Essentially nonlinear dynamos

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"On the generation of organised magnetic fields", Tobias, Cattaneo & Brummell, submitted 2010

-- Mathematically complex -- somewhat obscure -- almost impossible to follow

=> very Proctor-esque!

Large-scale(?) fields

Astrophysical motivation leads us to an interest in the creation of "large-scale" magnetic fields.

What do we mean by "large-scale"?

- ✓ Not completely random
- ✓ High degree of spatio-temporal organisation
- ✓ Spatial scales comparable to the object itself

e.g. Sun: *Large-scale (compared to Sun)* magnetic activity cycle

- ✓ Active region emergence and laws
- ✓ Signature of large-scale toroidal field

BUT also *small-scale (compared to object)* activity

- ✓ Still strong (signed) flux
- ✓ Mostly independent of cycle

Large-scale(?) fields: Mean Field Theory

Mainly due to mean-field theory, “large” and “small” has come to mean something slightly different:

Large-scale: magnetic field on scales much larger that the typical velocity scale

Small-scale: magnetic fields comparable or smaller than the typical velocity scale.

Clearly, mean field theory is a two-scale approach based on exactly the separation of these two scales.

System-scale fields

BUT mean field theory is a KINEMATIC concept.

What happens when the Lorentz force becomes involved and the situation is *DYNAMIC* and fully *NONLINEAR*?

– unless "large-scale" magnetic fields are force-free, there will be an accompanying velocity at the same scale!

– and then magnetic fields are not LARGE any more!

So, things are not so clear in the nonlinear context because the background flows get adjusted. (In the kinematic context, they cannot be adjusted)

Maybe more appropriate in the nonlinear context, to ask whether a field is generated at the largest available scale:

✓ a SYSTEM SCALE DYNAMO

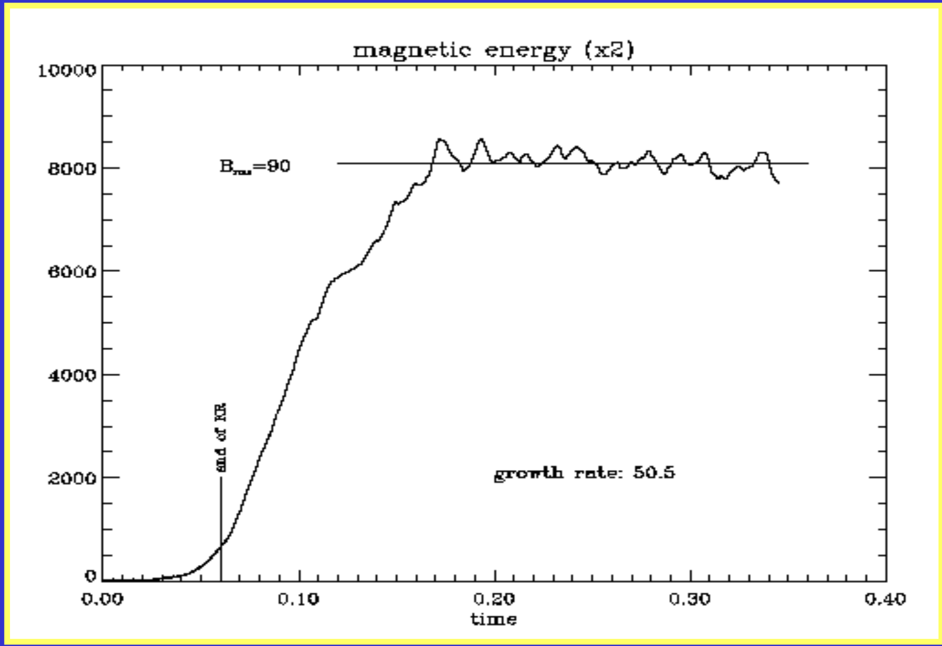
"Essentially kinematic" dynamos

Clearly, all depends on the role of the Lorentz force.

<u>Traditional view of dynamos:</u>

✓ Initial instability leads to kinematic growth of a linear eigenfunction

✓ Magnetic field grows until nonlinear interactions important, then Lorentz terms adjust the flow to saturate the growth.



e.g. Cattaneo 1999: turbulent Boussinesq convective dynamo

"Essentially kinematic" since works in the limit of vanishingly small initial field. Initial growth independent of initial magnetic field. Lorentz forces NOT important initially.

These types of dynamos have been relatively unsuccessful at producing large-scale organised field! Lack of success usually attributed to the fact that we cannot get to high enough Rm.

“Essentially nonlinear” dynamos

An alternative: *ESSENTIALLY NONLINEAR DYNAMOS*

The opposite extreme:

- ✓ Everything is driven by Lorentz forces, even initially.
- ✓ The very flows that are “dynamo flows” (i.e. amplify magnetic field, convert between components) are driven by the presence of the magnetic field
- ✓ Magnetic forces not only saturate growth but also CAUSE growth
- ✓ Requires the intervention of a finite size magnetic field (to create instability, drive flows nonlinearly)
- ✓ Does NOT WORK in the limit of vanishingly small magnetic field

THESE EXIST:

The following is an example that we found a while ago …

e.g. Shear-buoyancy dynamo

Interaction of seed field and velocity shear leading to magnetic buoyancy instability

Build one strong magnetic structure via shear

Have initial field that will diffuse if no dynamo

Velocity shear:

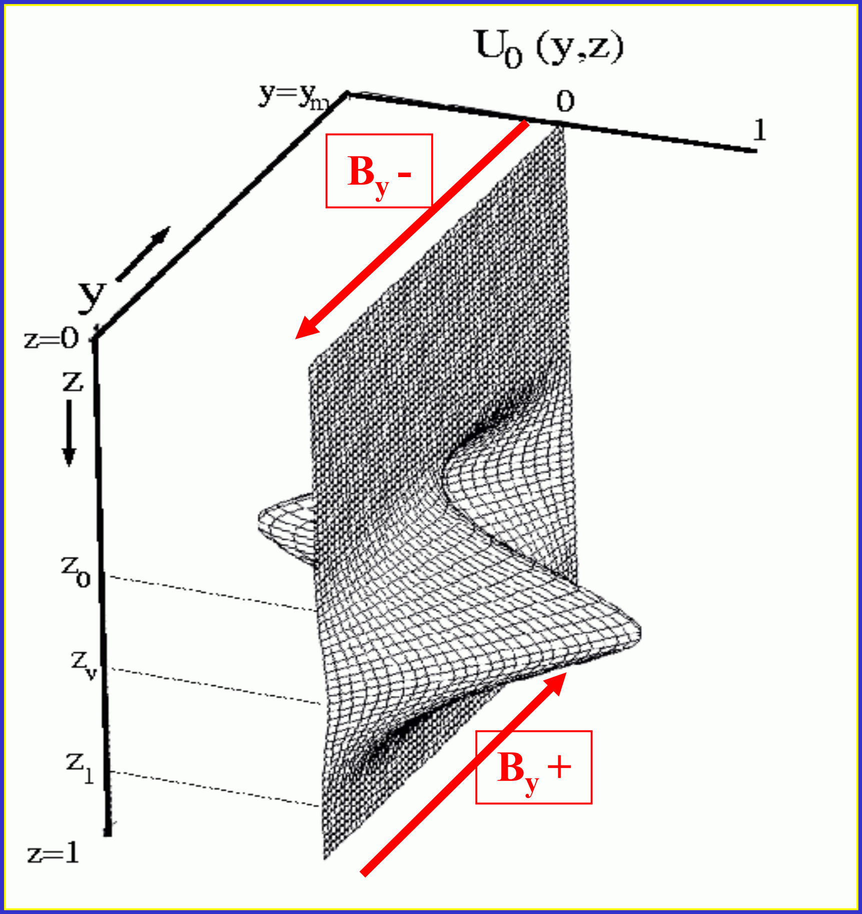
U(y,z) = f(z) [sawtooth(y)]

Magnetic field:

$$B_0=(0,B_y,0)$$

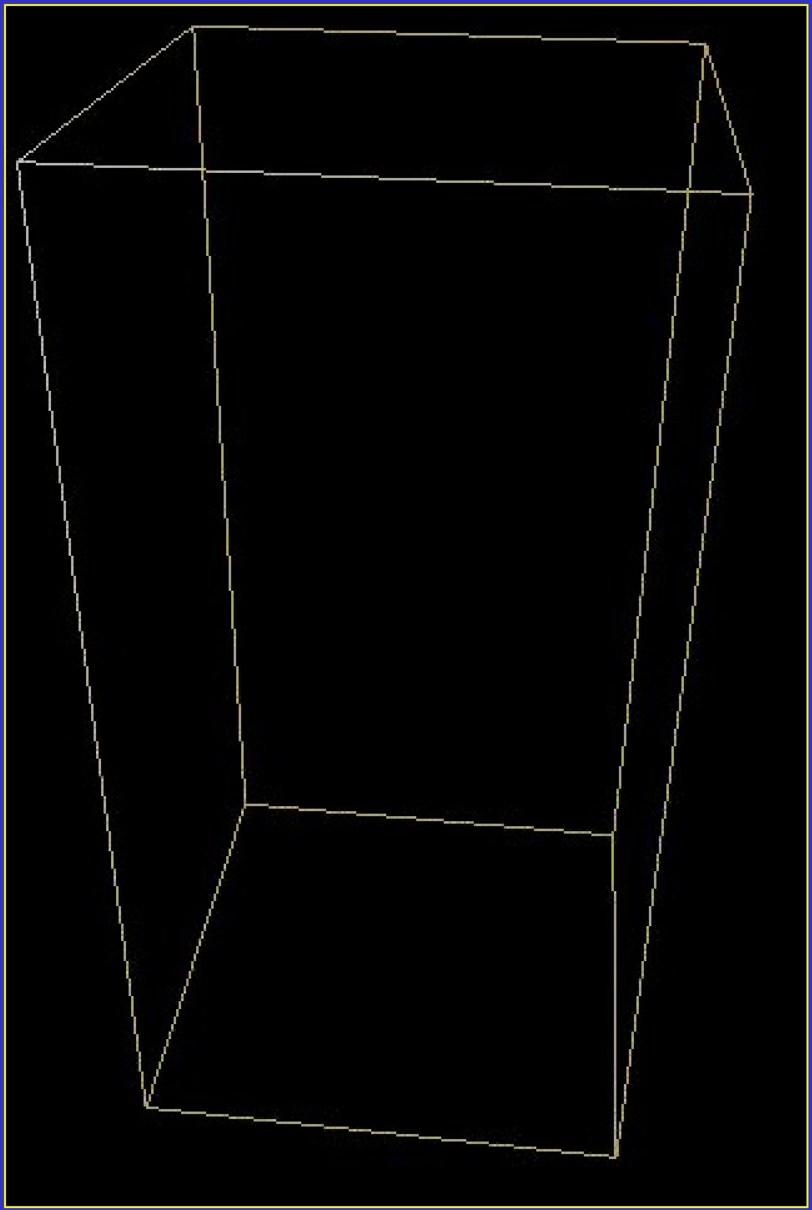
$$B_y = \begin{cases} +1 \ (z>0.5) \\ -1 \ (z<0.5) \end{cases}$$

Sawtooth profile

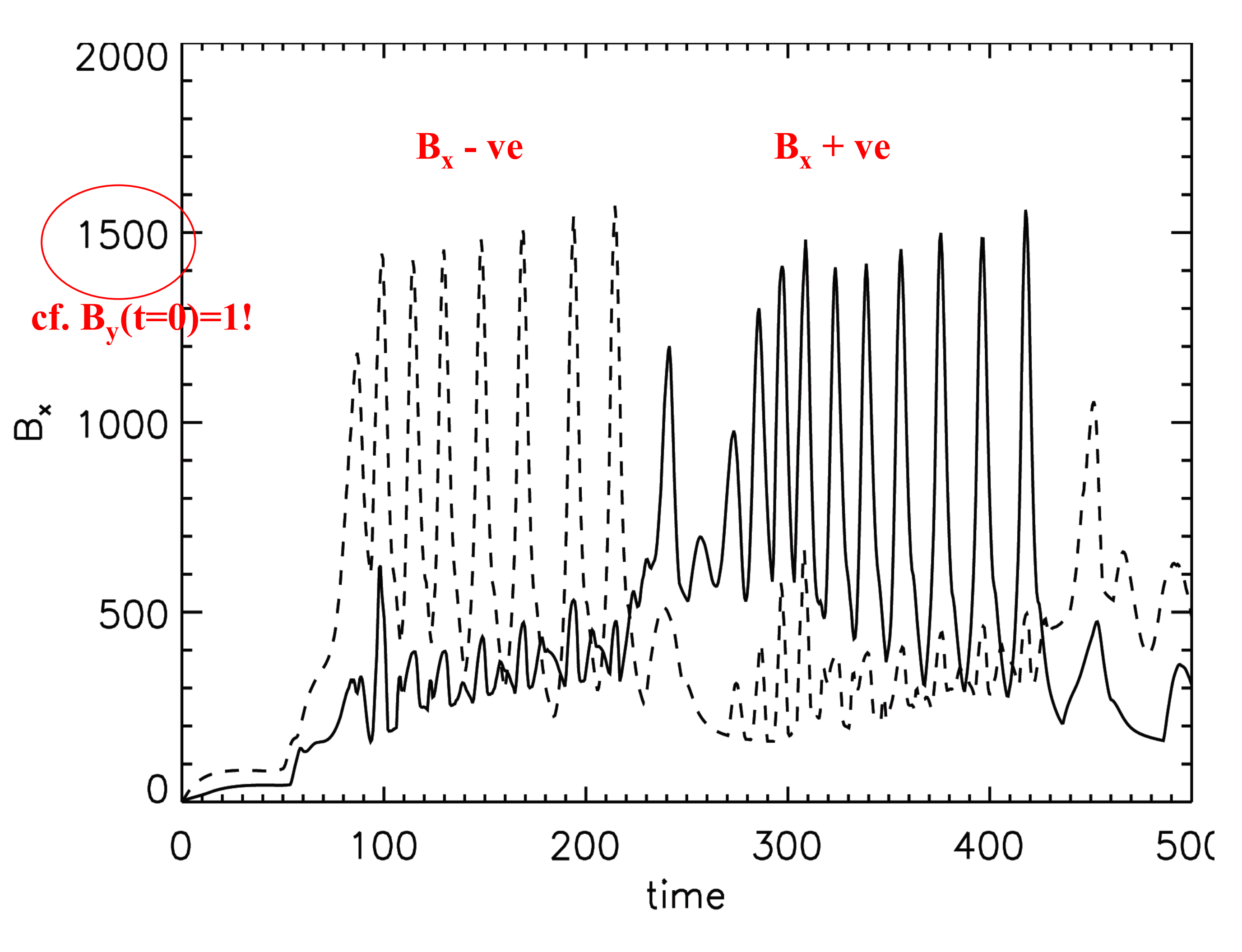


Cline, Brummell & Cattaneo 2003

Shear buoyancy dynamo: The movie …

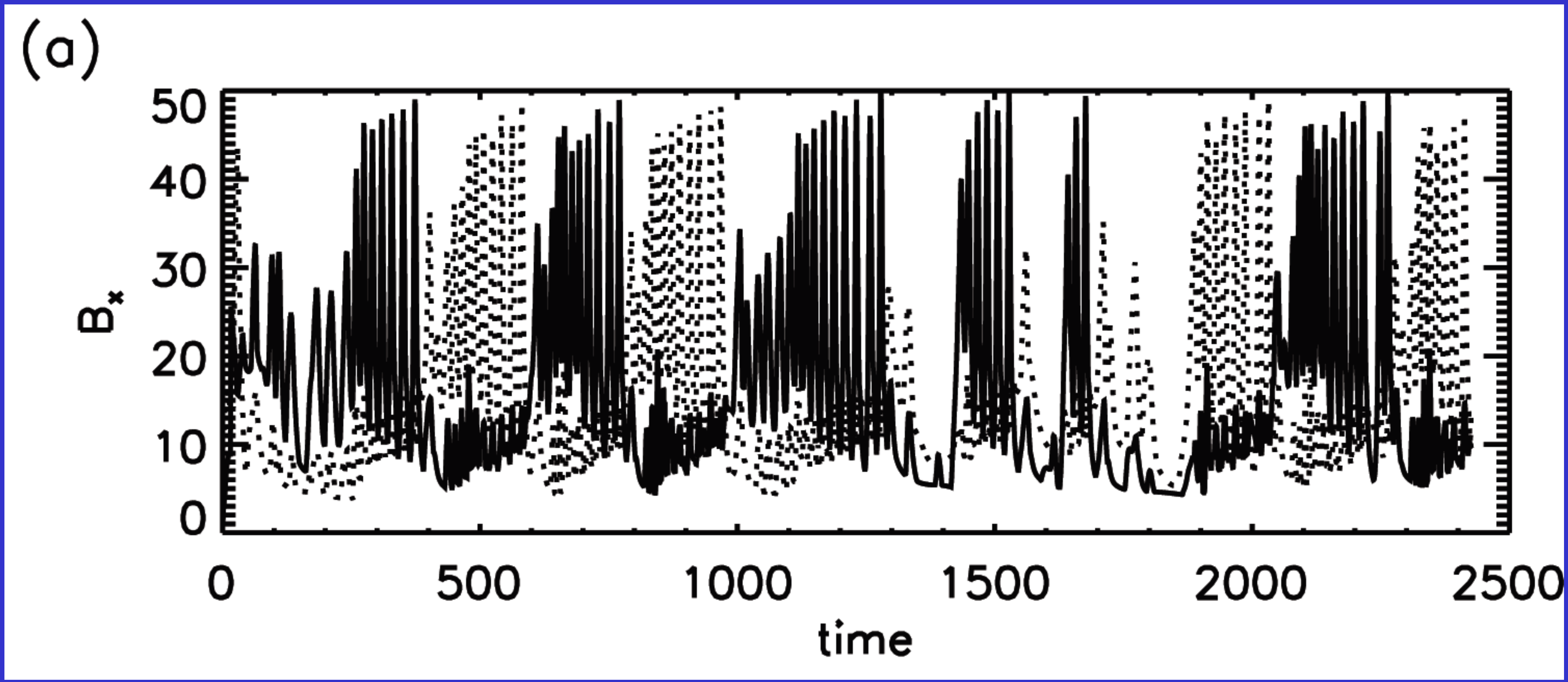


Shear-buoyancy dynamo



- Strong magnetic field maintained!
- Strong toroidal field is generated in a cyclic manner
- Polarity of the strong field reverses

Shear buoyancy dynamo: Longer time …

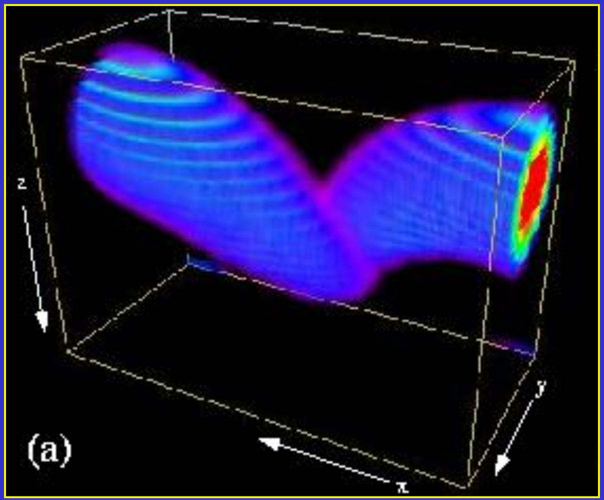


- Diffusion time ~ 300 time units
- => even more convincing is a dynamo
- Remarkably, also shows periods of reduced activity!

Mechanism: Field induced K-H instability

For sufficiently strong initial field:

Instability mechanism:



- ✓ Initial shear flow is not a dynamo!
- ✓ Initial field purely poloidal
- ✓ Poloidal field sheared -> toroidal
- ✓ Toroidal field is magnetically buoyancy
- ✓ Magnetic buoyancy induces roll-like poloidal flows
- ✓ These steepen the shear
- ✓ Shear then becomes unstable to Kelvin-Helmholtz type instability
- ✓ Shear modes in vertical and horizontal have phases such that they create helical flow
- ✓ Helical flow twists STRONG toroidal magnetic structure, giving rise to STRONG poloidal
- ✓ Feedback loop for dynamo created

Magnetic forcing of flows responsible for dynamo action

Shear buoyancy dynamo

A self-consistent dynamo driven solely by the action of shear and magnetic buoyancy. (Note: No Coriolis forces required!)

It is *NON-KINEMATIC, ESSENTIALLY NONLINEAR:*

- Original velocity (shear) NOT a dynamo flow
- Action of magnetic field adds dynamo component
- Needs a finite magnetic field to get it going

ASIDE: GOOD QUESTION: *WHAT IS THE ROLE OF TURBULENCE*?

- VERY LAMINAR! New question: No longer "Does turbulence do the trick?" but rather: "Does this still work in presence of turbulence?". Add noise to the dynamo simulations … (work in progress ☺ -- Seems to be robust at high Rm and to added noise

- NOT NEW: There are many examples of these things that have been around for a while (geodynamo: "strong field branch" ; Jean-Claude Thelen; Galloway - Archontis dynamo; tearing mode pinch in tokamacs; Rincon MRI subcritical dynamo instability; …)

BIG QUESTION: DO THESE HAVE DIFFERENT BEHAVIOUR AT HIGH Rm?

Simple model of END's?

Can we build a SIMPLER model that encapsulates the essence of the END?

- ✓ Would like to return to a KINEMATIC-type formulation, i.e. NOT solve the momentum equation; only solve the induction equation.
- ✓ But now need a velocity with a piece that depends on the magnetic field => a nonlinear induction equation.

Assume (kinetic) Reynolds number is low => velocity is linear functional of the forces (although forces might be nonlinear functions of magnetic field)

Require 3 parts to the forces and therefore the driven velocity:

- ✓ External force that drives a background (original) flow (dynamo or not), e.g. shear flow (common large-scale flow; thought to play a role; not a dynamo)
- ✓ Part of the Lorentz force that drives additional flow that contributes to dynamo action (cf. the helical K-H modes in shear-buoyant dynamo)
- ✓ Another part of the Lorentz force that drives a flow that is responsible for saturation of the dynamo (typically a small-scale flow)

Simple model of END's? Tobias, Cattaneo, Brummell, 2010

So, solve, in a multiply-periodic domain $(\partial_t - \eta\nabla^2)\,\mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B})$

with $\mathbf{u} = C_1\mathbf{u}_{shear} + C_2\mathbf{u}_{ss} + C_3\mathbf{u}_M$ $\quad C_i = c_i/(1 + \mu_i\langle B^2\rangle)$

and assume velocities independent of z

=> $\mathbf{B} = \Re(\hat{\mathbf{B}}(x,y)\exp ik_z z) = \mathbf{B}^c \cos k_z z - \mathbf{B}^s \sin k_z z$

$$\hat{\mathbf{B}} = (\mathbf{B}^C + i\mathbf{B}^S) = (b_1^C + ib_1^S, b_2^C + ib_2^S, b_3^C + ib_3^S)$$

1. $\mathbf{u}_{shear} = (\sin(2\pi y/Y_{mx}), \epsilon\sin(2\pi x/X_{mx}), 0)$

Shear velocity: non-dynamo

2. $\mathbf{u}_{ss} = (\partial_y\psi, -\partial_x\psi, 10.0\psi)$

$$\psi = 0.1\,(\cos k(x - \cos t) + \sin k(y + \sin t))$$

Small-scale velocity: CAN BE a dynamo!

3. $\mathbf{u}_M = (N_1\partial_y\psi^{NL}, -N_1\partial_x\psi^{NL}, N_2 w^{NL})$

$$\nabla^2\psi^{NL}(x,y,t) = \partial_y b_1^c(x,y,t) - \partial_x b_2^c(x,y,t)$$

$$w^{NL}(x,y,t) = b_2^c(x,y,t) + b_2^s(x,y,t)$$

Magnetically-induced velocity: non-linearly driven, right sort of parity, solenoidal

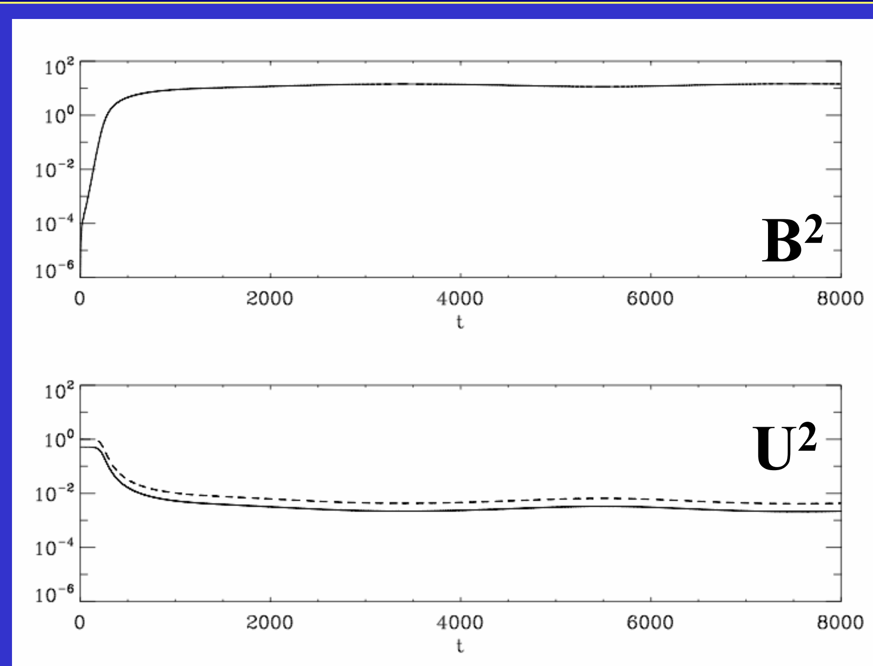
$$k = 3,\ k_z = 0.1,\ \varepsilon = 0.2,\ \eta = 0.01,\ X_{mx} = Y_{mx} = 6\pi,\ nx = ny = 768$$

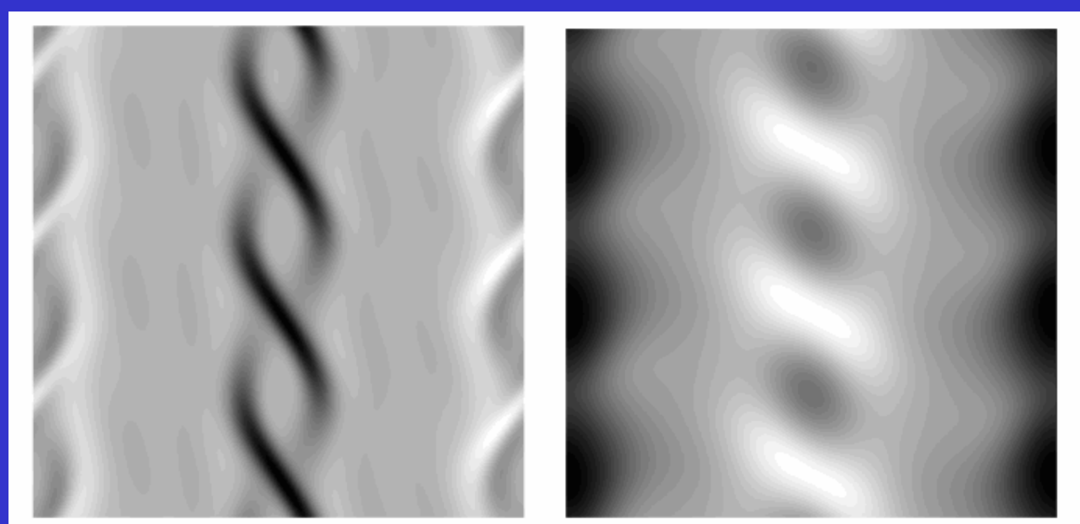
Results: Essentially kinematic

$c_1=1$ $c_2=1$ $c_3=0$: ($\mu_i=1$ unless otherwise stated)

- ✓ Shear and small-scale (potentially dynamo) velocity only
- ✓ No magnetically-driven flow
- ✓ Effect of nonlinearity is only to saturate growth

=> "essentially kinematic" dynamo





t=29 (early) t=4026 (saturated)

k=3

$k_x=1, k_y=0$

Results: Mixed kinematic and nonlinear

$c_1=1 \;\; c_2=0 \;\; c_3=1$:

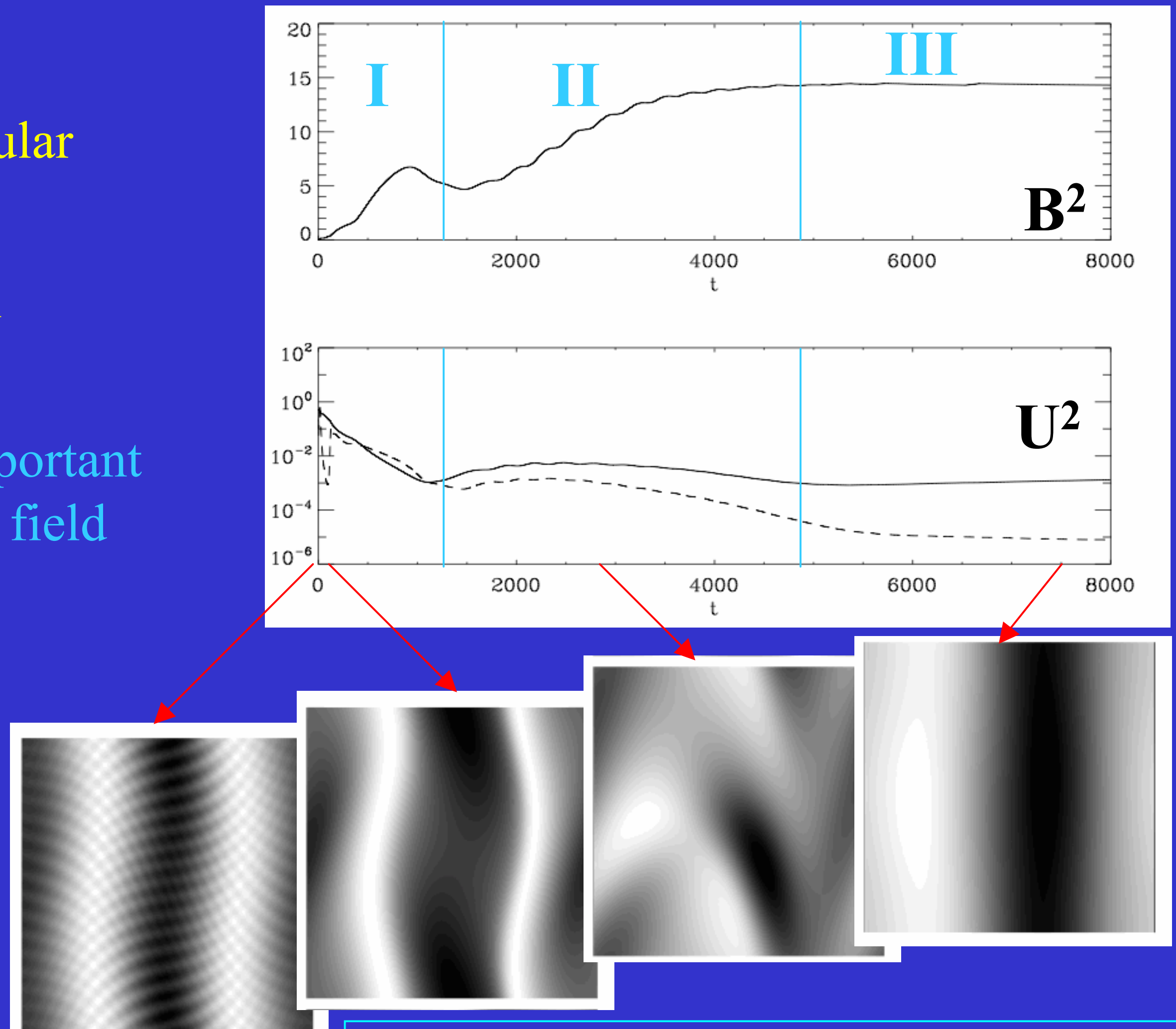
- ✓ No u_{ss}: small-scale, cellular flow
- ✓ u_M: magnetically-driven nonlinear flow
- ✓ => Initial conditions important - need substantial initial field
- ✓ $k_x=k_y=16$ initial field

<u>Three regions of results:</u>

I. Adjustment

II. Nonlinear growth (non-exponential)

III. Saturation (by quenching factors)



<u>Final state:</u> k=1 *"system scale"*; w small; *almost* planar flow; very little induction and dissipation

Results: Essentially nonlinear

$c_1=0$ $c_2=0$ $c_3=1$:

- ✓ No shear - maybe this causes *system-scale* dynamo so turn it off.
- ✓ No u_{ss}: small-scale, cellular flow
- ✓ Only u_M: magnetically-driven nonlinear flow

=> Initial conditions important again (a) System-scale i.c. (b) Small-scale i.c.

(a) $b_1^c = \sin y \sin x, \quad b_1^s = 0, \quad b_2^c = -\cos y \sin x \quad b_2^s = 0$

(b) $b_1^c = 0.5(\cos 5x + \cos 4y), \quad b_1^s = 0.35(\cos 5x + \cos 4y),$

$b_2^c = 0.5(\cos 5x + \cos 4y), \quad b_3^s = 0.45(\cos 5x + \cos 4y)$

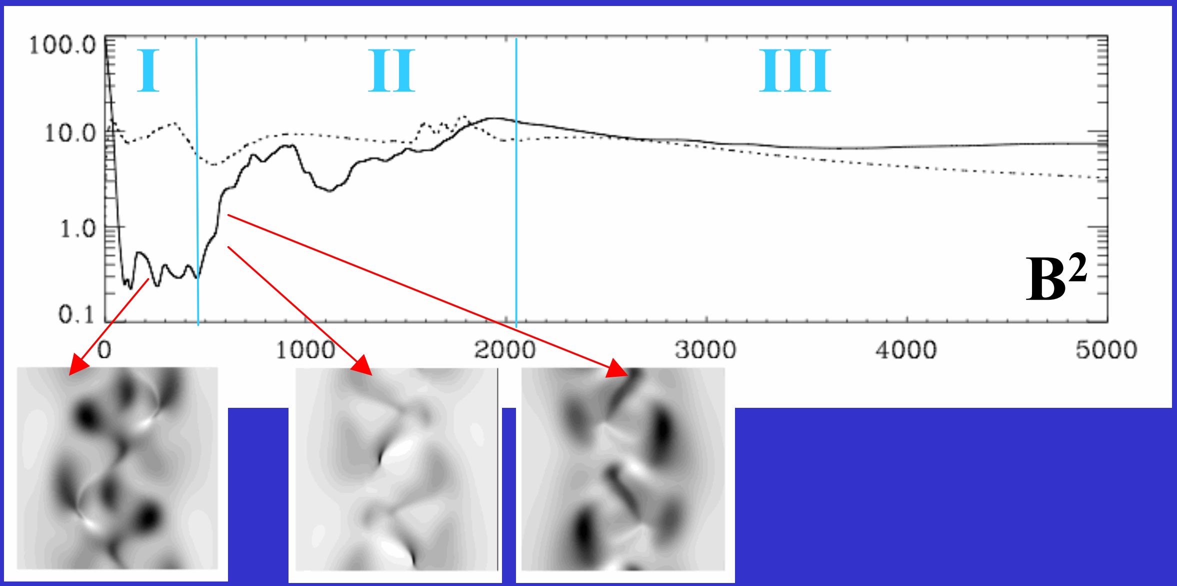
Results similar

(to each other and to previous results)

I. Adjustment

II. Nonlinear growth : Re-adjustments of scale in bursts; controlled by μ_i

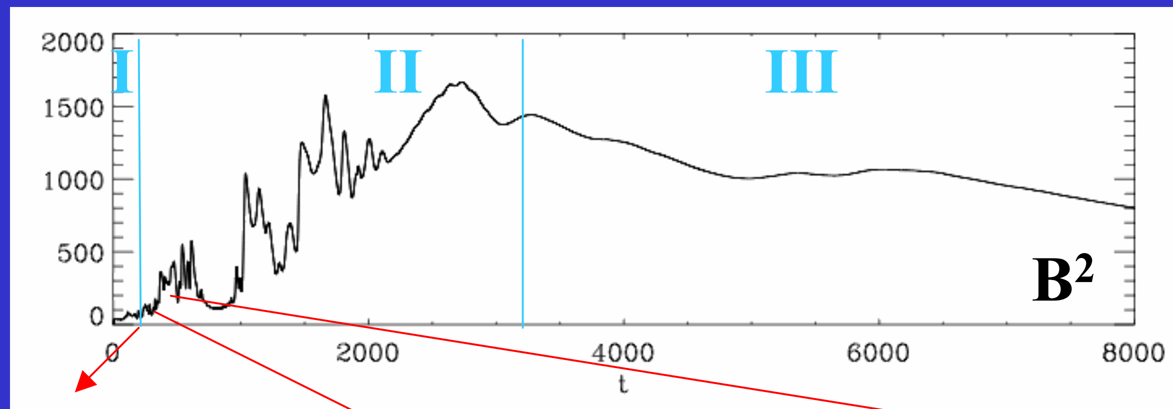
III. Saturation : System-scale, lower energy (no shear)

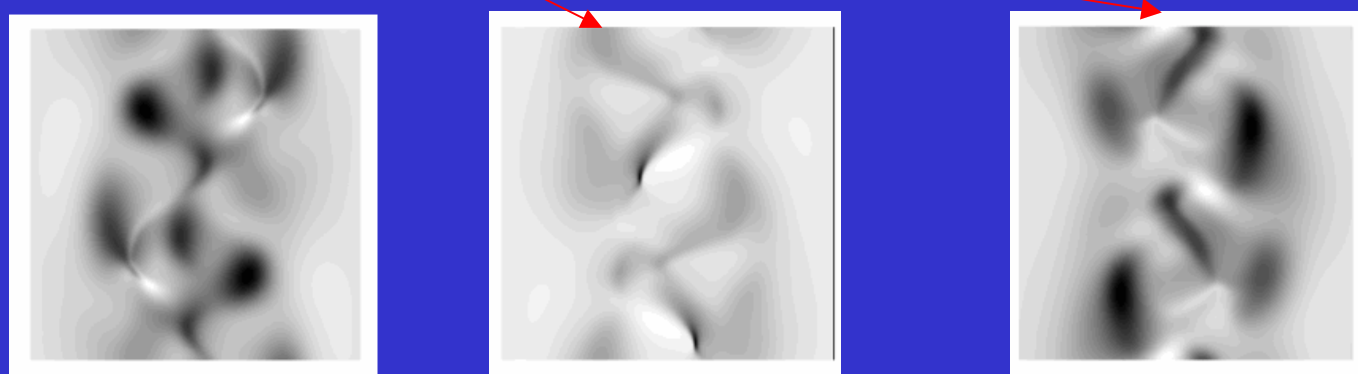


Results: Essentially nonlinear

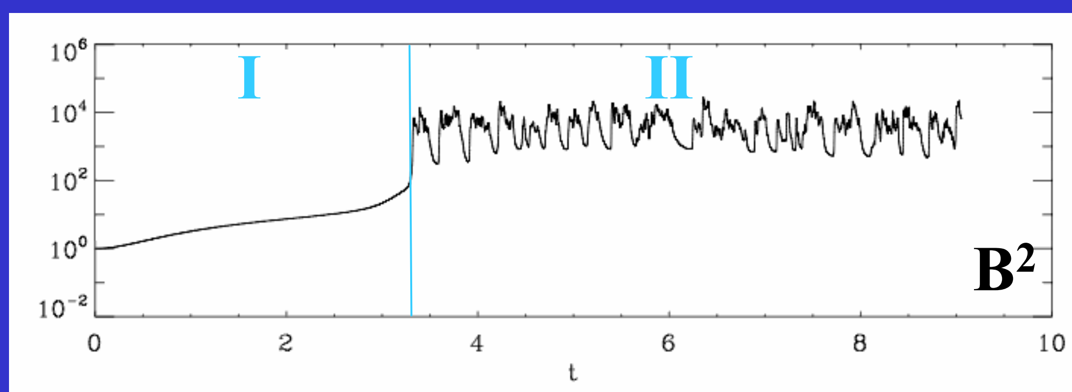
$c_1=0$ $c_2=0$ $c_3=1$:

$\mu_3=0.1$





$\mu_3=0.001$



Conclusions

Old concepts: "large-scale" and "small-scale" dynamos:

- ✓ useful in kinematic sense but much less clear in the nonlinear sense, since the velocity field may evolve as the Lorentz force becomes important

New concept: "system-scale" dynamo:

- ✓ compare characteristic scale of magnetic field to the scale of the object in question (which does not evolve [in general])

Nonlinear (dynamic rather than kinematic) regime:

Possibilities: essentially kinematic, essentially nonlinear (and mixture)

Can the "essentially nonlinear" do what the "essentially kinematic" dynamos have struggled to do -- produce system scale fields at large Rm.

Simple model:

- ✓ Encouraging: Nonlinear induction equation can have system-scale solutions
- ✓ Future work: different relationships u-B; 3D?