ABC Flows Then and Now

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Outline

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Fast dynamos and the ABC flows

Vainsteiin and Zeldovich 1972: two sorts of kinematic dynamo

Fast dynamos grow on the turnover time of the flow

- Slow dynamos grow on the diffusion timescale of the fluid object
- The latter is prohibitively long in most astrophysical situations, if a laminar diffusivity is used, so fast dynamos are required.
- Stretch, twist, fold cycle gives dynamo, whose field strength grows by a factor 2 each time --- fast dynamos exist, no problem!

Chaos stretches line elements of a flow exponentially fast and since magnetic fields are like line elements, chaotic flows should be good candidates for fast dynamos (chaos subsequently proved to be necessary by Klapper and Young 1995).

ABC flows are chaotic (Arnold 1965, Hénon 1966)

 $\mathbf{u} = (A \sin z + C \cos y, B \sin x + A \cos z, C \sin y + B \cos x)$

so how about testing these for dynamo action?

Most work since has concentrated on the A=B=C=1 case, following Arnold and Korkina (1984)

Galloway and Frisch (1984, 1986) used state-of-the-art (!) Cray-1 computers to produce the following results:





2. 1991: The infamous flow

Frustrations of trying to reach high Rm in 3-D: no chaos possible in 2-D

But...2-D + time-dependence can be chaotic

Try

u = (A sin($z + \cos \omega t$) +C cos($y + \sin \omega t$), A cos ($z + \cos \omega t$), C sin ($y + \sin \omega t$))

Modes proportional to exp(ikx) are uncoupled

(G.O. Roberts idea); see also Otani flow.

Another idea: 1:1:1 ABC with cosines omitted (sines flow, aka Kolmogorov flow)---twice as fast to compute





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3. A spherical version (Rainer Hollerbach, MREP, DJG)

Similar phasing ideas can be used in a spherical geometry---2.5D spherical shell with coupling between nearest-neighbour Legendre polynomials, four cell flow. Los Alamos Cray T3D took us up to Rm=500,000 by which time fastness began to look plausible.







4. Filamentary vs non-filamentary dynamos: adding dynamics

First write down the governing equations for an incompressible flow driven by a prescribed force field $\mathbf{F}(\mathbf{r}, [t])$:

$$\frac{\langle \mathbf{H} |}{\langle \mathbf{H} \rangle} 2 \langle \mathbf{H} | \mathbf{H} | \mathbf{H} \rangle \langle \mathbf{H} | \mathbf{H} \rangle \langle \mathbf{H} | \mathbf{H} \rangle \langle \mathbf$$

where we will use scaled units such that KT/Re, =1/Rm.

Classic filamentary: (1, 1, 1) ABC dynamo

 $F = v (\sin z + \cos y, \sin x + \cos z, \sin y + \cos x)$

Taking Re=5, Rm=400, at two times where the flow has become statistically steady. The total magnetic and kinetic energies are are of the





But...this is at low Re (chosen so underlying ABC flow wants to be stable). Does similar behaviour exist at high Re? Answer: no (numerically shown by Galanti, Pouquet & Sulem 1992).

Scaling argument (Galloway 2003): assume upper bound for ohmic dissipation of magnetic field is viscous dissipation in the absence of any field. Then

 $\frac{\text{Total ME1}}{\text{Total KERe}} - (\text{filaments of thickness Rm}^{-1/2})$ $\frac{\text{Total ME1}}{\text{Total KERe}} - \frac{\varphi \mathcal{K}}{\frac{1}{2}} \varphi^{1/2} \quad (\text{filaments of order 1 radius with Rm}^{-1/2} \text{ ges})$

This is bad news. Same conclusion reached by Brummell, Cattaneo & Tobias (2001), for time-dependent ABC forcing.

5. Interesting "toys" - the Archontis dynamo

Take \mathbf{F} = the viscosity v times (sin z, sin x, sin y). A dynamo results, known as the Archontis dynamo. At high Reynolds numbers this has **u** almost equal to **B** (or to -B). Both are close to half of the applied force, but remain distinct from this in the limit that the diffusivities tend to zero (Archontis 2000 PhD thesis; Cameron & Galloway MNRAS 2006; Archontis, Dorch & Nordlund 2007; Gilbert, Ponty & Zheligovsky 2010).





Evolution of KE and ME starting from small seed field. The upper curve shows the evolution of the cross-helicity, which is the integral of **U.B** over the box.

Tubes around heteroclinic orbits

Isosurface of |u-B| (0.75 of max)

Results for Re=Rm=200



A scaling argument: dynamos to order

Suppose we have any steady solution $\mathbf{B}_{\mathbf{0}}$ to the induction equation when solved with a velocity field U_0 and a magnetic diffusivity η_0 . We can now generate an equilibrium solution to the whole dynamo problem (including the momentum equation) for $\eta = \varepsilon \eta_0$. This is $U_1 = \varepsilon U_0 + B_0$, $B_1 = B_0$, $\mathbf{F} = \varepsilon^2 \mathbf{U}_0 \cdot \mathbf{W} \mathbf{U}_0 - \varepsilon \mathbf{V} \mathbf{W}^2 \mathbf{U}_0 + 2\varepsilon \mathbf{B}_0 \cdot \mathbf{W} \mathbf{U}_0$. This dynamo has the property that U tends to B as the diffusivity tends to zero. Note the stability of the resulting object is uncertain. However, experimentation with the Archontis dynamo shows that this idea works and that the results are often stable, with heuristic arguments to support this. Friedlander and Vishik have shown that the ideal MHD case is neutrally stable.

...Eg more ABC

We begin with a dynamo which is driven by an external forcing

$$\mathbf{F}_{\gamma} = \nu \begin{pmatrix} A \sin z + C\gamma \cos y \\ B \sin x + A\gamma \cos z \\ C \sin y + B\gamma \cos x \end{pmatrix}.$$
(9)

When $\gamma = 1$ this force is the well-studied ABC forcing (Galanti et al. 1992), and when $\gamma = 0$ and A = B = C it is the forcing for the Archontis dynamo. The former case, although well studied, is undesirable for the current study because the ABC flow is Beltrami: with $U \times \nabla \times U = 0$. Since $U \sim B$ dynamos involve a balance between $U \times \nabla \times U$ and $B \times \nabla \times B$ in the momentum equation, Beltrami flows form a special case. To have a system with less symmetry we have taken $\gamma = 0.125$ and A : B : C = 0.5 : 1 : 1. Results using this forcing are described in section 3.1, and in particular when $\eta = \nu = 1/100$ we find a stable, time-independent solution.

Some new work: the issue of low magnetic Prandtl number

The magnetic Prandtl number $p_m = v/\eta$, the ratio of viscous to magnetic diffusivity.

Various authors (Boldyrev, Cattaneo, and others at KITP Santa Barbara 2008 "Dynamo Theory") have claimed that dynamos cannot function when this ratio is low as inside the Sun and other stars. These worries arise from the use of turbulent models, usually involving the mean field approximation.

The Archontis dynamo works with $u = B \approx (\sin z, \sin x, \sin y) / 2$

at
$$v = 1/100, \eta = 1/400 (p_m = 4)$$

 $v = 1/100, \eta = 1/100 (p_m = 1)$
 $v = 1/400, \eta = 1/100 (p_m = 1/4)$

Consider v = 1/100 and increase η , thus making p_m smaller, but maintaining the forcing proportional to v. A general result from dynamo theory is that a dynamo cannot function when the magnetic Reynolds number Rm = UL/ η falls below a certain (flow-dependent) critical value – this is called the Backus bound. We find the Alfvénic dynamo persists up till n = 1/13, at which level the diffusion is big enough to prevent field growth, and the magnetic field dies out. However, the solution with no magnetic field now fails to satisfy the momentum equation and the flow becomes turbulent. We intend to study hysteresis effects and find out more about this. (DJG and David Lewis, in preparation)

6. Astrophysical relevance?

Are Alfvénic dynamos freaks? Or could they actually arise in astrophysical objects?

Basic idea: magnetic field and velocity are aligned and equal in magnitude.

But alignment is a slow (diffusion timescale) process, and typical boundary conditions involve non-alignment.

Solar dynamo: attempt to make persistent long-timescale **U**=+/-**B** dynamo in tachocline which feeds oppositely directed fields to base of convection zone every 11 years (Cameron and Galloway, in preparation---see KITP 2008 Dynamo Theory web site).

Some evidence from geodynamo and ASHmob simulations that strong field dynamos can emerge naturally... ... With thanks to collaborators MREP60, Uriel Frisch, Olga Podvigina, Rainer Hollerbach, Robert Cameron and David Lewis...