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$$\begin{aligned}\dot{A} &= 2DB - A, \\ \dot{B} &= iA - \frac{1}{2}iA^*\omega - B, \\ \dot{\omega} &= -iAB - \nu\omega,\end{aligned}$$

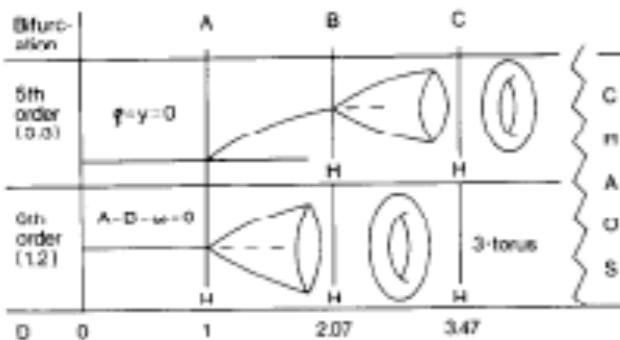


Fig. 5. Schematic bifurcation diagrams for the fifth and sixth order systems with $\sigma = 1$, $\nu = 0.5$. Hopf bifurcations are indicated by the letter H.

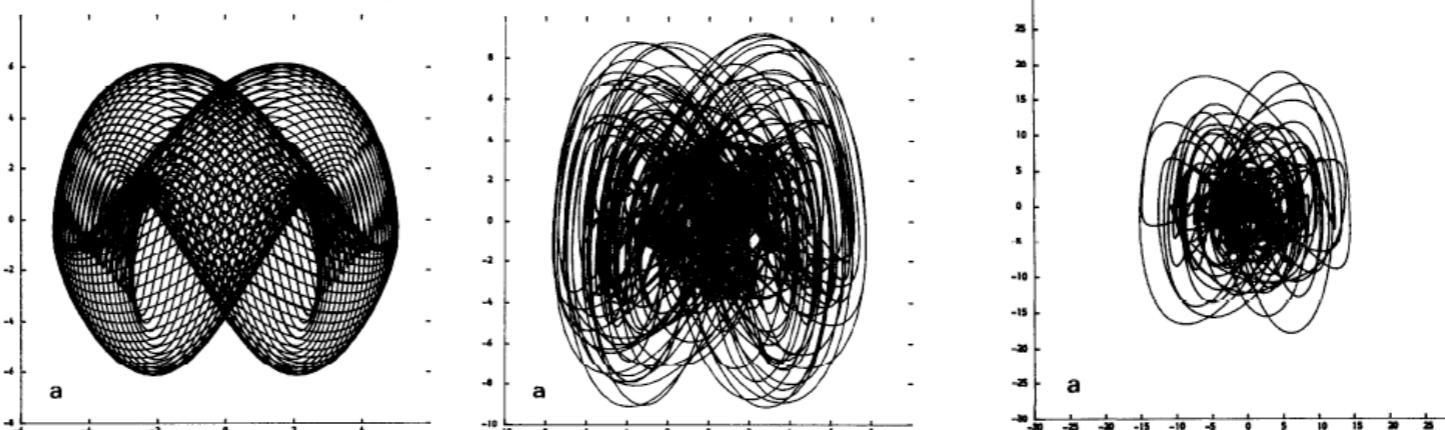


Fig. 1. a) A projection of a trajectory for the sixth order system (1.1) with $\sigma = 1$, $\nu = \frac{1}{2}$ and $D = 3.0$. The projection is onto the $B_i\omega_r$ -plane. b) The Poincaré map for the same plane as in a), taking the section $A_i = 0$.

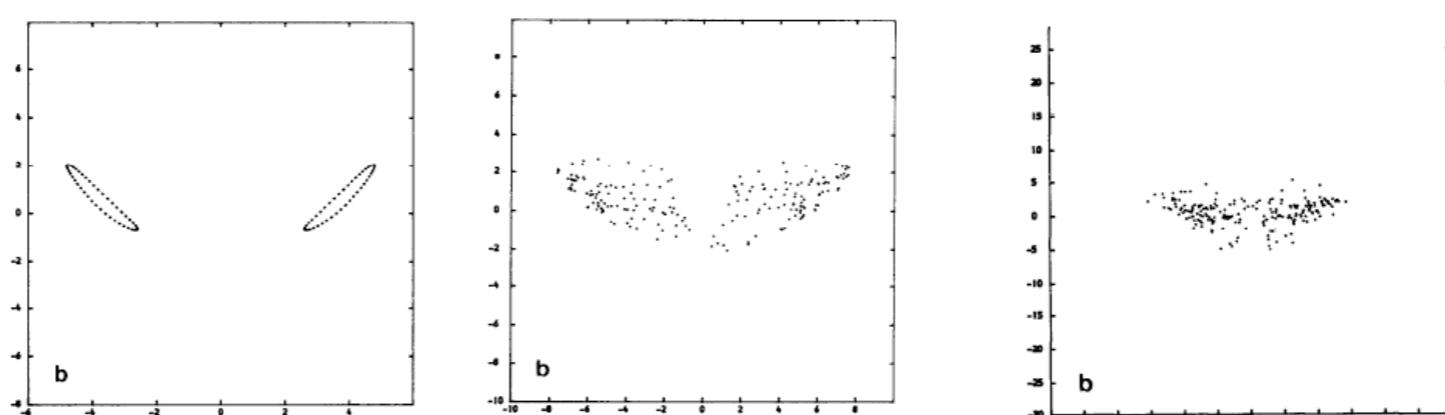


Fig. 2. a) As for 1a, but with $D = 3.6$; b) As for 1b, but with $D = 3.6$.

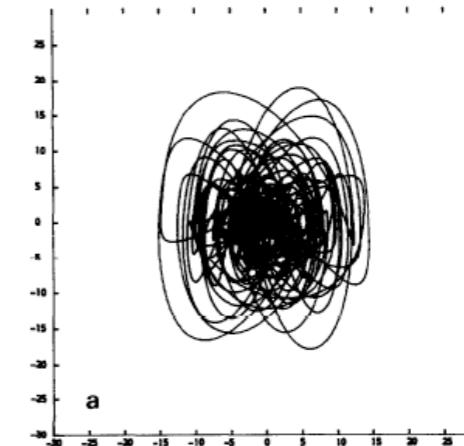


Fig. 3. a) As for 1a, but with $D = 8.0$; b) As for 1b, but with $D = 8.0$.

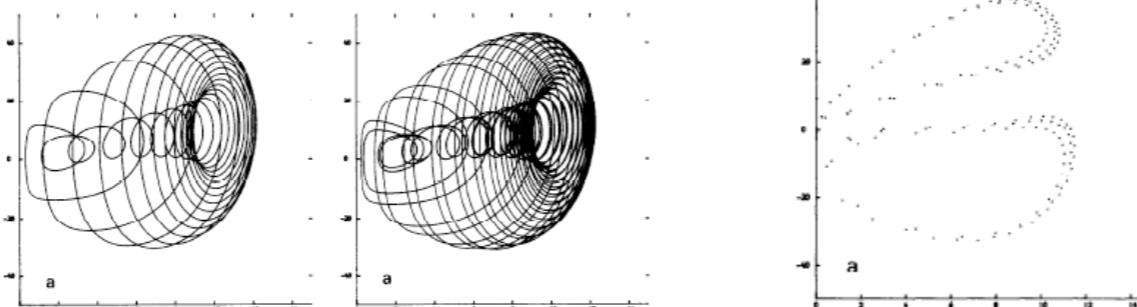


Fig. 11. a) As for 9a, but with $D = 3.80$. The frequency is now locked on a 25:1 ratio; b) The Poincaré map corresponding to 11a.

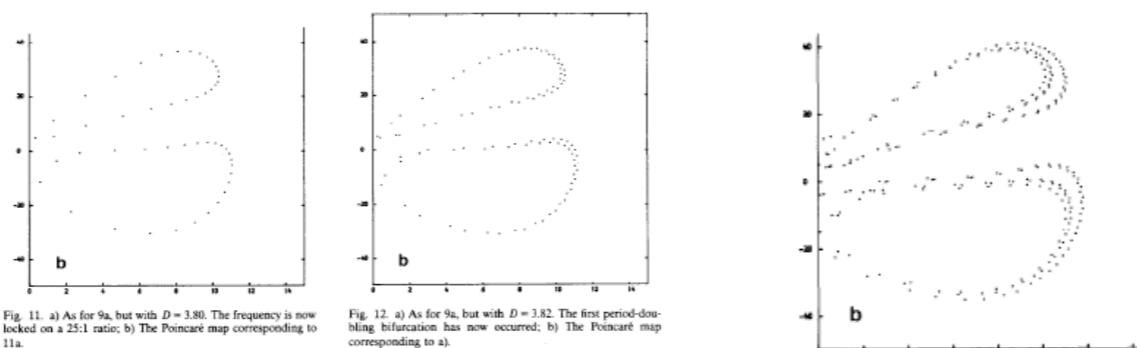
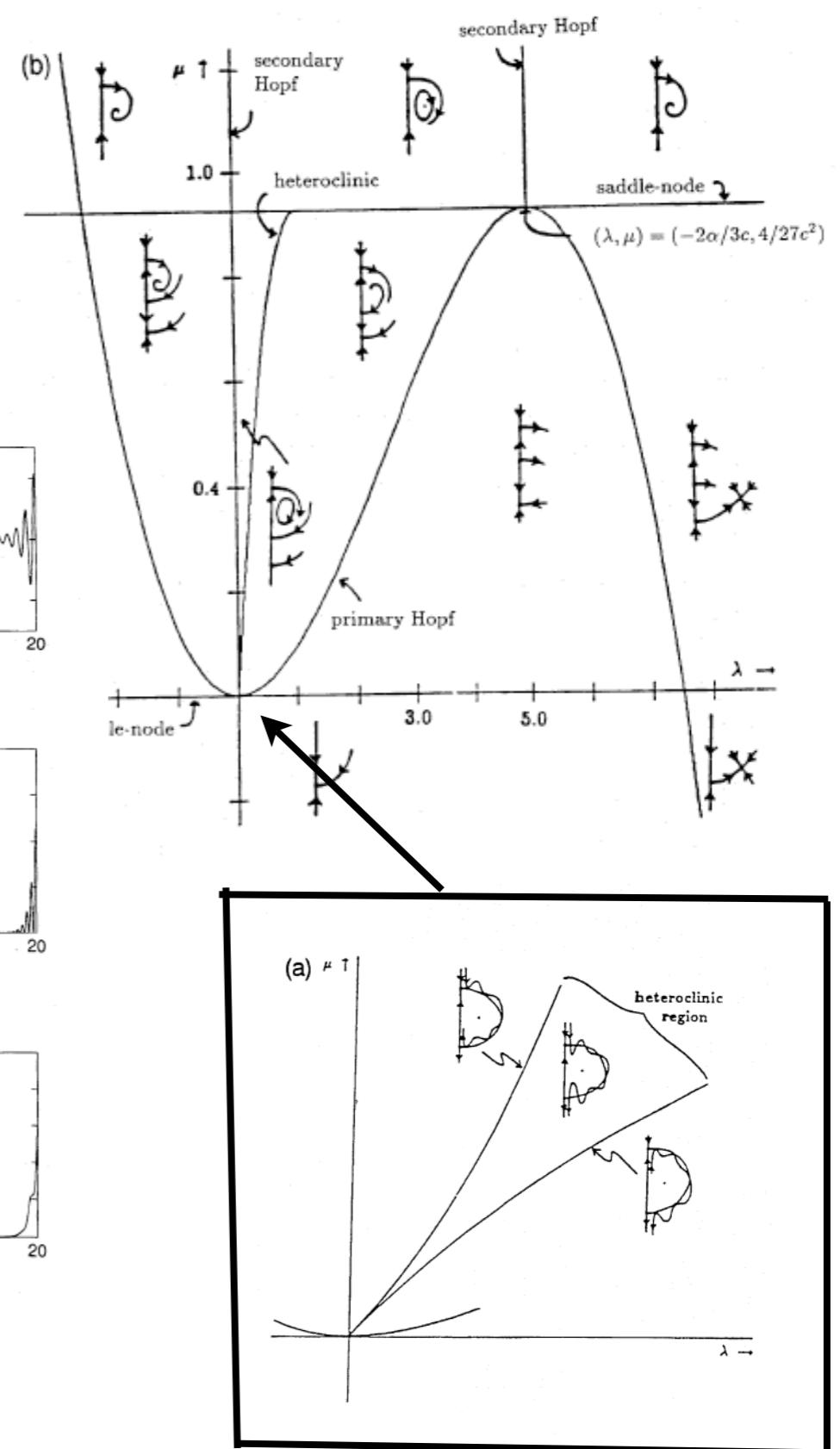
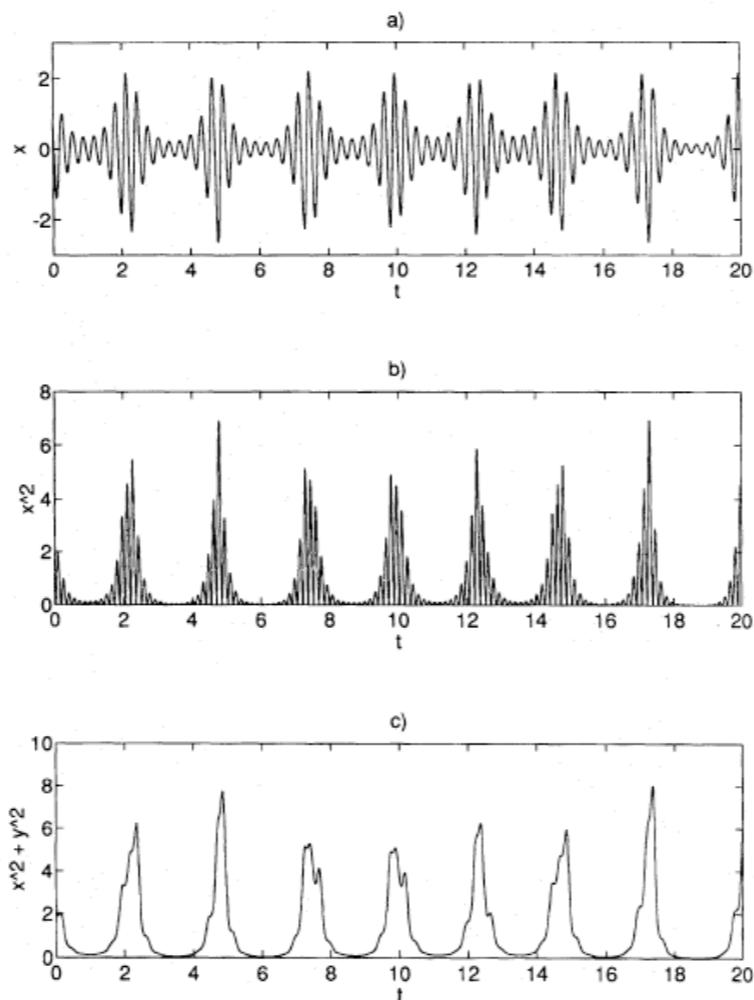
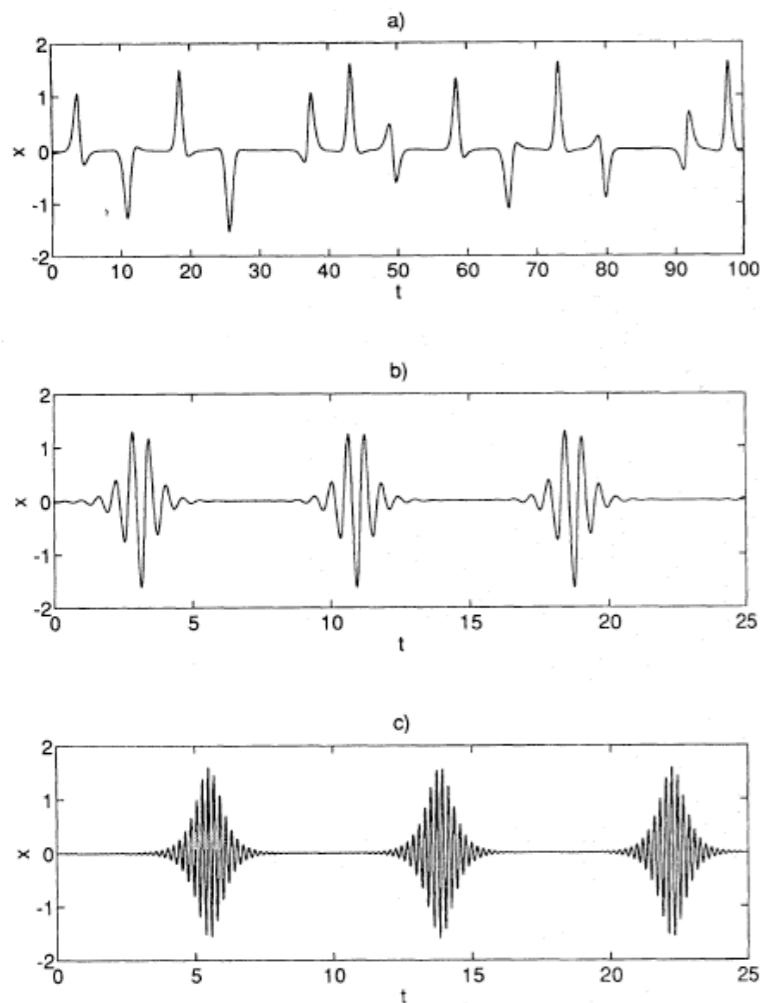


Fig. 12. a) As for 9a, but with $D = 3.82$. The first period-doubling bifurcation has now occurred; b) The Poincaré map corresponding to a).

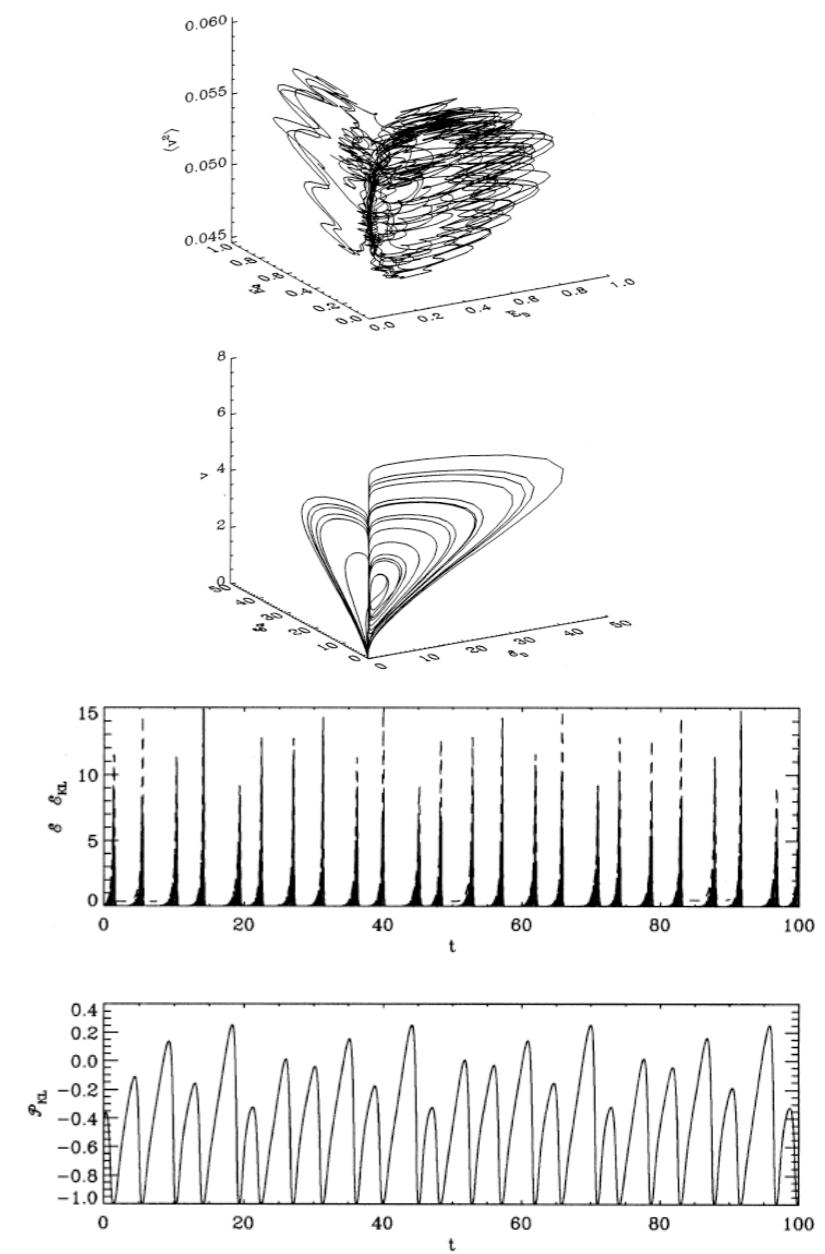
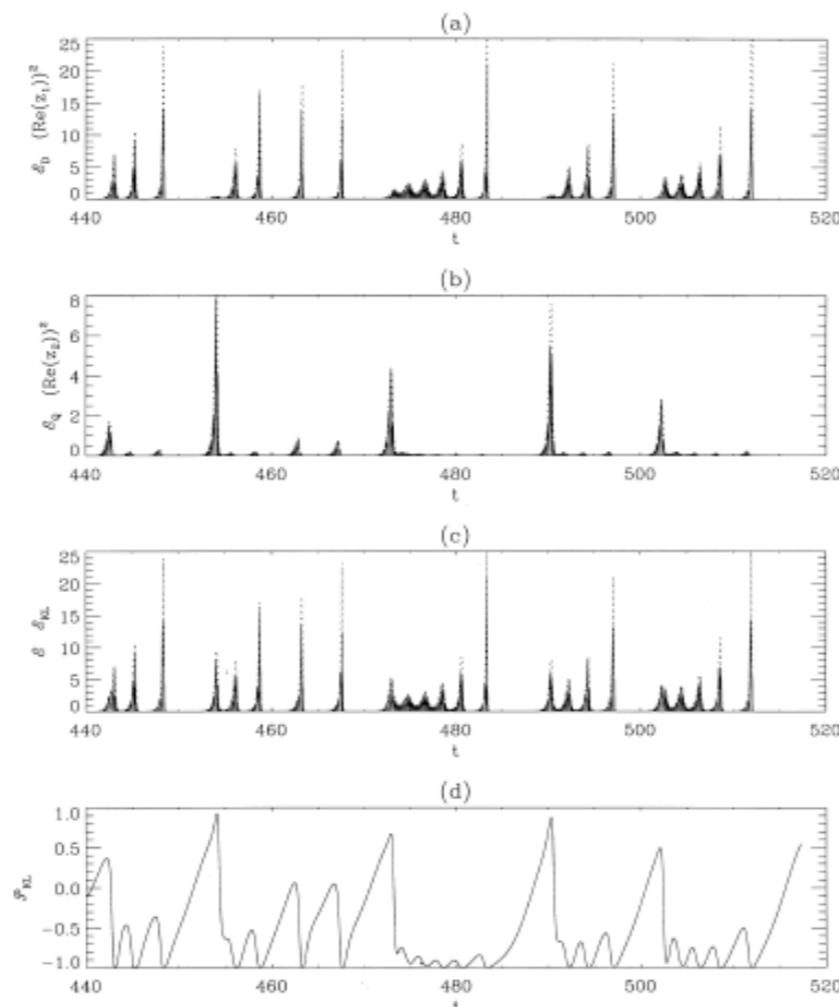
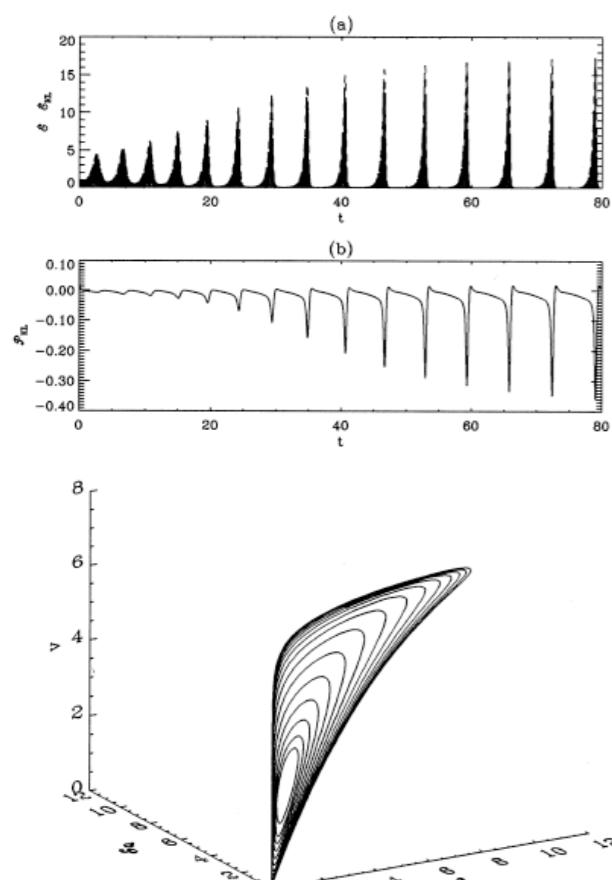
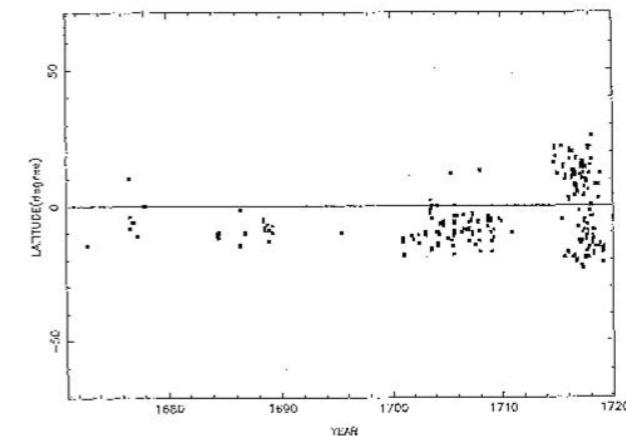
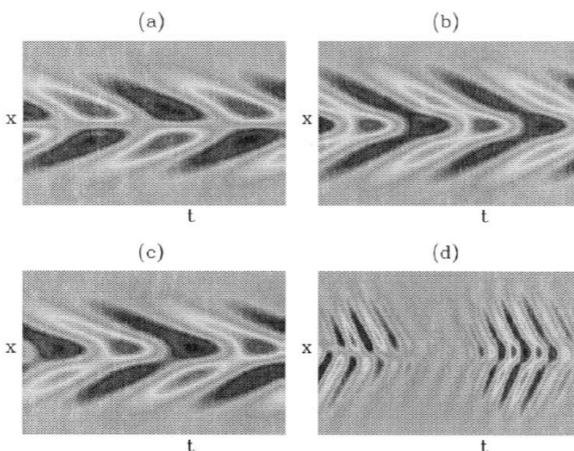
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$$\begin{aligned}\dot{r} &= \lambda r + a z r + d r^2 z \cos \phi, \\ \dot{\phi} &= \omega - d r z \sin \phi, \\ \dot{z} &= \mu - z^2 - r^2 + c z^3.\end{aligned}$$



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$$\begin{aligned}
 \dot{z}_1 &= (\mu + \sigma + i\omega_1) z_1 + a |z_1|^2 z_1 + b |z_2|^2 z_1 + c z_2^2 \bar{z}_1, \\
 &\quad + (\epsilon v + \delta v^2 + \kappa w^2) z_1 + (\beta + \gamma v) w z_2, \\
 \dot{z}_2 &= (\mu + i\omega_2) z_2 + a' |z_2|^2 z_2 + b' |z_1|^2 z_2 + c' z_1^2 \bar{z}_2 \\
 &\quad + (\epsilon' v + \delta' v^2 + \kappa' w^2) z_2 + (\beta' + \gamma' v) w z_1, \\
 \dot{v} &= -\tau_1 v + e_1 (|z_1|^2 + |z_2|^2), \\
 \dot{w} &= -\tau_2 w + e_2 (z_1 \bar{z}_2 + z_2 \bar{z}_1).
 \end{aligned}$$



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$$\begin{aligned}
\dot{z}_1 &= (\mu_1 + i\omega)z_1 - |z_1|^2 z_1 - \frac{c_2 + \mu_1}{\mu_2} x_2^2 z_1 + \frac{e_3 - \mu_1}{\mu_3} x_3^2 z_1 \\
&\quad + \epsilon_1 \bar{z}_1^5 + \epsilon_2 |z_1|^4 x_2 + \epsilon_3 z_1 x_2 x_3^3, \\
\dot{x}_2 &= \mu_2 x_2 - x_2^3 - \frac{c_3 + \mu_2}{\mu_3} x_3^2 x_2 + \frac{e_1 - \mu_2}{\mu_1} |z_1|^2 x_2 \\
&\quad + \epsilon_1 \operatorname{Re}(z_1^3) x_2^2 + \epsilon_2 x_1^5 + \epsilon_3 x_3^5, \\
\dot{x}_3 &= \mu_3 x_3 - x_3^3 - \frac{c_1 + \mu_3}{\mu_1} |z_1|^2 x_3 + \frac{e_2 - \mu_3}{\mu_2} x_2^2 x_3 \\
&\quad + \epsilon_1 \operatorname{Re}(z_1^3) x_2 x_3 + \epsilon_2 x_1^3 x_2 x_3 + \epsilon_3 x_2^5,
\end{aligned}$$

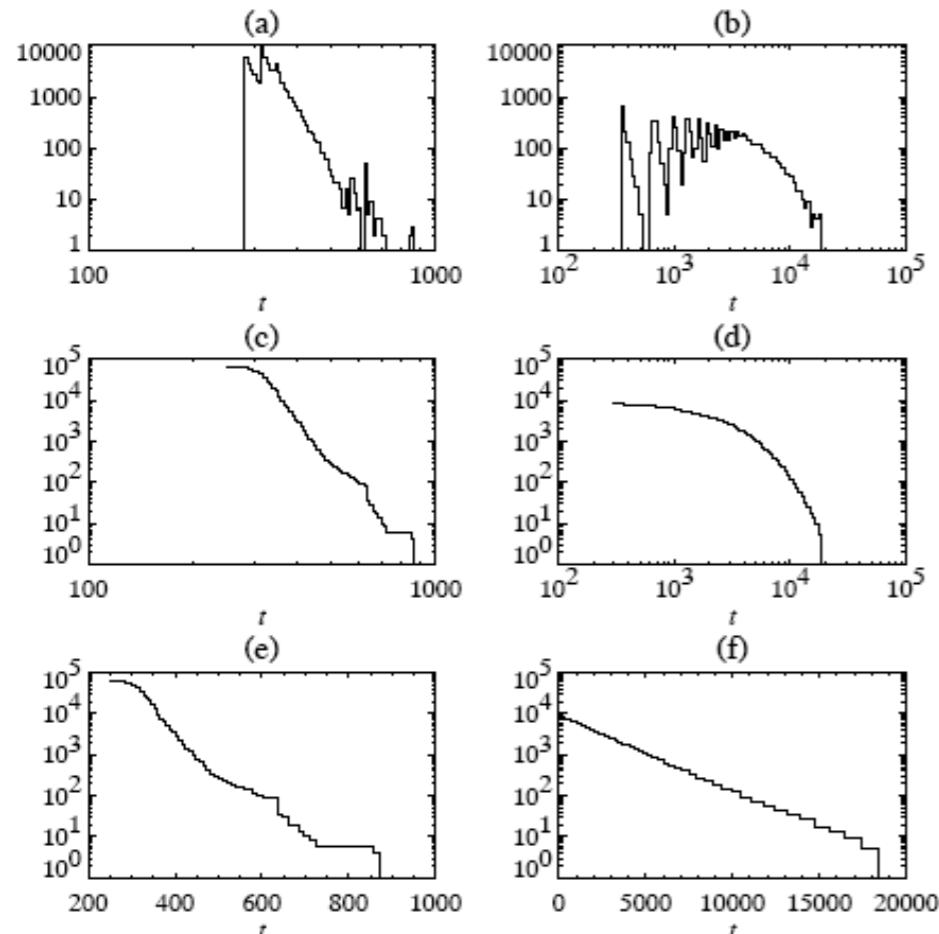


Figure 2. Distributions of durations of (a,c,e) excursions and (b,d,f) reversals. (a,b) histograms. (c,d) cumulative plot (log-log). (e,f) cumulative plot (log-linear).

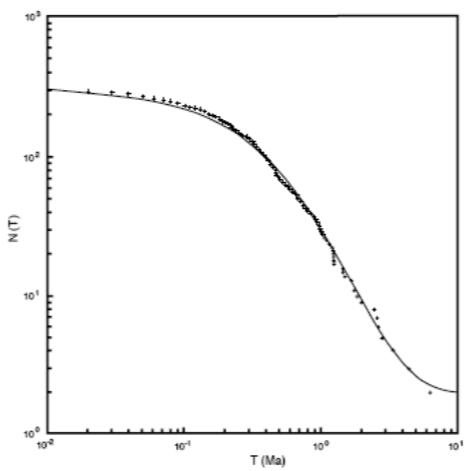


Figure 3. Distribution of reversal durations: cumulative plot (log-log) for the magnetic field (reproduced from [1]). Compare with figure 2(d).

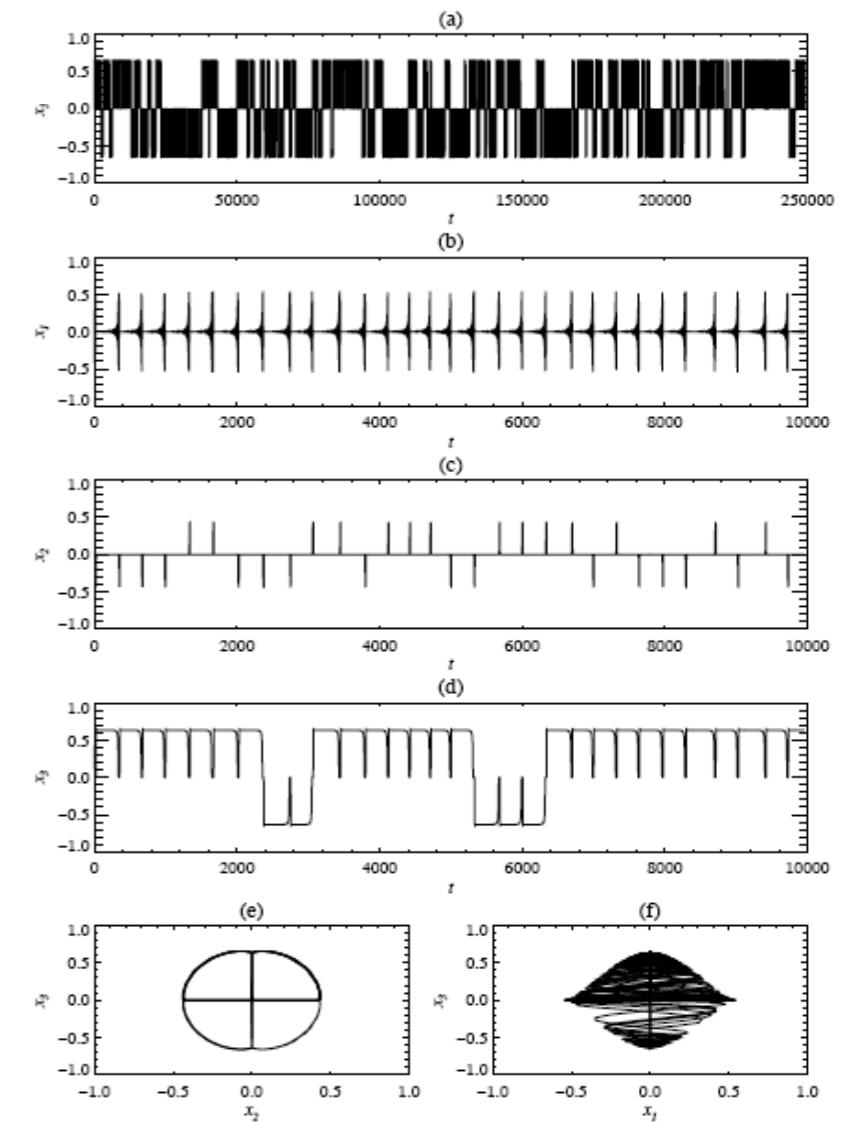


Figure 1. Behaviour of (6-8) for the parameters given in the text. (a) shows a long time series for the dipole mode x_3 , and (b-d) show the x_1 , x_2 and x_3 evolution in the first part of the time series. (e-f) show phase portraits: (e) x_3 vs. x_2 (f) x_3 vs. x_1 .

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