Kinematic dynamo in a reflection-invariant random field

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We obtain the excitation threshold, growth rate, and spatial properties of a growing magnetic field in a conducting medium with a prescribed random motion that is reflection-invariant on the average and is also isotopic and instantaneously correlated. The propagation velocity of an initially localized field is found. The behavior of the field in the subthreshold regime and the limits of applicability of the model are discussed.

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1. INTRODUCTION

Kazantsev[1] has proposed a model of magnetic-field generation in an instantaneously correlated isotropic reflectively invariant average random velocity field. The problem was reduced to solution of a second-order Schrödinger equation for the correlation function of the magnetic field. Bound states correspond in this equation to dynamo solutions that increase exponentially with time. Some generalizations of the Kazantsev models were obtained in Refs. 2–4.

The existence of bound states (of self-excitation) at large magnetic Reynolds numbers was noted in Ref. 1. Direct numerical calculations for a more realistically specified velocity field (without instantaneous correlation and with allowance for the reaction of the magnetic field on the motion) confirms the presence of self-excitation.[5]

The excitation threshold was first estimated in Ref. 6 and was later obtained in Ref. 4 using a different model. In this paper we refine and generalize the results of Ref. 6 concerning the excitation threshold, obtain the dependences of the growth rate and of the spatial form of the solution on the magnetic Reynolds number, and determine the propagation velocity of an initially localized field.

2. EQUATION FOR THE CORRELATION FUNCTION

The assumptions that the velocity field of an incompressible liquid is isotropic and has mirror symmetry mean that its correlation tensor can be represented in the form (see, e.g., Ref. 7).

$$\langle v_i(r_1,t)\, v_j(r_2,t')\rangle$$
$$=\frac{lv}{3}\left[F(r)\,\delta_{ij}+\frac{r}{2}\frac{dF}{dr}\left(\delta_{ij}-\frac{r_ir_j}{r^2}\right)\right]g(t-t'),$$
$$\mathbf{r}=|\mathbf{r}_1-\mathbf{r}_2|,\qquad i,j=1,2,3, \tag{1}$$

where the dimensional factor is separated, l is the correlation length, v is the characteristic velocity, and the angle brackets denote averaging over the ensemble of the realizations of the field velocity. The longitudinal correlation function $F(r)$ is dimensionless, so that $F(0)=1$. It has a positive Fourier transform and satisfies the usual requirement $F(r)\to 0$ as $r\to\infty$.[7] Let, for the sake of argument,

$$l=\int_0^\infty F(r)\,dr.$$

We introduce also the correlation function $f(r)$

$$\langle v_i(\mathbf{r}_1,t)\,v_i(\mathbf{r}_2,t')\rangle = lvf(r)\,g(t-t').$$

Obviously,

$$f=\frac{1}{3r^2}\frac{d}{dr}(r^3F).$$

In the instantaneous correlations approximation we have $g(t-t')=2\delta(t-t')$. In the problem of the turbulent kinematic dynamo the function $F(r)$ [or $f(r)$] is assumed specified. We shall consider below the characteristic forms of this function.

We examine first in detail the evolution of an initially homogeneous distribution of a magnetic field with zero mean value. In Sec. 4 we shall discuss how to describe the behavior of an inhomogeneous (localized) initial distribution of the magnetic energy.

We seek the equal-time correlation tensor of a divergence-free magnetic field in a form similar to (1):

$$\mathscr{H}_{ij}(r,t)=\frac{1}{3}\left[W(r,t)\,\delta_{ij}+\frac{r}{2}\frac{dW}{dr}\left(\delta_{ij}-\frac{r_ir_j}{r^2}\right)\right]. \tag{2}$$

The determination for the longitudinal correlation function $W(r,t)$ reduces to solution of a Schrödinger type equation with variable mass but without a complex factor in front of the time derivative[1]

$$\frac{1}{2}\frac{\partial\psi}{\partial t}=\frac{1}{2m(r)}\frac{\partial^2\psi}{\partial r^2}-U(r)\,\psi \tag{3}$$

and with a potential

$$U(r)=\frac{1}{2r}\frac{df}{dr}+\frac{1}{mr^2}-\frac{1}{8m^3}\left(\frac{dm}{dr}\right)^2. \tag{4}$$

The time and the coordinate are measured here in units of $\lambda\sqrt{2/v}$ and $\lambda\sqrt{2}$, where

$$\lambda=\left(-\frac{d^2f(0)}{dr^2}\right)^{-1/2},$$

and we have introduced the notation

$$\frac{1}{2m}=\frac{1}{R_m}+\frac{1-F(r)}{3}, \tag{5}$$
$$\psi(r,t)=\frac{r^2}{3\sqrt{2m}}W(r,t),$$

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where $R_m = lv/\nu_m$ is the magnetic Reynolds number and ν_m is the magnetic-diffusion coefficient. We introduce also a correlation function analogous to $f(r)$

$$\langle H_i(\mathbf{r}_1, t) H_i(\mathbf{r}_2, t)\rangle = w(r, t).$$

Then

$$w(r, t) = \frac{1}{3r^2}\frac{\partial}{\partial r}(r^3 W).$$

The smooth decrease of $F(r)$ guarantees a decrease of the potential U at infinity. In accord with the dynamo-problem requirement that there be no external sources, we assume that $w(r,t)$ at least does not increase as space becomes infinite.

We note that the substitutions $d\rho = (2m)^{1/2}dr$ and $\varphi = (2m)^{1/4}\psi$ reduce (3) to an equation with constant mass. The potential U, however, is then expressed in terms of ρ not explicitly but parametrically.

3. CALCULATION OF THE EXCITATION THRESHOLD AND OF THE GROWTH RATE

Since the coefficients of Eq. (3) do not depend on the time, a growing solution can be sought in the form

$$\psi(r, t) = e^{2\gamma t}R(r),$$

where γ is the magnetic-field growth rate. This reduces the problem to finding the eigenvalues and the eigenfunctions for one variable

$$d^2R/dr^2 + 2m(E-U)R = 0, \tag{6}$$

where $E = -\gamma$.

The boundary condition $R(0) = 0$ follows directly from the definition of ψ (see (5)) and from the fact that $w(0,t) = 1e^{2\gamma t}$ (i.e., $R \sim r^2$, $r \to 0$). For $E < 0$ ($\gamma > 0$) the equation has solutions that grow or decrease exponentially (like $\exp[\pm(2m|E|)^{1/2}r]$) as $r \to \infty$. But since the condition that $w(r,t)$ decrease as $r \to \infty$ means that $R(r)$ cannot increase more rapidly than r^2, only solutions $R(r)$ that decrease exponentially at infinity need be considered. It follows therefore, in particular, that for the solution that increases with time we have

$$\int_0^\infty wx^2\,dx = 0, \tag{7}$$

i.e., $w(r)$ is of alternating sign. The condition (7) means[6] that in a growing solution the spectrum of the magnetic energy $\langle H^2\rangle/2 = \int M(k)dk$ takes at small wave numbers the form $M(k) \sim k^4$, as it should for solenoidal fields.[7]

The problem (6) was solved numerically by reverse iterations.[8,9] The eigenvalues and eigenfunctions at each step $s = 1,2,\ldots$ of the iterations were calculated from the equations

$$E^{(s+1)} = E^{(s)} + \frac{1}{2m(r_*)}\frac{\tilde{R}^{(s)}(r_*)}{R^{(s+1)}(r_*)}, \tag{8}$$

$$d^2R^{(s+1)}/dr^2 + 2m(E^{(s)} - U)R^{(s+1)} = 2m\tilde{R}^{(s)},$$

$$\tilde{R}^{(s)} = R^{(s)}\left[\int_0^\infty (R^{(s)})^2\,dr\right]^{-1/2}.$$

Here r^* is the value of R at which $|R^{(s+1)}|$ is a maximum. We note that at each step $R^{(s+1)}$ increases in inverse proportion to the difference $E - E^{(s)}$, but by normalizing the function $R^{(s)}$ [i.e., by transforming to $\tilde{R}(s)$] we avoid large numbers in the calculations, and three or four iterations suffice to obtain a normalized eigenfunction.[9] The second equation was approximated by using a three-point scheme on a grid uniform in $r^{1/2}$. The obtained difference equations were solved by the run-through method. In the course of the solution we used the asymptotic forms of (6) at zero and at infinity:

$$R(r) = \begin{cases} r^2\left(1 - R_m\dfrac{E+1}{10}r^2\right), & r \to 0 \\ \exp[-(2m|E|)^{1/2}r], & r \to \infty \end{cases}. \tag{9}$$

For $R^{(0)}$ and $E^{(0)}$ we can use in (8), for example, the quasiclassical approximation.

Since the equation for the Fourier transform $w(k,t)$ is self-adjoint in $\mathscr{L}_2$ space, with a certain weight,[1,3] the Fourier transform $w(k,t)$, which corresponds to the ground state, is positive. Taking into account the boundary condition at $r = 0$, we conclude that the $w(r,t)$ corresponding to the ground state is the correlation function of the field.

As the spatial correlation function of the velocity we consider two characteristic forms:

$$F_1 = \exp(-{}^3/_5 r^2), \qquad F_2 = \exp(-{}^3/_5 \zeta^2), \tag{10}$$

where

$$\zeta(r) = \begin{cases} r, & 0 < r < \varkappa \\ (r/\sqrt{\mathrm{Re}})^{1/3}, & \varkappa < r < \infty \end{cases}.$$

The factor $\frac{3}{5}$ in (10) was chosen to make $f(r) \sim 1 - r^2$ at small r. It can be shown that it does not matter whether $\zeta(r)$ is smooth or not at the point $\varkappa$. These functions satisfy the necessary conditions $F(0) = 1$, $F(\infty) = 0$, have positive Fourier transforms, and are normalized by the condition $f''(0) = -2$. The function F_1 has one characteristic scale, say l. The function F_2 has two characteristic scales, $\varkappa = \mathrm{Re}^{-1/4}$ and $l \sim \mathrm{Re}^{1/2}$, which simulate the Kolmogorov and the basic energy-carrying scales of hydrodynamic turbulence with Reynolds number Re. We recall that our length is measured in units of the so-called Taylor scale λ (Ref. 7) multiplied by $\sqrt{2}$. This scale is not connected with any particular characteristic point of the spectrum of the motions, and determines $f''(0)$, i.e., the slope of the pulsational velocity near small r [see Fig. 1, where $\zeta(r) \sim \delta v$].

The form of the potential $U(r)$ for $F_1(r)$ and for four different values of R_m is shown in Fig. 2. Obviously, at small R_m there are no discrete levels. The first level appears at

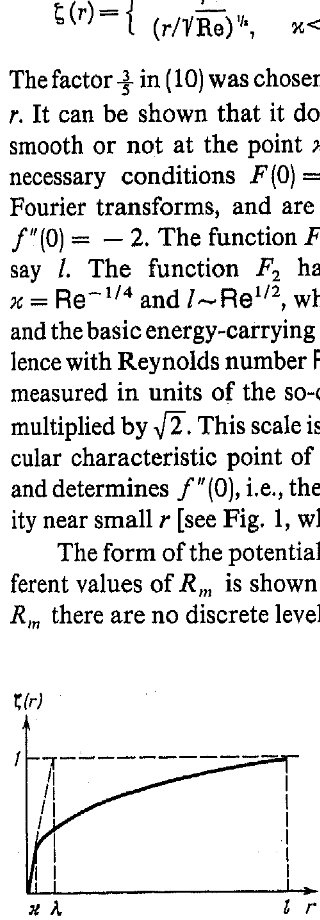

FIG. 1. Plot of the function $\zeta(r)$ that simulates the functional velocity.

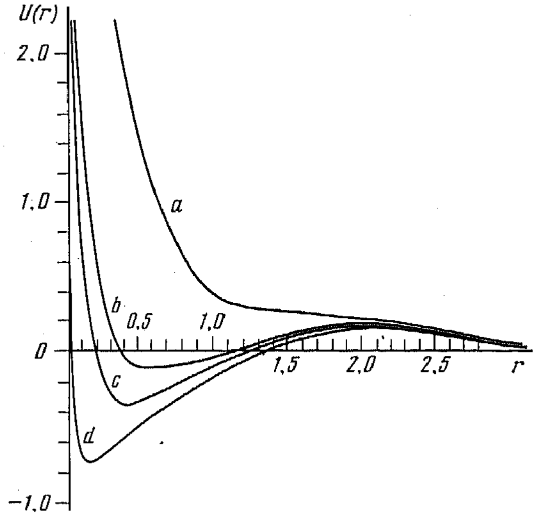

FIG. 2. Potential for $F_1 = \exp(-3r^2/5)$, $R_m = 4$ (*a*), 25 (*b*), 53, (*c*) 10^4 (*d*).

$$(R_m)_{cr} \approx 53. \tag{11}$$

The correlation function $w(r,t)/w(0,t)$ for two values of R_m is shown in Fig. 3. It "hugs" the ordinate axis with increasing R_m, but there is always a tail with $w(r) < 0$. To find the limiting form of $w(r)$ it is convenient to use the substitution $d\rho = dr\sqrt{2m}$ (see Sec. 2). In the limit of large R_m we obtain for $\psi(\rho)$ an equation that does not contain R_m. Such a form of the correlation indicates that although the field becomes concentrated to small scales $\sim R_m^{-1/2}$ with increasing magnetic Reynolds number, there is always an anticorrelation tail in the large scales. Thus, as the magnetic lines approach one another they turn and add up. Consideration of fields in small scales only is therefore insufficient. An analysis of the results demonstrates by the same token the presence of large-scale magnetic structures (of the same scale as the velocity field). The smallness of the amplitude of the $w(r)$ tail, however, is evidence that these structures occupy a small volume (alternation).

Figure 4 shows the dependence of the growth rate $\gamma(R_m)$ on the magnetic Reynolds number for F_1. We note

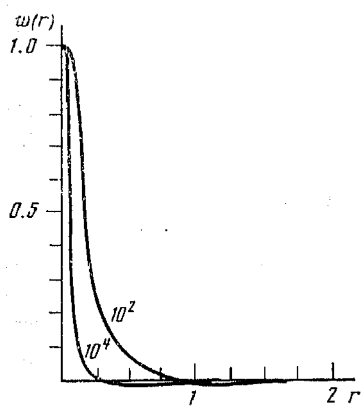

FIG. 3. Correlation function of magnetic field for two values, $R_m = 10^2$ and 10^4, (indicated in the figure) and $F_1(r)$.

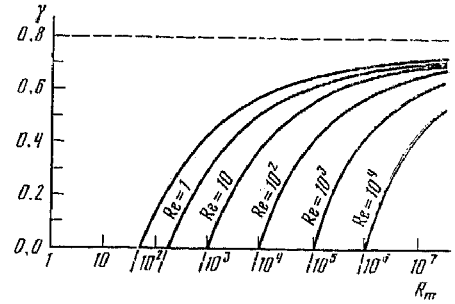

FIG. 4. Rate of exponential growth of magnetic field as a function of the magnetic Reynolds number for F_1 and F_2 with different values of Re.

that as at the intercept with the abscissa axis ($\gamma = 0$) the $\gamma(R_m)$ curve has a finite derivative. This is due to the presence of potential barrier similar to the centrifugal one, inasmuch as at large r we have according to (4)

$$U(r) \approx (mr^2)^{-1} \approx 2/3r^2.$$

For the same reason, the $\gamma(R_m)$ curve is uninterruptedly continued into the subcritical region $R_m < (R_m)_{cr}$ (dashed in Fig. 4), where it has the meaning of the real part of the complex energy (quasistationary states). Obviously, $d\,\mathrm{Im}\,\gamma/dR_m \to 0$, $R_m \to (R_m)_{cr}$. This agrees with the known quantum-mechanical analysis of the spectrum of the Schrödinger equation.[10]

At $R_m \gg 1$ the field growth rate, in dimensional units, is

$$\gamma = \left(\gamma_0 \frac{l}{\lambda\sqrt{2}}\right) \frac{v}{l} \leqslant 0.8\,\frac{v}{l}, \tag{12}$$

where $\gamma_0 \approx 0.7$ according to numerical calculations (see Fig. 4).

In the case described by the correlation function $F_2(r,\mathrm{Re})$, the parameter Re appears in addition to R_m. Of practical interest in our problem is the situation with $R_m \gg 1$ and $\mathrm{Re} \gg 1$. A potential well apears only at $R_m > 100$ Re and is located at small r (of the order of the scale $R_m^{-1/2}$). Its maximum depth (as $R_m \to \infty$) does not depend on Re and is again equal to $-\frac{4}{5}$. On the (Re, R_m) plane of Fig. 5, the straight line (at $\mathrm{Re} > 10$)

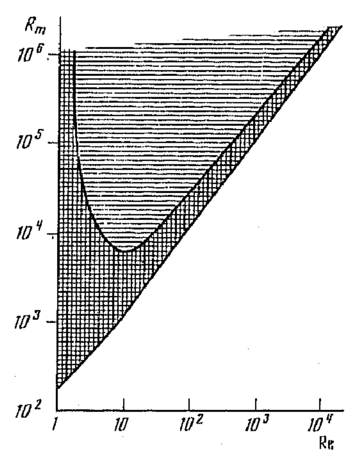

FIG. 5. The region of self-excitation of the magnetic field in the $F_2(r,\mathrm{Re})$ model lies above the straight line (single-hatched). The region where the model is realistic ($\gamma < v/l$) is cross-hatched).

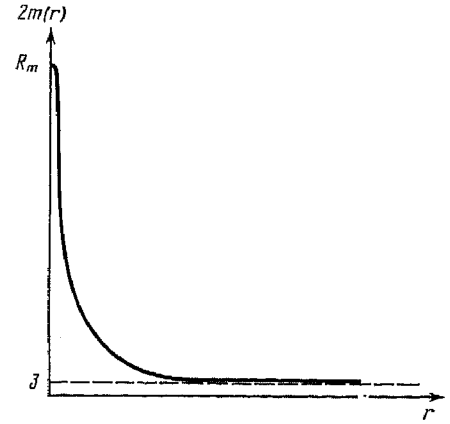

FIG. 6. Effective mass (which is the inverse of the turbulent-diffusion coefficient) as a function of r.

$$\lg R_m = \lg \mathrm{Re} + 2$$

separates the region of self-excitation of the field (on the left). The field growth rate is in this case much faster than (12): $\gamma \sim (v/l)\mathrm{Re}^{1/2}$, since $l/\lambda \sim \mathrm{Re}^{1/2}$.

To conclude this section we note that the results can be qualitatively understood by using the quasiclassical approximation. Although the use of this approximation is not verified formally (in particular, the eigenfunction is not at all similar to the quasiclassical one), it does yield the dependence of the excitation condition on the parameters of the problem. This is possible because at large R_m the quasiclassical integral $\int \sqrt{2m(E-U)}\,dr$ tends, albeit weakly (as $\ln R_m$) to infinity because of the growth of the effective mass (see Fig. 6). The estimate obtained[6] for the threshold in the quasiclassical approximation differs from the value (11) calculated above by not more than 20%.

4. GROWTH OF THE FIELD WITH TIME IN THE ABSENCE OF SELF-EXCITATION

When the magnetic Reynolds number is lower than the critical value there is no self-excitation and the spectrum is continuous and positive. However, as first noted in Ref. 11 with two-dimensional turbulence as the example, at sufficiently large R_m (but smaller than critical) the evolution of the initial field does not reduce to a smooth damping, and the magnetic energy can increase with time. In the two-dimensional case the effect is explained as follows.

Imagine a magnetic field of very large scale. This field hardly diffuses initially, and only becomes tangled under the influence of turbulent diffusion, with the scale decreasing and the single-component vector potential conserved. Therefore the magnetic-energy density, which is proportional to the square of the vector-potential gradient, increases until ohmic dissipation comes into play.

In the two-dimensional case the potential $U(r)$ is always positive (see the Appendix) and takes the form of a centrifugal potential $2mU = 3/4r^2$, i.e., there is no potential barrier. the spectrum is therefore real and positive at all R_m. The increase of the magnetic energy with time in a two-dimensional isotropic turbulence was demonstrated by numerical experiments by Pouquet[12] and by Orszag and Tang,[13] who specified at the initial instant a smooth field distribution with an energy spectrum of the type $k\exp(-k)$.

The effect of the growth of the magnetic energy with time can be understood also with the aid of Eq. (3). This equation, obtained assuming homogeneity and isotropy in the mean, is of course not suitable for the description of the behavior of the initial homogeneous field. It is possible, however, to consider a field whose characteristic scale is still large enough (of the order of the characteristic length l of the velocity field) and its distribution can at the same time be regarded as isotropic.

We are dealing here with a nonstationary problem described by Eq. (3) with initial condition $\psi|_{t=0} = \psi_0(r)$. By virtue of the conditions imposed on $w(r)$, the initial function $\psi_0(r)$ should vanish at $r\to\infty$ and $r=0$. Inasmuch as in the region $r \lesssim R_m^{-1/2}$ the potential $U(r)$ takes the form of the "wall" $2/R_m r^2$, the function $\psi(r,t)$ can be taken in this region to be the parabola $r^2 w(0,t)$, where $w(0,t)$ is the magnetic energy. It suffices therefore to show that $\psi(r,t)$ increases at the point $r_m R_m^{-1/2}$. By the same token, we are interested in the solution of an equation of the heat-conduction type at the point r_m. The presence of a centrifugal potential at large r can, as usual, be taken into account by changing the phase of the ψ-function.

Let us specify the initial function $\psi_0(r)$ in the form of a distribution with a maximum at $r = r_1 \sim 1$. This distribution begins to spread out towards larger and smaller r. The spreading over large distances is determined by the turbulent coefficient of diffusion and is rapid, as $t^{-1/2}\exp(-r^2/4\nu_T t)$. On going from r_1 to smaller distances, however, the diffusion coefficient $\nu_T = (2m)^{-1}$ decreases and tends to ν_m as $r\to 0$.

The magnetic energy (which is proportional to $\psi[r_m,t)]$ thus increases with time because of the exponential spreading of the initial distribution $\psi_0(r)$ which is concentrated at large r. Since the diffusion coefficient (the mass m) is variable, this growth is lengthened. Estimates show that the characteristic growth time depends logarithmically on the magnetic Reynolds number. After reaching the maximum the energy decreases in power-law fashion.

We note that a scalar quantity, say temperature, cannot increase with time, since there is no "potential wall" here and the maximum of the correlator of the scalar proportional to $\psi(r,t)$ is located at $r=0$, i.e., it can only decrease on spreading.

In the three-dimensional case, at sufficiently large magnetic Reynolds numbers, but lower than critical, the magnetic energy can likewise increase with time. The arguments presented above are by themselves sufficient to prove this statement. In contrast to the two-dimensional case, there is a potential barrier in the three-dimensional one. Therefore at certain R_m close to critical an additional growth of the ψ-function is possible not only in the region of the viscous scale, but also in the velocity-field scale. This growth is due to the onset of a quasistationary state. Indeed, since Eq. (3) coincides with the Schrödinger equation when the substitution $t\to it$ is made, the quantum-mechanical behavior of $\psi \sim \exp(iEt)$, $E = E_0 - i\Gamma/2$ (Ref. 14) corresponds to

$$\psi \sim \exp(-2Et)\sin(\Gamma t+\delta),$$

i.e.,at the appropriate phase ψ can increase with time because of the initial increase of the sine function.

5. EVOLUTION OF LOCALIZED DISTRIBUTION UPON SELF-EXCITATION

So far we have assumed that the distribution of the initial magnetic energy is uniform, and we were interested in the behavior of the correlation function $w(r,t)$, which depends only on the distance between the points r_1 and r_2 at which the values of the field are chosen. Let us discuss (as suggested by Ya. B. Zel'dovich) the evolution of an initially inhomogeneous distribution of the magnetic field. The spectrum of this problem is always continuous: its upper bound is determined by the growth rate γ.

At a uniform distribution of the velocity, the change of the magnetic energy at a given point of space $\tilde{r} = r_1 = r_2$

$$\mathscr{H}(\tilde{r}, t) = \langle H_i(r_1, t) H_i(r_2, t)\rangle$$

is determined by the growth of the field on account of the dynamo process, and by its transport by the turbulent (and ohmic) diffusion. Over sufficiently long times, therefore, when the characteristic dimension of the inhomogeneities becomes larger than the magnetic-field correlation length, we have

$$\partial\mathscr{H}/\partial t = D\Delta\mathscr{H}+2\gamma\mathscr{H}, \quad D=\nu_m+\nu_T(\infty)=1/2m(\infty). \qquad (13)$$

An equation of this type (with the nonlinearity taken into account) was first studied by Kolmogorov, Petrovskiĭ, and Piskunov, and is widely used in combustion theory.[15,16] The principal feature of the solutions of Eq. (13) is that the surfaces $\mathscr{H}(\tilde{r},t) =$ const propagate with a velocity determined by D and γ. It is known [15] that this velocity is equal (in our notation) to $2(2\gamma D)^{1/2}$. This is easiest shown by starting with a δ-localized energy distribution $\mathscr{H}(\tilde{r},0) = \mathscr{H}_0\delta(\tilde{r})$. We seek a solution in the form $\mathscr{H} = n(\tilde{r},t)\exp(2\gamma t)$. We obtain then for n simply the diffusion equation. Consequently

$$\mathscr{H}(\tilde{r}, t) = \frac{1}{(4\pi Dt)^{3/2}}\exp\left(2\gamma t - \frac{\tilde{r}^2}{4Dt}\right).$$

The argument of the exponential can be represented in the form

$$2\gamma t - \frac{\tilde{r}^2}{4Dt} = -\frac{1}{4Dt}[\tilde{r}-2(2\gamma D)^{1/2}t][\tilde{r}+2(2\gamma D)^{1/2}t],$$

whence it is seen in fact that the surface $\mathscr{H} =$ const propagates at a velocity $2(2\gamma D)^{1/2}$. We note that the result remains the same when account is taken of the nonlinearity, which merely limits the growth of the amplitude.

6. REMARKS ON THE REALIZABILITY OF THE CONSIDERED VELOCITY-FIELD MODEL

Let us discuss the degree to which the foregoing results can be applied to real turbulent flows of a conducting liquid.

For flows with a longitudinal correlation close to $F_1(r)$, i.e., containing in fact a single characteristic scale, the model studied above can be readily used to determine the excitation threshold and the growth rate, inasmuch as even the limit $\gamma^{-1} > l/v$.

The situation becomes considerably more complicated when we consider a correlator $F_2(r,\text{Re})$ with two greatly differing scales. In this case the model is self-consistent only near the excitation threshold, since $\text{Re}\gamma$ (R_m, Re) increases with Re as $\text{Re}^{1/2}$. The self-consistency region $\gamma < v/l$ of the δ-correlated model is cross-hatched in Fig. 5. These conclusions depend very strongly on the form of the function $\zeta(r)$ in the region $\varkappa \lesssim r < l$. If it is assumed that ζ is proportional not to $r^{1/3}$ but, say, to $r^{\alpha/2}$, i.e., $F(r) \approx 1 - Ar^{\alpha}$, the field self-excitation conditions at $\alpha \gtrsim 1.3$ (Ref. 1) turn out in the model considered to be independent of the relation between Re and R_m.

It is natural to expect that allowance for the finite character of the correlation times of the velocity field leads to an integral equation for $w(r,t)$, and this equation goes over into the differential equation (3) only in the limit of short correlation times. Actually, certain model forms of such an equation were used in Refs. 2 and 4. In the model of Ref. 4, the field self-excitation conditions turned out to be independent of the relations between[1)] Re and R_m. The generation picture is close to that obtained for the case of $F_1(r)$. It can be assumed that an effective "averaging" of the correlation function $F_2(r,\text{Re})$ takes place in the integral equation, so that this function acts in fact like $F_1(r)$. We note that the choice of the exponential form of F_1, see Eq. (10), is not essential in principle. Calculations with the decreasing power-law function $(1+r^2)^{-1}$ and with the alternating-sign function $(1-r^2/2)\exp(-r^2/2)$ show that the deviation from $F_1 = \exp(-3r^2/5)$ is only quantitative (the threshold value of the magnetic Reynolds number changes somewhat).

We have assumed that the spatial annd temporal variables in the velocity correlator (1) are separable. We note that this separability is not obligatory as, e.g., in the Chandrasekhar turbulence theory (see Ref. 7).

Summarizing, we can conclude that a reflection-invariant random velocity field is capable of acting as a hydromagnetic dynamo at relatively small Reynolds numbers, attainable not only under astrophysical but also under laboratory conditions.

We thank Ya. B. Zel'dovich and A. P. Kazantsev for discussions and helpful remarks.

APPENDIX

The correlation tensors of a two-dimensional isotropic divergence-free velocity field and of a magnetic field are of the form (cf. Refs. 1 and 2)

$$\langle v_i v_k\rangle = \frac{1}{2}\left[F\delta_{ik}+r\frac{dF}{dr}\left(\delta_{ik}-\frac{r_i r_k}{r^2}\right)\right],$$

$$\langle H_i H_k\rangle = \frac{1}{2}\left[W\delta_{ik}+r\frac{dW}{dr}\left(\delta_{ik}-\frac{r_i r_k}{r^2}\right)\right], \qquad i,k=1,2;$$

$$f = \frac{1}{2r}\frac{d}{dr}(r^2F), \qquad w = \frac{1}{2r}\frac{\partial}{\partial r}(r^2W).$$

Here and below we retain the three-dimensional notation.

It can be shown, proceeding just as in the three-dimensional case, that

$$\psi=W\exp\left\{\frac{1}{2}\int_{r_0}^{r}\frac{6R_m^{-1}+3F(0)+F(r)-4f(r)}{2R_m^{-1}+F(0)-F(r)}\frac{dr}{r}\right\}=\frac{r^{3/2}}{m}W,$$

$$\frac{1}{2m}=\frac{1}{R_m}+\frac{1-F(r)}{2},\qquad U(r)=\frac{3}{8mr^2}.$$

$U\to 3/(4R_m r^2)$ as $r\to 0$ and $U\to 3/8r^2$ as $r\to\infty$. If $R_m\to\infty$, then $U\to 3/16$ as $r\to 0$, i.e., unlike in the three-dimensional case the potential is everywhere nonnegative.

[1)] To avoid misunderstandings when results of different studies are compared, it must be borne in mind that the definitions of the magnetic Reynolds number vary. In Ref. 5, for example, the magnetic Reynolds number differs from that calculated by us for $F_1(r)$ by a factor $2/\pi$, and in Ref. 6 the results are given for a magnetic number $2^{1/2}\lambda v/\nu_m = R_m(2^{1/2}\lambda/l)$.

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