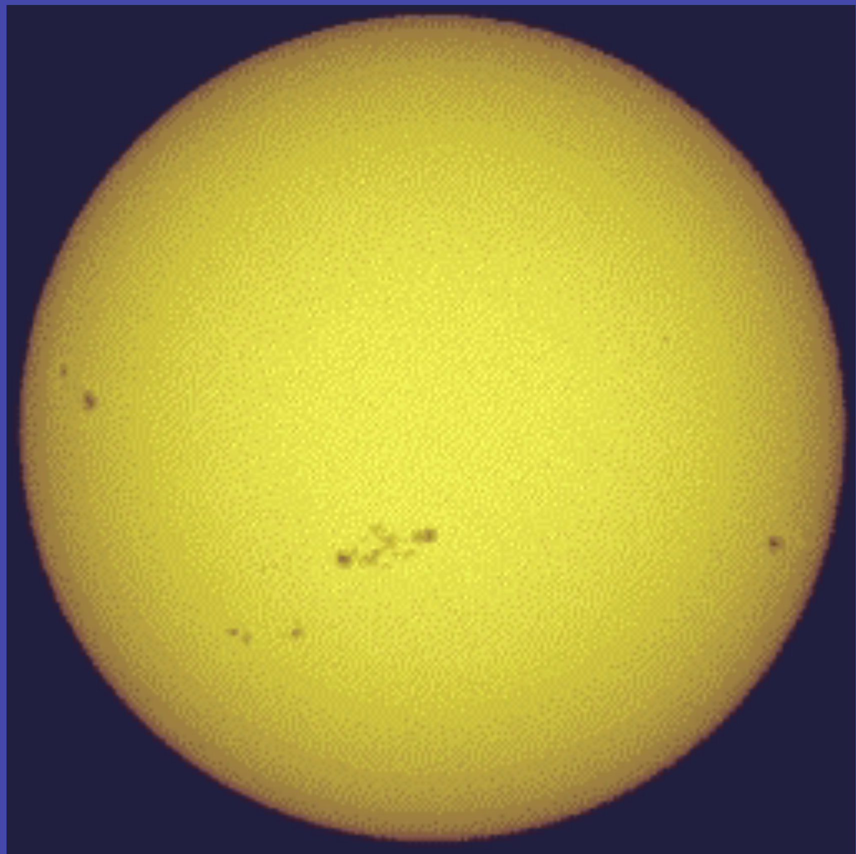
Pattern formation in rapidly oscillating peculiar A stars

Douglas Gough
Institute of Astronomy
Department of Applied Mathematics and Theoretical Physics
University of Cambridge

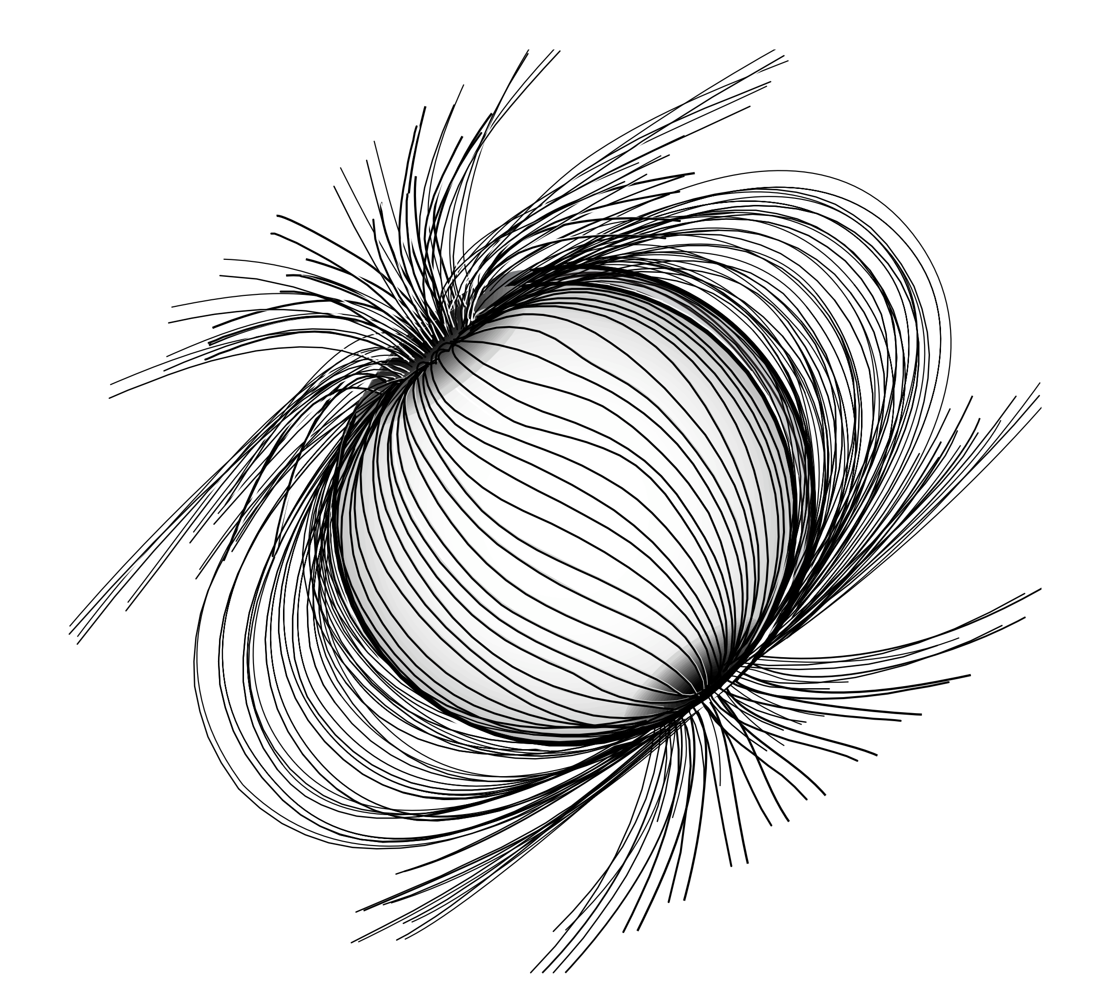


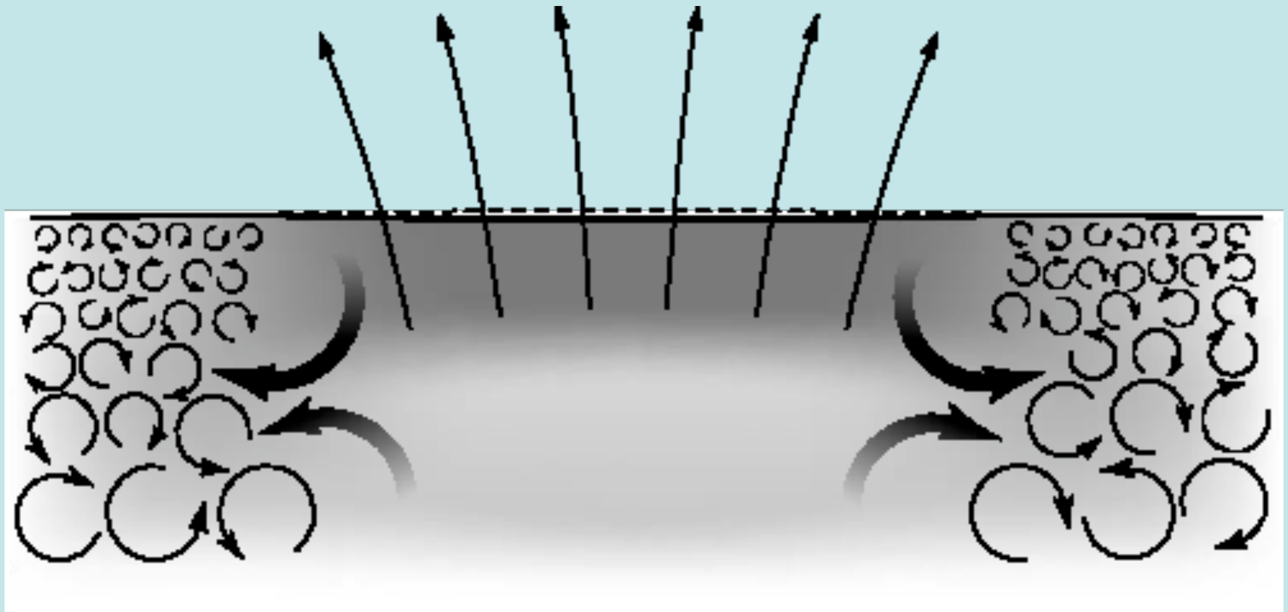
MREPfest: Cargese, 21 September 2010

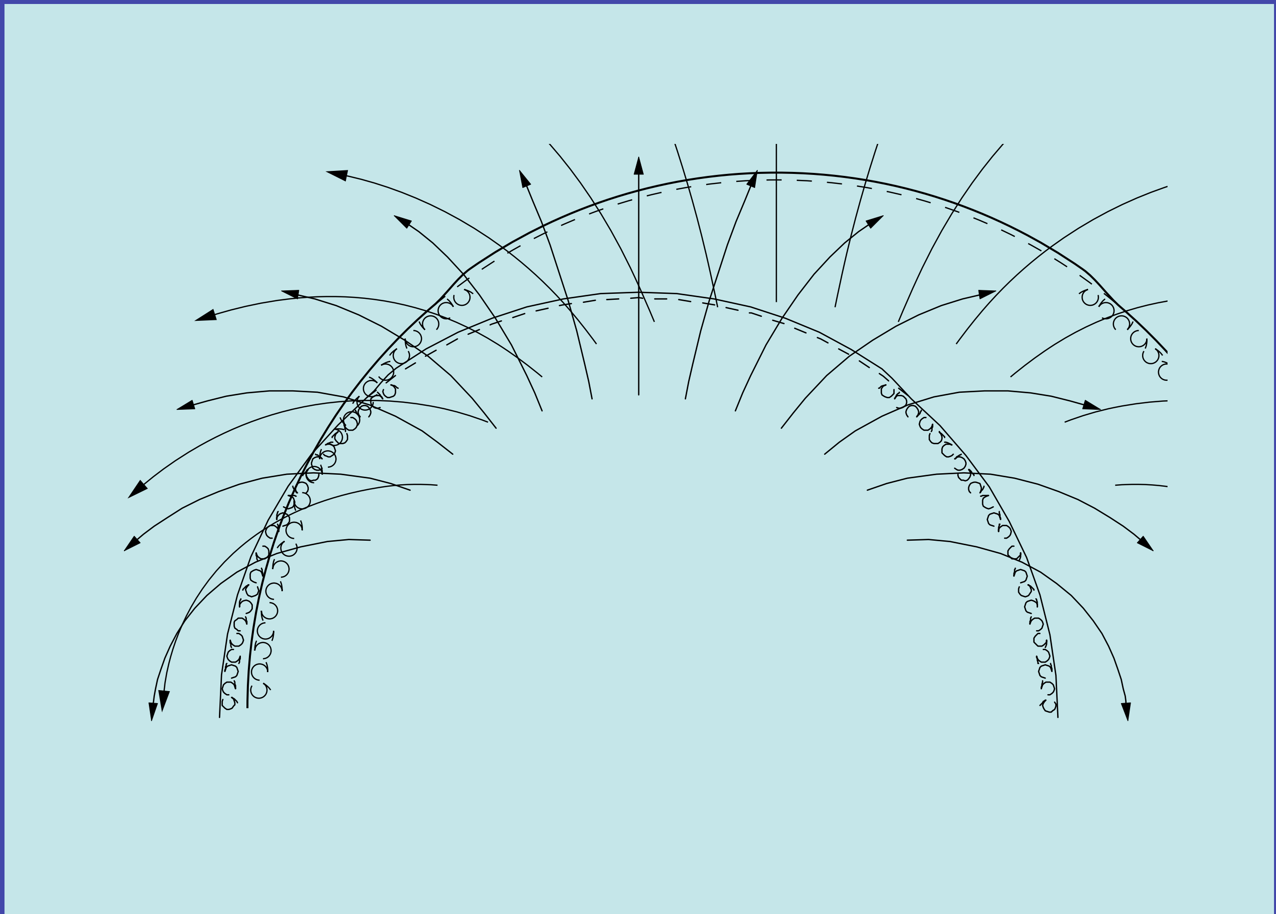
Photosphere



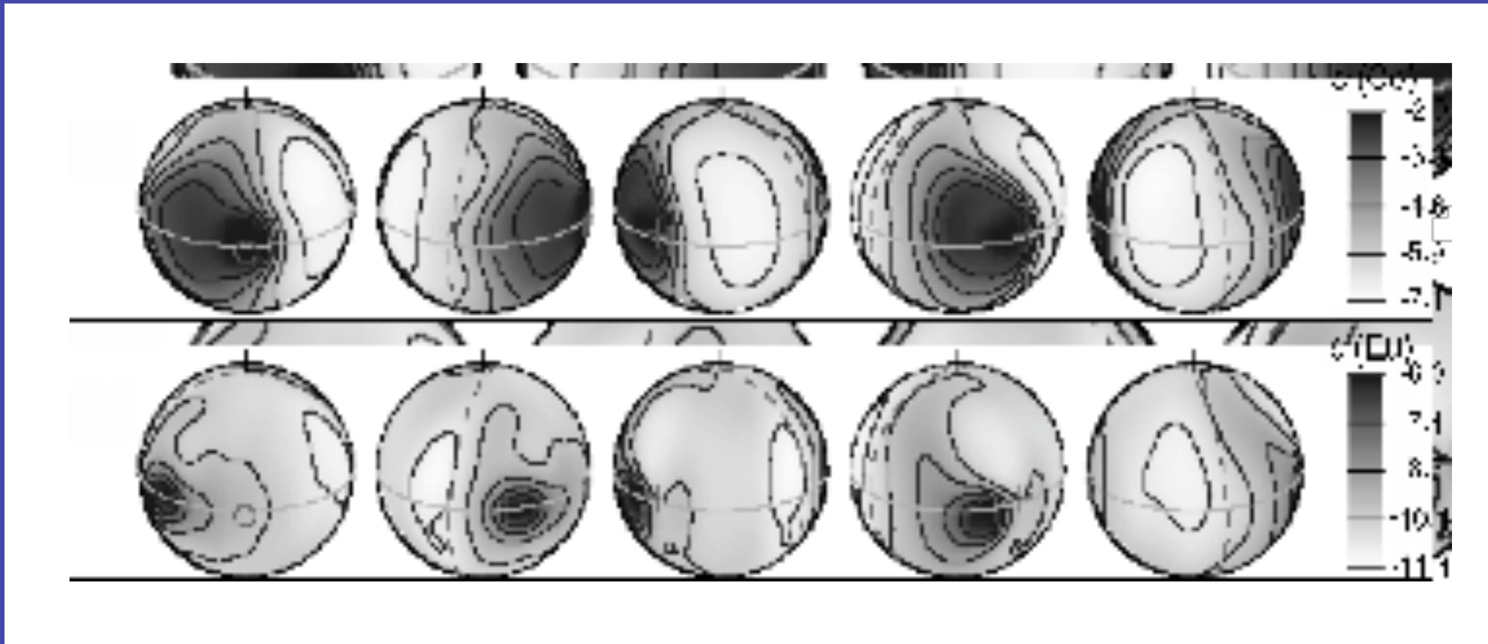
An roAp star







A Doppler-imaged roAp star



$$\psi_{nlm}(r,\theta,\phi,t) = \Psi_{nl}(r)\ P_l^m(\cos\theta)\cos(m\phi - \omega t)$$

$$\mathcal{L}\Psi_k + \mathcal{R}\Psi_k - \lambda_k\Psi_k = 0$$

where

$$\lambda_k = \omega_k^2$$

$$\Psi_k = \sum_m a_{km}\Psi_{nlm}^{(0)}$$

$$\mathcal{L}\Psi_{nlm}^{(0)} - \lambda_k^{(0)}\Psi_{nlm}^{(0)} = 0$$

$$\mathcal{R}\boldsymbol{a}_k - \lambda_k^{(1)}\boldsymbol{a}_k = \boldsymbol{0}$$

$$\mathcal{R}_{m'm} = \langle\Psi_{nlm'}^{(0)}, \mathcal{R}\Psi_{nlm}^{(0)}\rangle$$

$$\mathcal{R} = -C\Omega\mathrm{A} + \omega_{\Omega 2}\mathrm{B}$$

where, for dipole modes,

$$\mathrm{A} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathrm{B} = \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}$$

$$\mathcal{L}\Psi_k + \mathcal{R}\Psi_k - \lambda_k \Psi_k = 0$$

where

$$\lambda_k = \omega_k^2$$

$$\Psi_k = \sum_m a_{km} \Psi_{nlm}^{(0)}$$

$$\mathcal{L}\Psi_{nlm}^{(0)} - \lambda_k^{(0)} \Psi_{nlm}^{(0)} = 0$$

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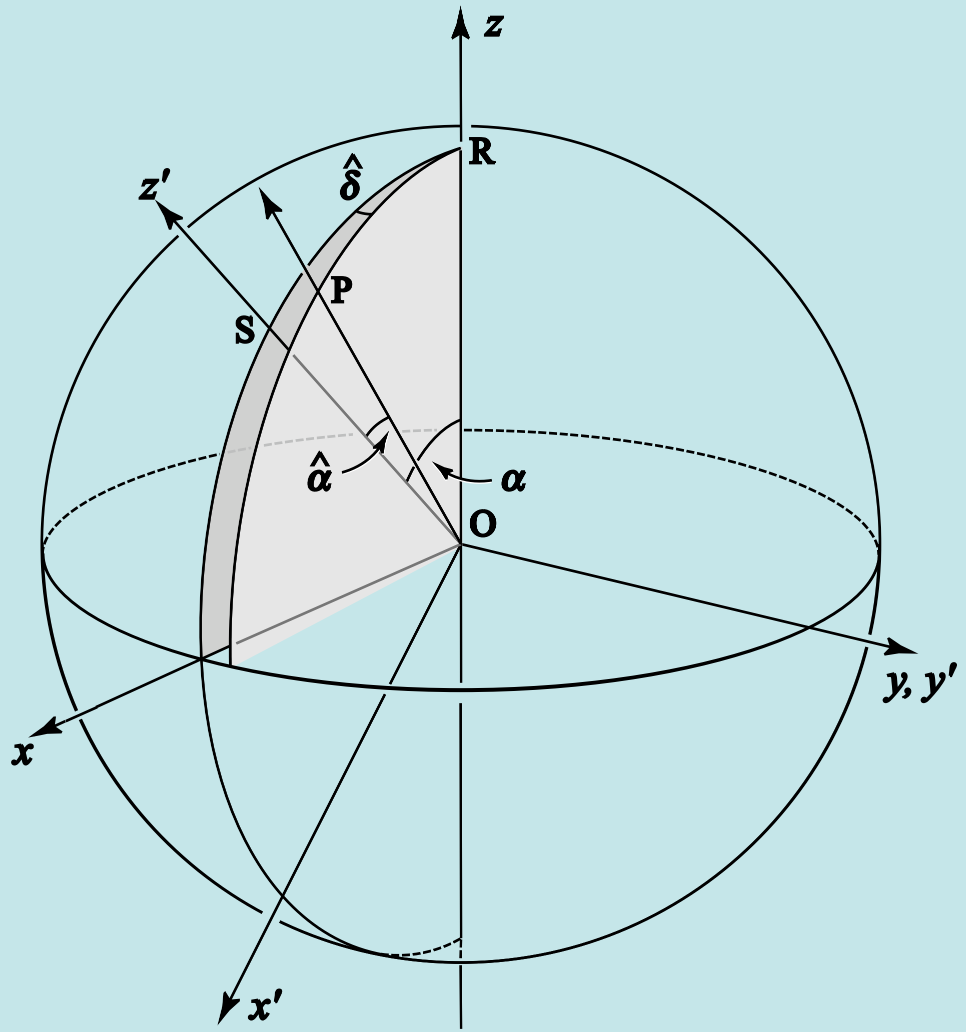
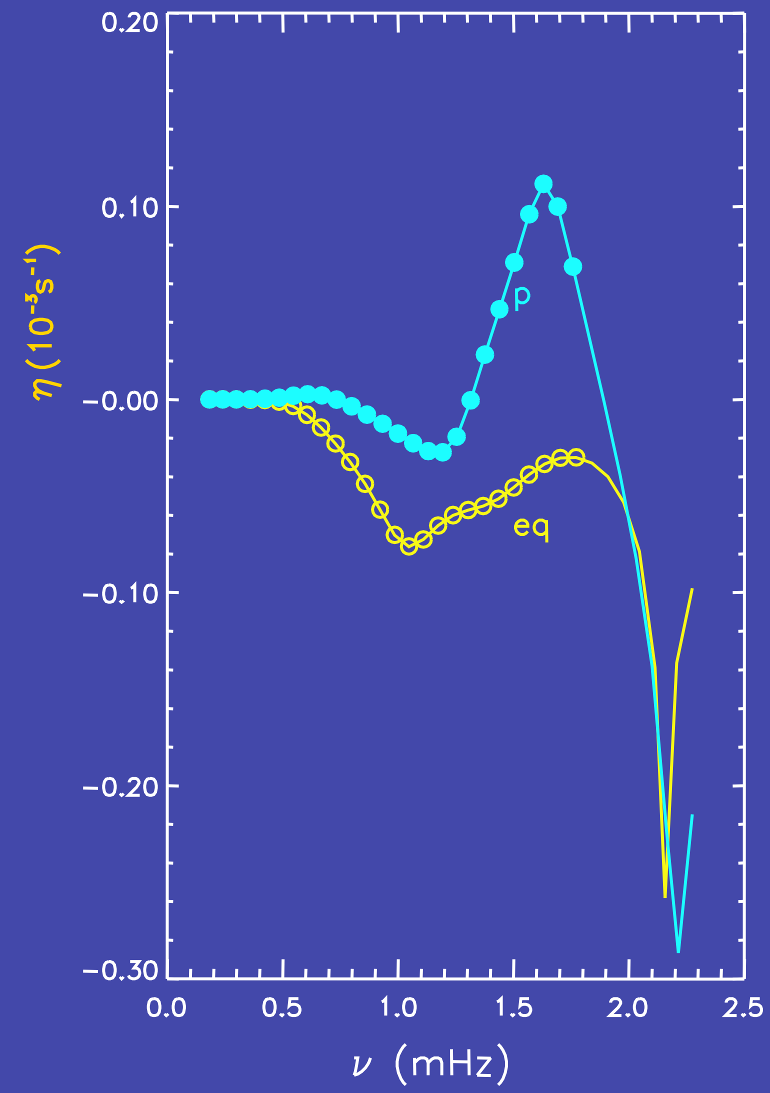
$$\mathcal{R}_{m'm} = \langle \Psi_{nlm'}^{(0)}, \mathcal{R}\Psi_{nlm}^{(0)} \rangle$$



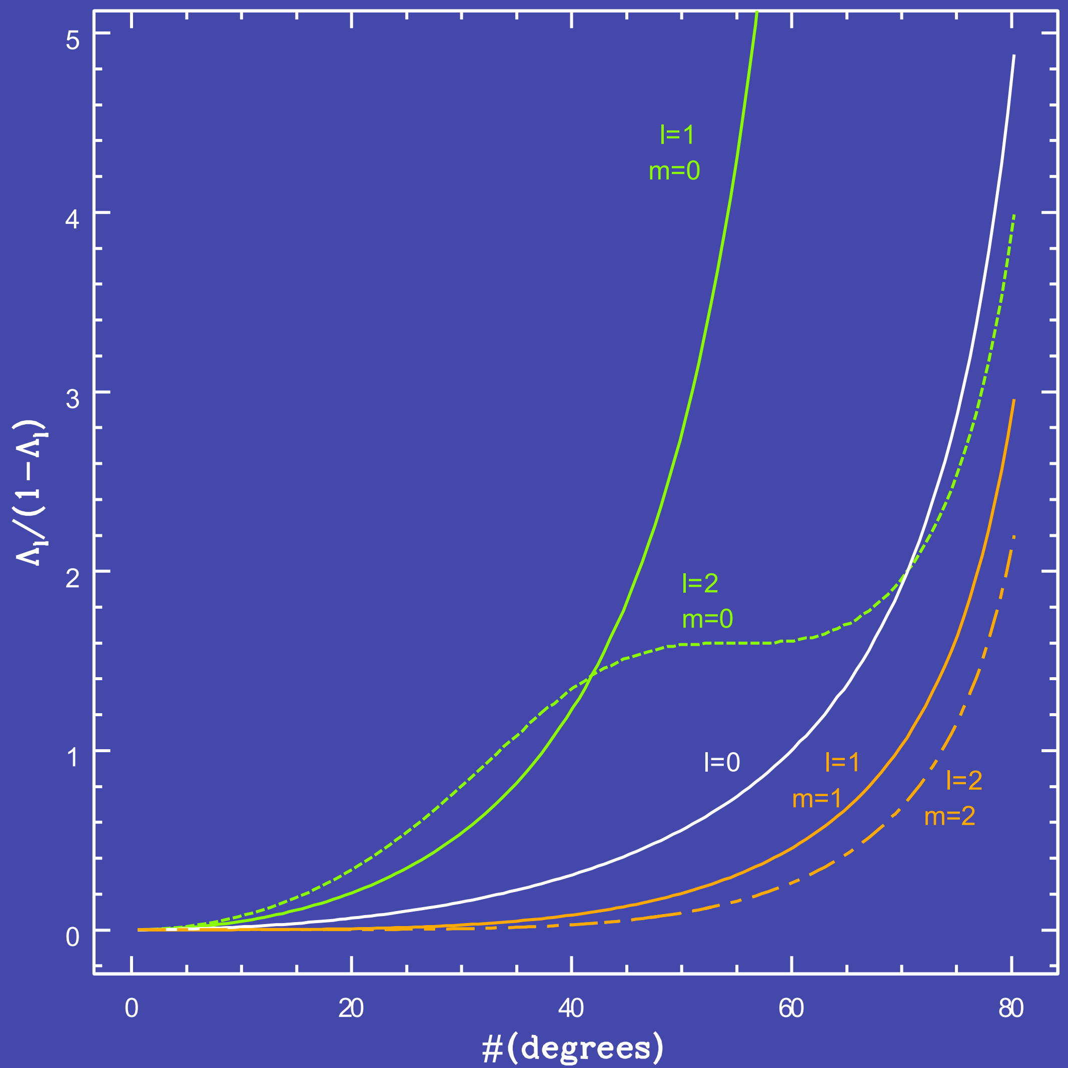
z
R
$\hat{\delta}$
z′
P
S
$\hat{\alpha}$
α
O
y, y′
x
x′

Contributions to the growth rate η



Growth rates: relative polar contribution Λ_l and the effect of spot size ϑ

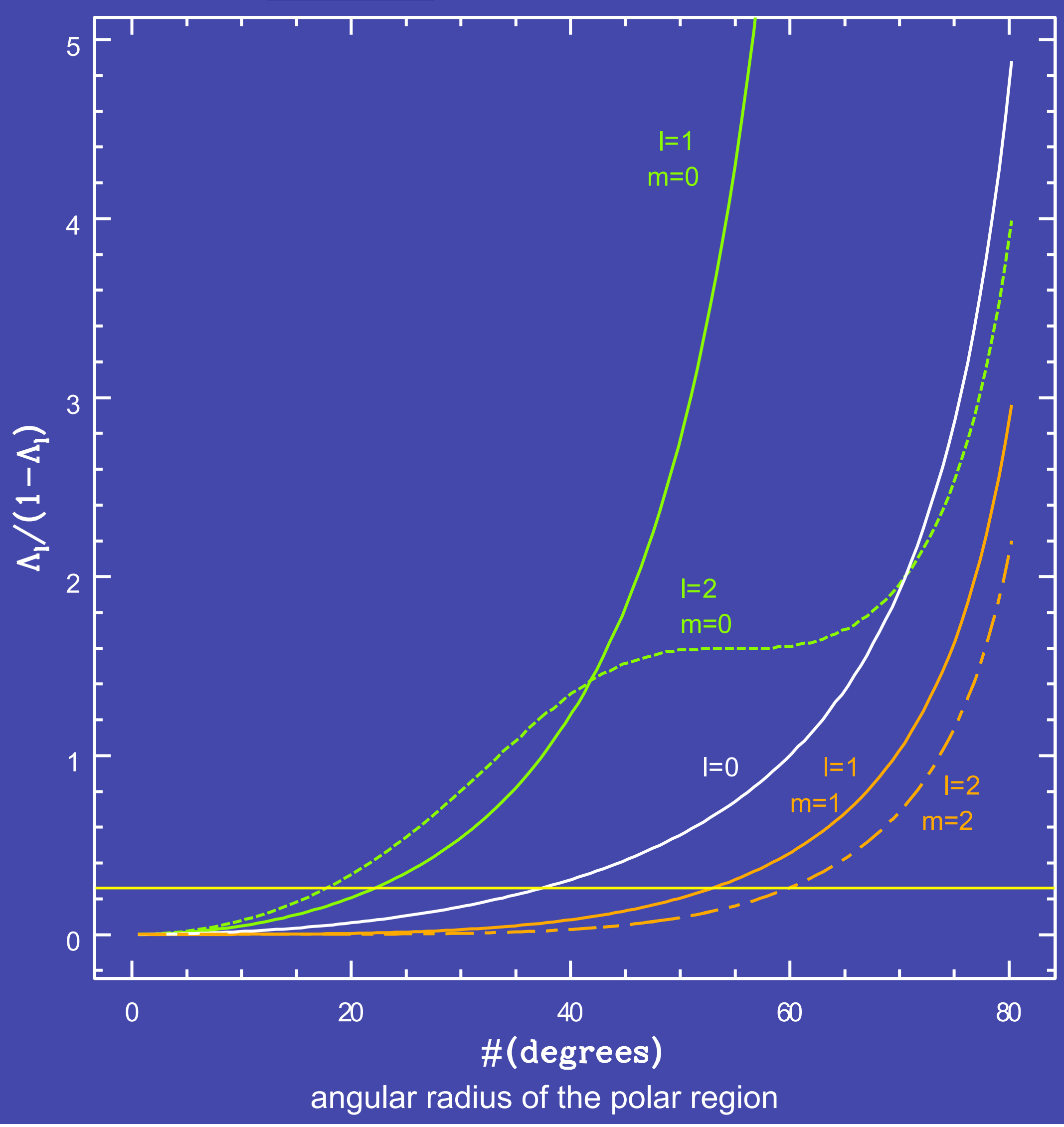
The larger $\Lambda_1/(1-\Lambda_1)$, the greater the effect of polar excitation

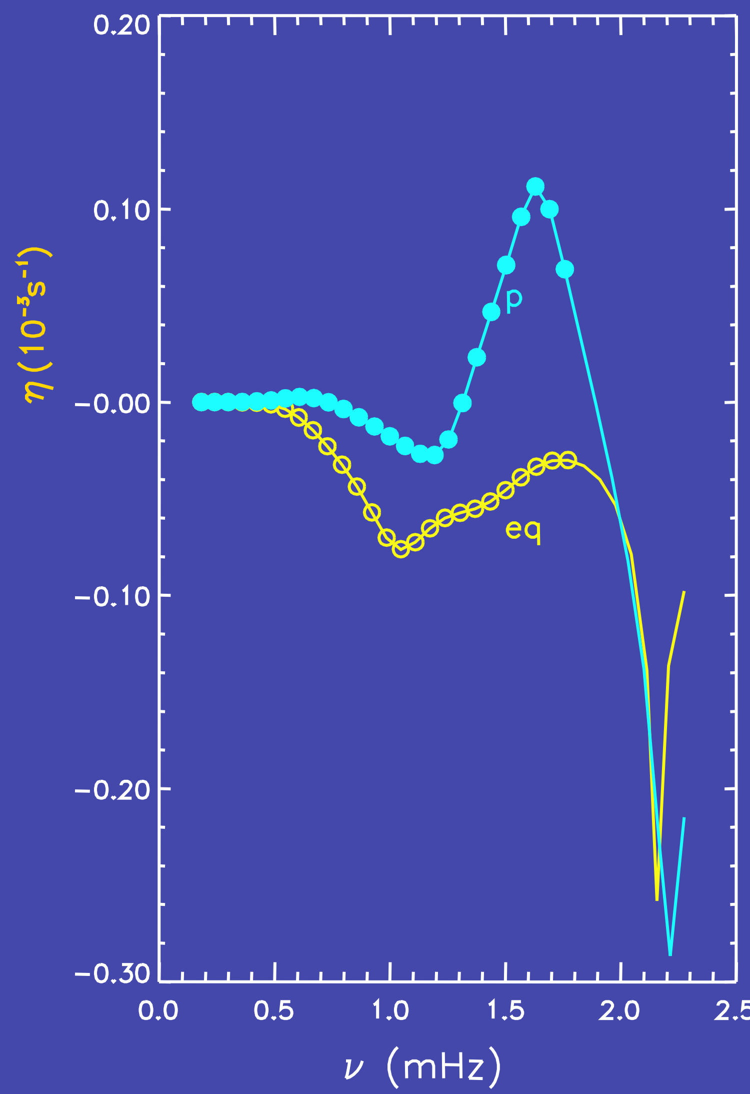


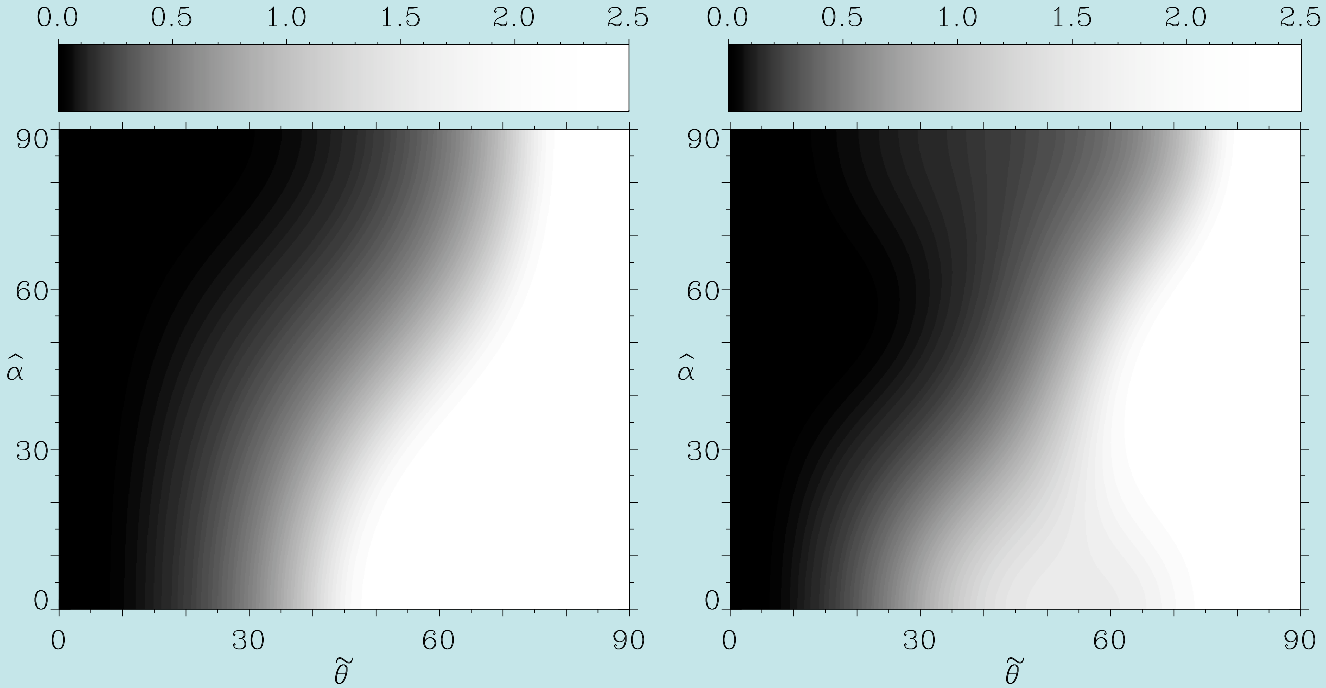
angular radius of the polar region

Growth rates: relative polar contribution Λ_l and the effect of spot size

The larger $\Lambda_l/(1-\Lambda_l)$, the greater the effect of polar excitation

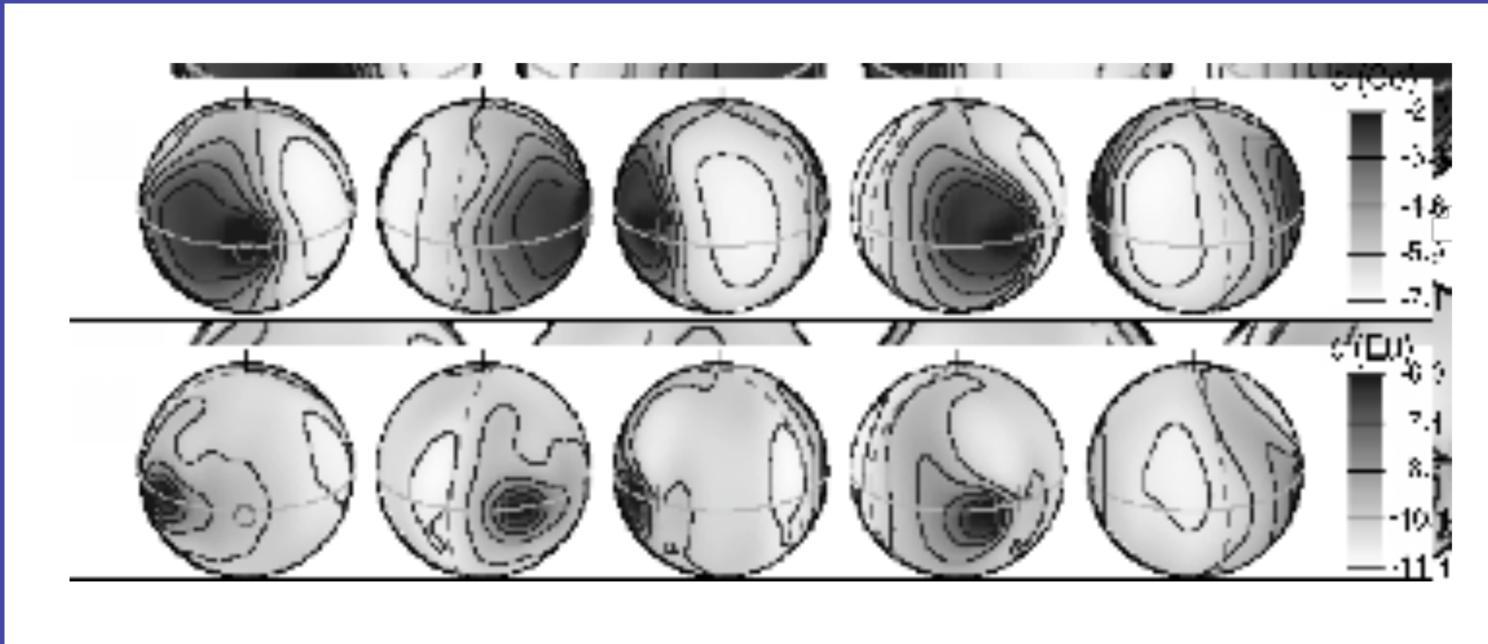




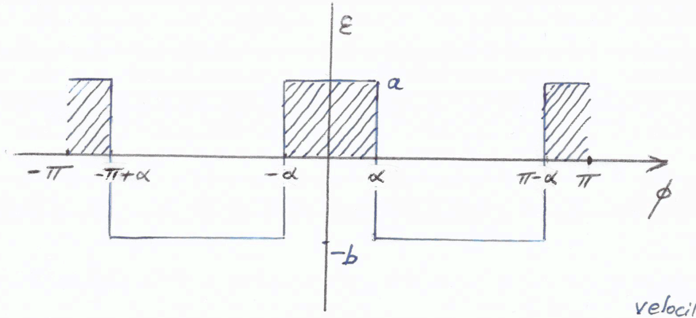


0.0 0.5 1.0 1.5 2.0 2.5
90
60
30
0
$\hat{\alpha}$
0 30 60 90
$\tilde{\theta}$
0.0 0.5 1.0 1.5 2.0 2.5
90
60
30
0
$\hat{\alpha}$
0 30 60 90
$\tilde{\theta}$

A Doppler-imaged roAp star



Model of roAp-star wave



velocity u
pressure p
density ρ

$$\rho \frac{Du}{Dt} = -\frac{\partial p'}{\partial \phi} \qquad \frac{D}{Dt} = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi}$$

precession rate Ω

$$\frac{D\rho'}{Dt} + \rho \frac{\partial u}{\partial \phi} = 0$$

f' = Eulerian fluctuation of f

δf = Lagrangian fluctuation of f

$$\delta p = c^2 \delta\rho + p\left(\frac{\partial \ln p}{\partial s}\right)_\rho \delta s$$

$$= c^2\delta\rho + (\gamma_3 - 1)\rho T \delta s$$

$$\frac{d\delta f}{dt} = \frac{Df'}{Dt}$$

$$c^2 = \gamma_1 \frac{p}{\rho} \qquad \gamma_1 = \left(\frac{\partial \ln p}{\partial \ln \rho}\right)_s$$

$$\gamma_3 - 1 = \left(\frac{\partial \ln T}{\partial \ln \rho}\right)_s$$

$$T\frac{Ds'}{Dt} = E\left(\frac{\delta\rho}{\rho} + \eta\frac{\delta T}{T}\right)$$

$$= \Lambda(\phi)\frac{\delta\rho}{\rho}$$

$$\Lambda = [1 + (\gamma_3 - 1)\eta]E$$

quasiadiabatic approximation

$$\boxed{\frac{D}{Dt}\left(\frac{D^2\rho'}{Dt^2} - c^2\frac{\partial^2\rho'}{\partial\phi^2}\right) = 2c^2\frac{\partial^2}{\partial\phi^2}(\varepsilon\rho')}$$

$$\begin{aligned}\varepsilon(\phi) &= \frac{\gamma_2 - 1}{2c^2}\Lambda(\phi)\\ &= \frac{\gamma_2-1}{2c^2}\left[1+(\gamma_2-1)\eta\right]E(\phi)\end{aligned}$$

Approximate solution

$$\begin{aligned}\rho'(x,t) &\simeq A(t)\cos\omega t\cos[m(\phi - \Omega t)]\\ &\equiv A(t)\,\psi(\phi, t)\end{aligned}$$

$m = \frac{\omega}{c}$ integer

A slowly varying: $\quad |\ddot{A}\psi| \ll \left|\dot{A}\frac{D\psi}{Dt}\right|$

$$\frac{\dot{A}}{A} = \frac{\frac{\partial^2}{\partial\phi^2}(\varepsilon\psi)}{\frac{\partial^2}{\partial\phi^2}\psi}$$

$$\varepsilon = \begin{cases} +a & |\phi| < \alpha \\ & |\pi - \phi| < \alpha \\ -b & \text{otherwise}\end{cases}$$

Approximate (local) dispersion relation: $(\rho' \propto e^{-i\omega t})$

$$\omega \simeq \pm mc + i\varepsilon$$

m integer

ε = growth rate

$$\boxed{\frac{D}{Dt}\left(\frac{D^2\rho'}{Dt^2} - c^2\frac{\partial^2\rho'}{\partial\phi^2}\right) = 2c^2\frac{\partial^2}{\partial\phi^2}(\varepsilon\rho')}$$

$$\varepsilon(\phi) = \frac{\gamma_2 - 1}{2c^2}\Lambda(\phi)$$

$$= \frac{\gamma_2 - 1}{2c^2}\left[1 + (\gamma_2 - 1)\eta\right]E(\phi)$$

Approximate solution

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$$\frac{\dot{A}}{A} = \frac{\frac{\partial^2}{\partial\phi^2}(\varepsilon\psi)}{\frac{\partial^2}{\partial\phi^2}\psi}$$

$\varepsilon = +a$ $\quad |\phi| < \alpha$

$|\pi - \phi| < \alpha$

$-b$ otherwise

$$A(t) \simeq A_0 \exp\left[-\lambda(t-t_0) + \mu\left\{ \sin[2m(t-t_0) + 2m\phi_0] - \sin(2m\phi_0) \right\} \right]$$

$$\lambda = b - 2(a+b)/\pi$$

$$\mu = \frac{a+b}{2m^2\Omega\pi} \sin(2m\alpha)$$

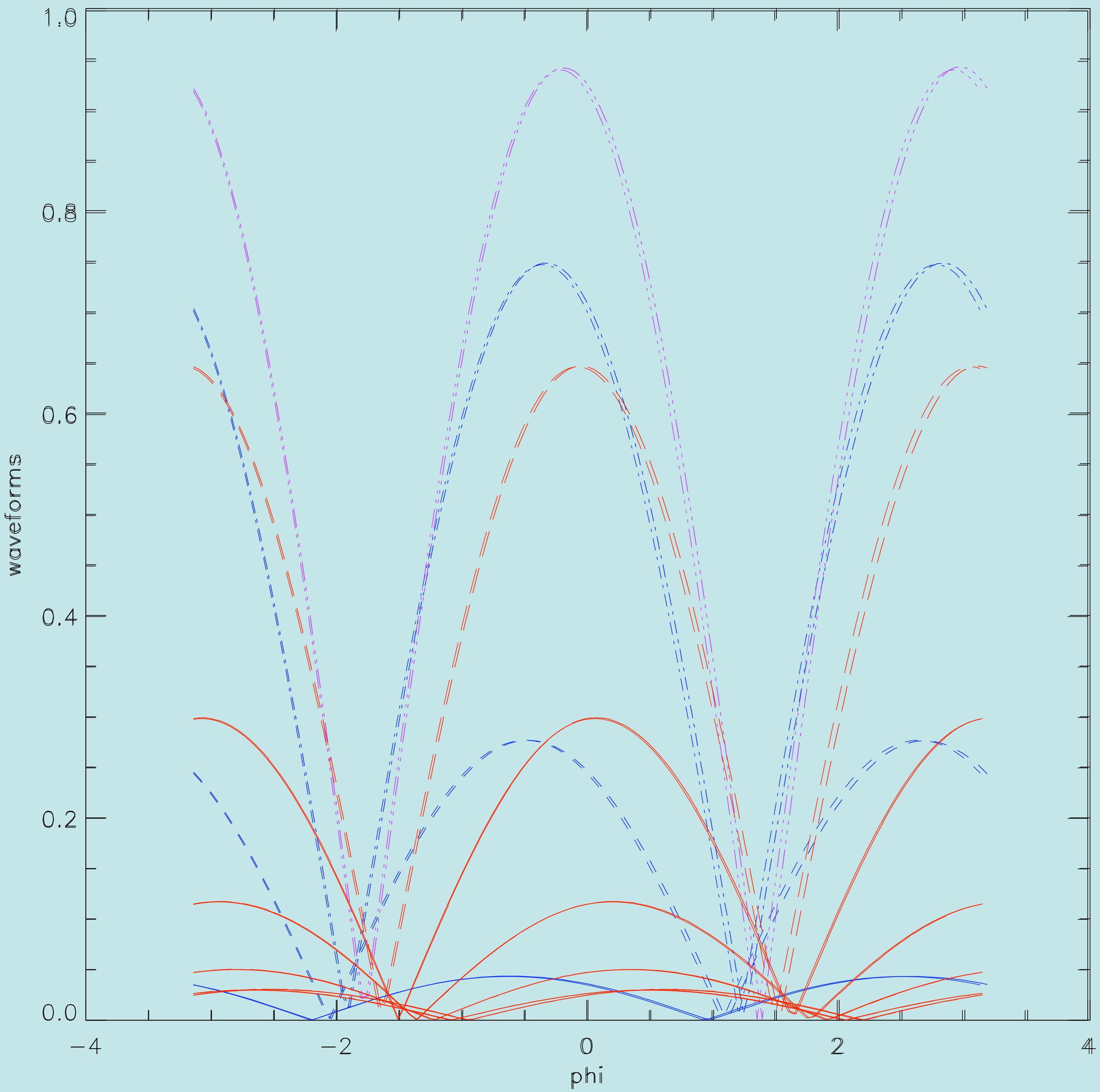
$A(t)$ reaches a (single) maximum when

$$t = t_m = -\frac{\phi_0}{\Omega} + \frac{1}{2m\Omega} \cos^{-1}\left(\frac{\lambda}{2m\Omega\mu} \right)$$

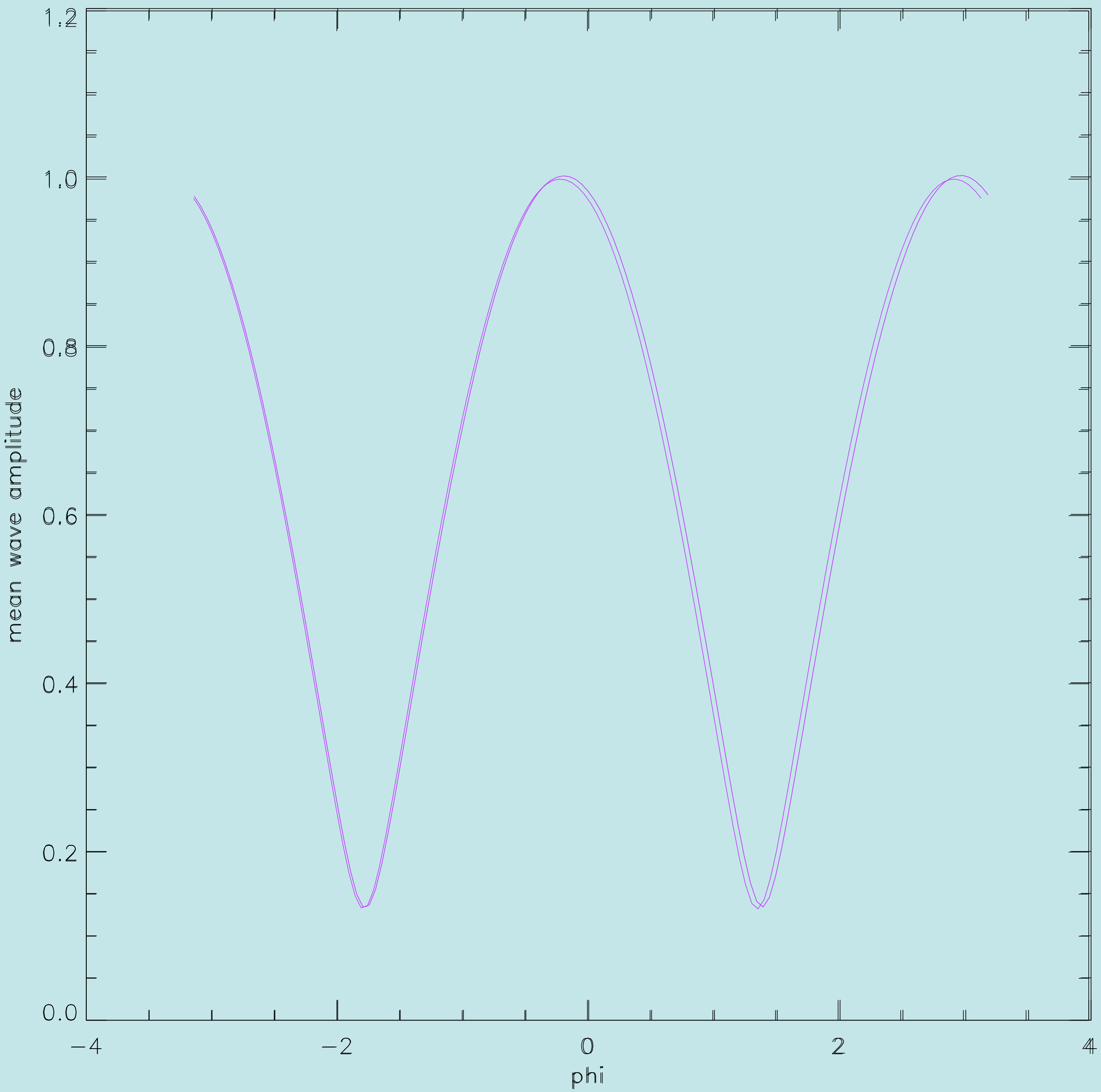
provided $\lambda < 2\Omega m\mu$

i.e. $a > \left(\frac{\pi}{\beta} - 1 \right) b$

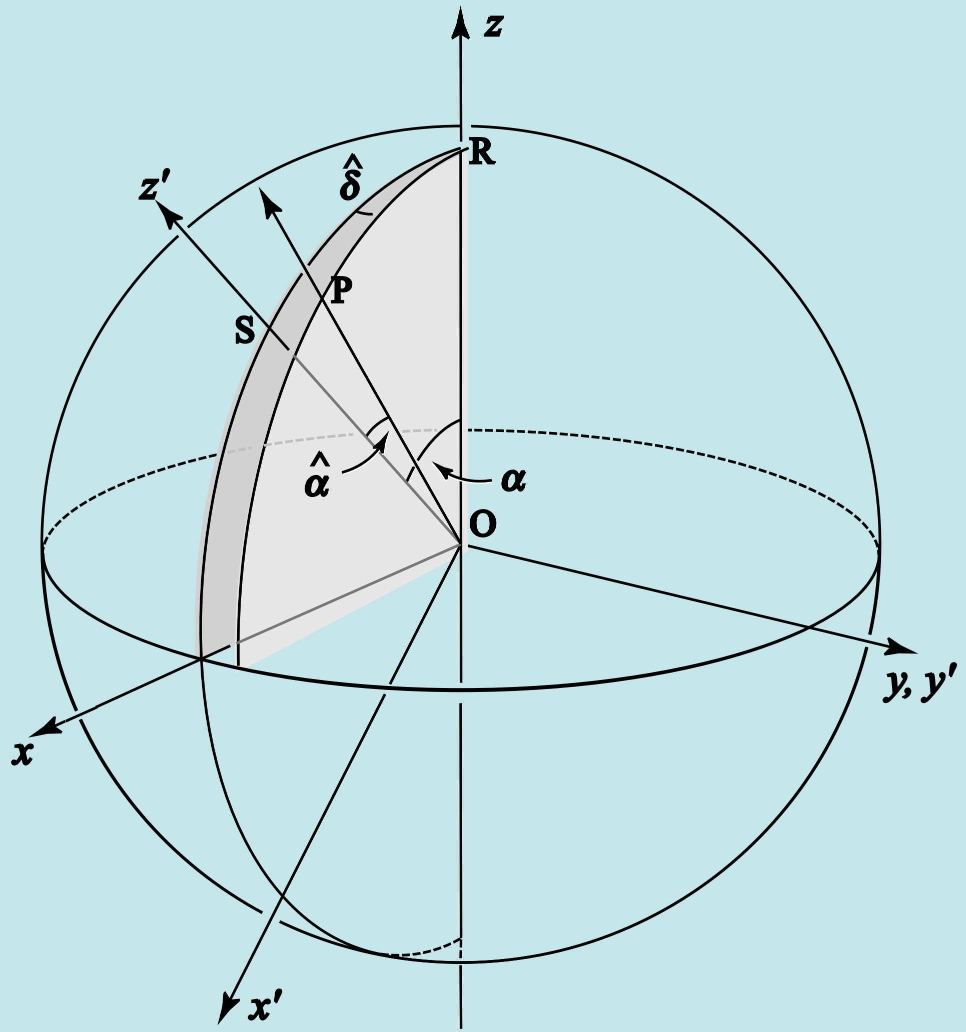
where $\beta = \frac{\sin(2m\alpha)}{m} + 2\alpha$



1.0
0.8
0.6
0.4
0.2
0.0
waveforms
−4
−2
0
2
4
phi



mean wave amplitude
phi
0.0
0.2
0.4
0.6
0.8
1.0
1.2
−4
−2
0
2
4



z
R
$\hat{\delta}$
z′
P
S
$\hat{\alpha}$
α
O
y, y′
x
x′

$$\nabla^2\Psi + \frac{\omega^2}{c^2}\Psi = 0 \qquad \text{in } \mathcal{V}$$

$$\mathbf{n}\ .\nabla\Psi + \alpha\Psi = 0 \qquad \text{on } \mathcal{S}$$

$$\omega^2 = \frac{\int_{\mathcal{V}} |\nabla\Psi|^2 \mathrm{d}V + \int_{\mathcal{S}} \alpha|\Psi|^2 \mathrm{d}S}{\int_{\mathcal{V}} c^{-2}|\Psi|^2 \mathrm{d}V}$$

$$\frac{\Delta\omega}{\omega} = \frac{\int_{\mathcal{S}} \Delta\alpha|\Psi|^2 \mathrm{d}S}{2\omega^2 \int_{\mathcal{V}} c^{-2}|\Psi|^2 \mathrm{d}V}$$