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Dynamo processes: the interaction of turbulence and magnetic fields

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This chapter reviews recent research on the interaction of magnetic fields with MHD turbulence, with particular application to the question of the influence of Lorentz forces on the efficiency of large-scale field generation.

1.1 Scales for Solar Magnetic Fields

The solar magnetic field outside the radiative core exists on a great range of length and time scales; these embrace all sizes from that of the disc itself to that of the diffusion length scales of a few km, well below present observational resolution. While it is the largest scales that force themselves on our attention, due to the visibility of sunspots and associated coronal structures, and the coherence of the solar cycle, it is not clear whether these large-scale fields control, or are controlled by, the small-scale fields that have much greater total energy. While the cycle is clearly global in nature, the "magnetic carpet" of small-scale field structures that appear in quiet regions would seem to be a local manifestation of dynamo action due to turbulent stretching.

Linear dynamo theory, in particular the "mean-field" or "α-effect" models, has proved amazingly successful in predicting aspects of the solar cycle such as the butterfly diagram. In fact some of this 'success' has nothing to do with the physics employed, but derives from the symmetry of the underlying geometry. As Knobloch (1994) convincingly argues, all manifestations of oscillatory wavelike processes in a finite domain with reflexional symmetry will take the form at onset of travelling wave structures with either dipole or quadrupole symmetry. In this severe view all that remains is to determine the direction of travel of the wave crests. Furthermore, the physics of the mechanism is by no means fully understood, even in the kinematic limit where Lorentz forces are ignored. While the mechanism for production of toroidal field by differential rotation is fairly secure, there is difficulty in obtaining a convincing and detailed explanation of the return of coherent field to great depths, as is now demanded by 'interface' models of the dynamo (Parker 1993). The favoured thinking at the present time is that the action of the three-dimensional turbulence acts to pump field down to a level where it can be acted on by the zonal flow. Even within this paradigm, however, there is disagreement as to whether the source of the helical flows with strong vertical component are just turbulent, as in the original Parker (1955) picture, or whether the process involves magnetic buoyancy and is essentially nonlinear. (see e.g. Thelen 1997). Large-scale photospheric fields, such as sunspots and pores, have important dynamics of their own, which is outside the scope of this paper. But the cyclical nature of the appearance of bipolar regions and the regularity of their latitudinal distribution indicate that they are part of of the same large-scale process, and are not the products of autonomous dynamo action.

Finally, we come to the magnetic carpet, the small-scale fields clearly shown in recent TRACE observations (Title 2000). These fields are if anything more prominent during times of reduced sunspot activity; they show little or no cyclical behaviour and are seen well away from active regions. It seems likely that these fields are caused by local dynamo action. When trying to understand such dynamo action at very high magnetic Reynolds numbers ($R_m \equiv UL/\eta$, where U, L are velocity and length scales and η is the magnetic diffusivity), it is important to escape from the preconceptions induced by kinematic dynamo studies and the MHD of steady flows. In a turbulent flow the field lines are stretched at almost all locations, as nearby trajectories of the flow particles separate exponentially. Thus there is a strong mechanism for enhancing magnetic energy locally. In the fully developed state, although there will be some intermittency due to cancellation caused by folding of the trajectories, the fluid will be permeated with field to a much greater extent than would appear at the surface. This is because, at a boundary, areas of surface particles are not conserved. We therefore expect to find tangled fields with a fractal dimension between 1 and 2, even in the limit $\eta \to 0$. These fields will exert a significant dynamical influence on the flow, and so can be expected to be at equipartition levels. Such a "magnetic fondue" can be glimpsed in the magnetic carpet, but the basic arguments apply to all scales where the magnetic Reynolds numbers are large, and where the turnover time (which, rather than the diffusion time, is the appropriate time for growth of the fields) are not too large. The mechanisms at larger scales are likely to be affected by Coriolis forces; however the magnetic carpet, and numerical simulation, (Cattaneo 1999, Cattaneo

& Hughes 2001)show that large-scale rotation is not necessary for dynamo action. The problem, therefore, is to understand how the large-scale observed fields can be generated (the "α" part of an α-ω dynamo), in these magnetically dominated flows. The answers are still highly controversial. This paper reviews recent theoretical ideas and associated numerical work in an attempt to throw light on the difficulties.

1.2 Field structure in kinematic dynamos at large $\mathbf{R_m}$

The α-effect, or mean field dynamo, has long been a mainstay of theories of the solar cycle, and it is still widely used today. The text by Moffatt (1978) gives an excellent overview of early applications, while more recent references can be found in Weiss (1994). There are two basic assumptions; that there is scale separation between 'mean' and 'fluctuating' fields; and that the averaged e.m.f. induced by the small-scale fields is a local function of the mean magnetic field and its derivatives. The first assumption would seem reasonable, but the second is harder to justify in the interesting case where the magnetic Reynolds number is very large, even on the smallest scales. (When the small-scale $R_m \ll 1$ then a rigorous theory can be constructed; see e.g. Moffatt's book). There are several important consequences of large R_m (Galloway & Proctor 1992, Cattaneo *et al* 1995):

- Field structures are highly intermittent, with length scales $\sim R_m^{-\frac{1}{2}}$.
- These structures do not depend much on the value of R_m, but on the topology of the flow pattern. Only the thickness of the structures depends on R_m.
- These small-scale fields can be *self sustaining*; that is, there is a small-scale dynamo.

The smallest scales of the field appear very rapidly in kinematic computations at high R_m – in fact after a few turnover times L/U. However the growth rate of dynamo disturbances does depend on R_m, but typically appears to reach a limiting value independent of R_m as $R_m \to \infty$, though precise computation becomes very difficult owing to to the small length scales involved. This limiting growth rate is typically of order L/U; these are known as *fast dynamos* (see, e.g. Childress & Gilbert 1995). In spite of the difficulty in resolving the smallest structures, we find that scaling laws for the eigenfunctions are established accurately at much lower R_m and can be accurately calculated. Such laws give a power-law distribution for integrated quantities such as $R_1 \equiv |\langle \mathbf{B} \rangle|^2 / \langle |\mathbf{B}|^2 \rangle$, which $\sim R_m^{\gamma}$, where γ is a constant of order unity depending on the flow, as shown in Figure 1.1.

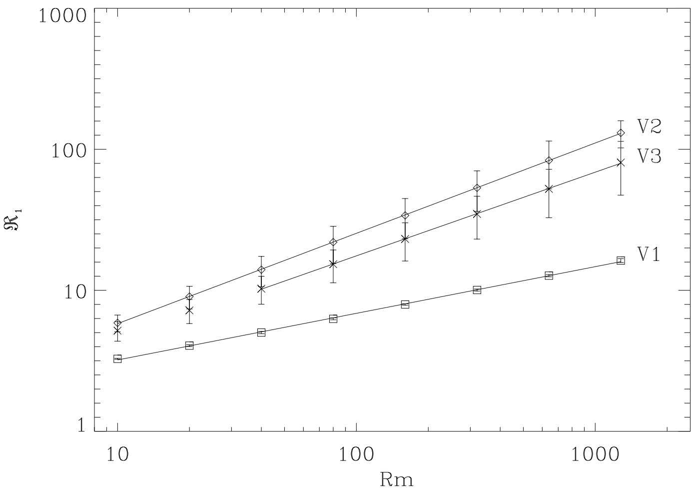

Fig. 1.1. Behaviour of the quantity R_1 defined in the text as a function of R_m for three different dynamo flows (from Cattaneo *et al.* 1995). The greater the slope, the greater the energy in the small-scale fields.

These power laws demonstrate that the field distribution is fractal in nature; and indeed if γ is not too small then at large R_m the smallest scales of field are dominant, as is perhaps to be expected. One final aspect of these kinematic fast dynamos deserves attention. When the dynamo field exists on essentially the same scale as the velocity field, helicity of the flow is not necessary for efficient dynamo action. This shows that the fact that the magnetic carpet fields are on too short a timescale to notice the Sun's rotation does not rule out dynamo action as their cause. Such fields will not work as mean-field dynamos (see below), because for them helicity is essential. If buoyancy is the principal driving mechanism then helicity cannot be introduced directly; it follows that for an efficient mean field dynamo we require either rotation or inhomogeneity (giving gradients of large-scale helicity).

1.3 Dynamical equilibration of small-scale dynamos

How large can a small-scale dynamo field get before the growth of the field is halted by the dynamical effects of the Lorentz force? In the solar context, where the viscosity is small, we expect such effects to occur when the magnetic energy density $|\mathbf{B}|^2/2\mu_0$ is comparable with the kinetic energy density

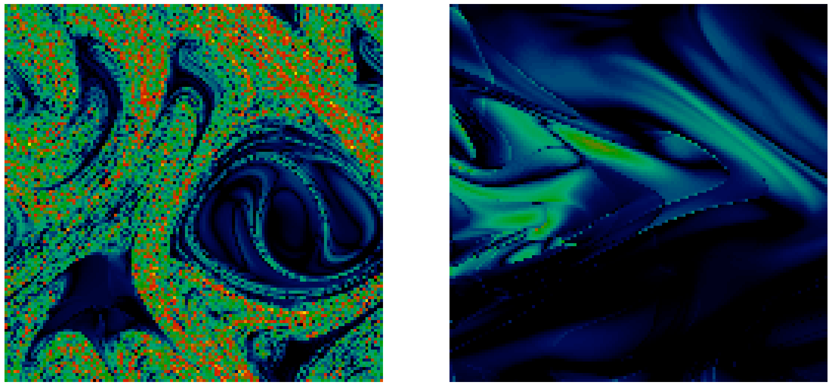

Fig. 1.2. Finite time Liapunov exponents for a simple quasi-two dimensional dynamo (after Cattaneo *et al.* 1996). Lighter shades indicate greater stretching. (a) Kinematic case, (b) dynamic case when Lorentz forces have reduced the stretching properties of the flow.

$\rho|\mathbf{u}|^2$ (equipartition). This expectation is confirmed by the results of several calculations of model dynamos, and by the full MHD simulation of a convective dynamo by Cattaneo (1999). At high values of R_m, as the field amplitude grows, we must pass from a growth rate comparable with the turnover time to one which is zero! How is this accomplished? One mechanism, which would hold for spatially constrained flows, would be for the kinetic energy to be reduced, thus reducing the magnetic Reynolds number towards the critical value. This is most unlikely to happen when the kinematic R_m is far above critical, since that would demand a huge reduction in the kinetic energy. Instead, these systems equilibrate in a much more subtle way, which is almost invisible in the Eulerian statistics. An example is given for a simplified model by Cattaneo, Hughes & Kim (1996), and examples of finite-time Liapunov exponents for the kinematic and dynamic cases are shown in Figure 1.2 They reduce their efficiency as a dynamo by altering their stretching properties, so that the Liapunov exponents go down, leading to less efficient energy growth, leaving cancellation effects to mop up such growth as remains. (It is possible that in some cases the cancellation is enhanced, rather than the stretching reduced. But the detailed results produced to date do not show this. Such enhancement is more likely to be a consequence of two-dimensionalization of the flow induced by a large-scale field.) How long does it take for equilibration to be achieved? The growth rates at high R_m are fastest for the smallest scales of motion, so one could expect that each scale might become dynamically active after a time proportional to its turnover time. Magnetic energy reaches equipartition successively at longer and longer scales. Finally we have "MHD turbulence" with Lorentz forces important at all scales. The crucial question for the coherent dynamo involved in the solar cycle is: can fields which have a scale much greater than that of the turbulent flows grow at a substantial rate? Thus we need to address the dynamical effect of the Lorentz force on mean field growth.

1.4 Growth and equilibration of mean fields

In this section we discuss the way in which large-scale ("mean") fields can arise as a result of small-scale fluctuating motion. We first note that the distinction between large and small scales is only clear when the small-scale turbulence is homogeneous. Any systematic large-scale inhomogeneity will ineluctably lead to components of the Fourier spectrum of the field on the same scale. These are of a different nature, however, from the independently generated fields that form the cycle. The effects of the small-scale on the large-scale fields may be seen by writing the magnetic field $\mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}$ and the velocity field $\mathbf{U} = \overline{\mathbf{U}} + \mathbf{u}$; then the induction equation for time derivative of the mean field $\overline{\mathbf{B}}$ becomes

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\overline{\mathbf{U}} \times \overline{\mathbf{B}}) + \nabla \times \boldsymbol{\mathcal{E}} - \nabla \times \eta \nabla \times \overline{\mathbf{B}} \,, \text{where } \; \boldsymbol{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}} \,. \quad (1.1)$$

In order to get significant mean fields on a relevant (i.e. non-diffusive) timescale we need the "α-effect" $\boldsymbol{\alpha}$, defined by the *ansatz* $\boldsymbol{\mathcal{E}} = \boldsymbol{\alpha} \cdot \overline{\mathbf{B}}$, to be of order $|\mathbf{u}|$, i.e. independent of η, the magnetic diffusivity. While $\boldsymbol{\alpha}$ is straightforward to calculate when the small-scale R_m is small (see, e.g. Moffatt 1978), it is much harder to see how to proceed when the small-scale R_m is large. In the Parker (1955) picture field lines are twisted and rotated by a helical "cyclonic event". For events shorter than the turnover time we can say that $\boldsymbol{\mathcal{E}}$ is proportional to $-\mathcal{H}$, where $\mathcal{H}$ is the helicity. But if such an event persists longer than a turnover time the constant of proportionality may change sign due to multiple rotations. Thus even the sign of the effect is not certain, and there are other problems associated with the possible nonlocal dependence of $\boldsymbol{\mathcal{E}}$ on $\overline{\mathbf{B}}$. Nonetheless, one can imagine an experiment in which a *uniform* magnetic field $\mathbf{B}_0$ permeates a region of homogeneous MHD turbulence. There is no large scale dynamo but $\boldsymbol{\mathcal{E}}$ can be calculated as a function of $\mathbf{B}_0$. It is crucial to understand how $\boldsymbol{\alpha}$ depends on $\mathbf{B}_0$. We expect it to reduce with increasing field ("α-quenching"), but when do Lorentz forces become important and initiate this quenching? There is considerable controversy over this question. To fix ideas, define B_E, the equipartition field strength, as $(\mu_0\rho\overline{|\mathbf{u}^2|})^{\frac{1}{2}}$. Then we can all agree that because of the symmetry under sign change of $\overline{\mathbf{B}}$, we expect some functional dependence for large R_m of the form

$$\boldsymbol{\alpha}(\overline{\mathbf{B}}) = \mathcal{F}(R_m^a|\overline{\mathbf{B}}|^2/B_E^2). \tag{1.2}$$

The controversy resides in the value of the exponent a. If $a \ll 1$ then the large-scale fields can reach equipartition values with relative ease, while if a is not small the mean field mechanism shuts down when $|\overline{\mathbf{B}}|$ is still well below B_E, making the timescales for the production of large-scale fields inordinately long.

Before looking at recent simulations which cast light on the value of a, we first deal with the formula for $\boldsymbol{\alpha}$ in MHD turbulence originally put forward by Pouquet, Frisch & Léorat (1976) and revisited by Blackman & Field (2000). We begin with fluctuating magnetic and velocity fields $\mathbf{b}$, $\mathbf{u}$. Then a uniform field $\overline{\mathbf{B}}$ is added, and this has the effect of changing the fluctuating fields to $\mathbf{b}+\mathbf{b}'$, $\mathbf{u}+\mathbf{u}'$, where $\mathbf{b}'$, $\mathbf{u}'$ obey the equations

$$\left.\begin{aligned} \frac{\partial \mathbf{u}'}{\partial t} &= -\nabla p + \frac{1}{\mu_0\rho}\overline{\mathbf{B}}\cdot\nabla\mathbf{b} \\ \frac{\partial \mathbf{b}'}{\partial t} &= \qquad\qquad \overline{\mathbf{B}}\cdot\nabla\mathbf{u} \end{aligned}\right\} + \text{small(?) diffusion terms} \tag{1.3}$$

To find the mean e.m.f. proportional to $\overline{\mathbf{B}}$, we can assume isotropy, so that $\boldsymbol{\alpha}_{ij} = \alpha\boldsymbol{\delta}_{ij}$. Thus we have (the dots denoting time derivatives)

$$\begin{aligned} \boldsymbol{\mathcal{E}} = \alpha\overline{\mathbf{B}} &= \overline{\mathbf{u}\times\mathbf{b}'} + \overline{\mathbf{u}'\times\mathbf{b}} \\ &\approx \int_0^{\tau_c}\left(\overline{\mathbf{u}\times\dot{\mathbf{b}}'} + \overline{\dot{\mathbf{u}}'\times\mathbf{b}}\right)dt \quad \text{where } \tau_c \text{ is a "correlation time"} \end{aligned} \tag{1.4}$$

If τ_c is short cf. other timescales then we can use (1.3) to obtain

$$\alpha \approx -\frac{\tau_c}{3}\left(\overline{\mathbf{u}\cdot\nabla\times\mathbf{u}} - (\mu_0\rho)^{-1}\overline{\mathbf{b}\cdot\nabla\times\mathbf{b}}\right) . \tag{1.5}$$

There are many assumptions made in this derivation, not least the one that equates correlation times for velocity and magnetic fields. Nonetheless the expression (1.5) does have the satisfying characteristic that if the "turbulence" takes the form of Alfvén waves, for which $\mathbf{u} = \pm\mathbf{b}/\sqrt{\mu_0\rho}$, then $\boldsymbol{\mathcal{E}}$ must vanish. Unfortunately the formula has been interpreted by many authors as giving a model of the effects of large imposed fields on α, with $\mathbf{u}$, $\mathbf{b}$ considered as the actual fields. In fact the formula can be justified only for *small* $|\overline{\mathbf{B}}|$, since equations (1.3) can then be linearized; and where the field

$\mathbf{b}$ has nothing to do with $\overline{\mathbf{B}}$ but is preexisting. Nonetheless it is useful as a guide to the initial growth rate of a large-scale field in the presence of MHD turbulence. It should be emphasised that the induction equation remains linear irrespective of the effects of the Lorentz force, and so the last term in (1.5) can *only* arise from magnetic fields that do *not* owe their existence to the imposed field $\overline{\mathbf{B}}$. This is not the situation considered by Moffatt (1978) and others.

Whether or not the above result remains true for large imposed fields, there remains the crucial question posed above: what is the form of the function $\mathcal{F}$ defined in (1.2), and what in particular is the crucial exponent a? In general terms we expect that $\mathcal{F}(X)$ decreases with X, and $\sim X^{-\beta}$ as $X \to \infty$, with $\beta \geq 1$. The existence of large-scale fields of significant amplitude suggests that a is small, while numerical calculations of idealized problems suggest that $a \sim 1$, which must lead to significant problems with the large-scale fields. In consequence these calculations have been criticized as inapplicable to real MHD turbulence. Nonetheless there are several theoretical reasons for supposing a significant, and the critics have not yet found a definitive solution to the difficulty.

The theoretical backing for a to be significant is provided by Gruzinov & Diamond (1994, 1995). They consider a situation in which magnetic and velocity fields are statistically stationary. This implies that the time derivative of the mean magnetic helicity $\overline{\mathbf{a}\cdot\mathbf{b}}$ vanishes, where $\mathbf{a}$ is the magnetic potential defined by $\mathbf{b} = \nabla\times\mathbf{a}$, $\nabla\cdot\mathbf{a} = 0$. The equations for $\mathbf{a}$ and $\mathbf{b}$ are

$$\left.\begin{aligned} \frac{\partial \mathbf{a}}{\partial t} &= (\mathbf{u}\times\overline{\mathbf{B}}) + (\mathbf{u}\times\mathbf{b}) - \nabla\chi - \eta\nabla\times\mathbf{b} \\ \frac{\partial \mathbf{b}}{\partial t} &= \nabla\times(\mathbf{u}\times\overline{\mathbf{B}}) + \nabla\times(\mathbf{u}\times\mathbf{b}) - \nabla\times(\eta\nabla\times\mathbf{b}) \end{aligned}\right\} \tag{1.6}$$

where χ is the electrostatic potential. Setting $\dfrac{\partial}{\partial t}(\overline{\mathbf{a}\cdot\mathbf{b}}) = 0$, we obtain after some manipulation

$$\overline{\mathbf{B}}\cdot(\overline{\mathbf{u}\times\mathbf{b}}) = \overline{\mathbf{B}}\cdot\boldsymbol{\mathcal{E}} = -\eta\,\overline{\mathbf{b}\cdot\nabla\times\mathbf{b}},$$

and so we have the exact result (not depending on any assumptions concerning small R_m or short correlation times)

$$\alpha = -|\overline{\mathbf{B}}|^{-2}\eta\,\overline{\mathbf{b}\cdot\nabla\times\mathbf{b}}. \tag{1.7}$$

It should be noted here that the field $\mathbf{b}$ is now the total small scale field; there is no approximation involving small $|\overline{\mathbf{B}}|$. Gruzinov & Diamond use (1.7) in combination with (1.5) to give a relation between α and $|\overline{\mathbf{B}}|$ of the form

$$\alpha = \alpha_0(1 + R_m|\overline{\mathbf{B}}|^2/B_E^2)^{-1},$$

where α_0 is the kinematic α-effect that holds when Lorentz forces are negligible. which suggests that $a = 1$. Although this result is very appealing, it must be recalled that the definitions of $\mathbf{b}$ in (1.5) and (1.7) are not obviously compatible. Further calculations establish that the part of $\boldsymbol{\mathcal{E}}$ proportional to gradients of $\overline{\mathbf{B}}$ (the 'turbulent diffusivity') only depends on $|\overline{\mathbf{B}}^2|/B_E^2$.

The physical picture that backs up the theory has been elaborated by Cattaneo & Hughes (1996), and recently given support by Brandenburg (2001). The basic idea is simple. The dynamical effects of the magnetic field on the flow must be felt when the Lorentz forces become significant. In flows of astrophysical interest, $R_m \gg 1$ even on the small scales, and in this case $|\mathbf{b}| \gg |\overline{\mathbf{B}}|$. In fact when the fields are sufficiently weak the growth of small-scale field is limited by diffusion in regions of flow convergence, and so we expect $|\mathbf{b}| \sim R_m^{\frac{1}{2}}|\overline{\mathbf{B}}|$ if the field is in sheets. When we have flux tube type structures, the amplification factor is larger but the dynamical effects smaller. In either case, when the small-scale field reaches the equipartition value we expect a significant change in the dynamo process. Thus the physical picture predicts $a \approx 1$. Although the small-scale field is highly intermittent the crucial mechanism of dynamo generation occurs precisely where the small-scale fields are being produced – and so such intermittency is unlikely to affect the value of a significantly.

These ideas have their origin in simpler studies in two dimensions (e.g. Vainshtein & Cattaneo 1992) investigating the effects of the Lorentz force on the diffusion rate of an imposed large scale field. Here there is no dynamo, but similar considerations suggest that the stretching properties of the flow are affected, leading to a decrease in the turbulent diffusivity, In that work it is argued that the conservation of the mean square magnetic potential in the absence of diffusion, together with the requirement that the turbulent diffusion have a value independent of η, requires the small-scale magnetic field to exist on diffusive length scales. There is a clear analogy in three dimensions with the conservation of magnetic helicity. This leads via (1.7) to the requirement of magnetic fields on diffusive scales in order that α not depend on the diffusivity. It is notable that there is no similar conservation law for mean square potential in three dimensions, and that the turbulent diffusivity in this case is affected much less by the imposed field (one can see that unknotted field lines can slip through highly conducting material in 3D without affecting the flow much). One would expect the helicity constraint to have an effect on this process; the situation remains unclear. The reduction in the α-effect occurs on this view because the Lorentz forces prevent the smallest scales of the magnetic field from reaching diffusion levels. In addition, when the magnetic Prandtl number is of order unity, as may be appropriate for the Sun, the MHD turbulence spectrum may contain a significant proportion of Alfvén waves, for which $\mathbf{u}$ and $\mathbf{b}$ are parallel and which thus give no contribution to $\boldsymbol{\mathcal{E}}$. When the magnetic Prandtl number is very large, as may be the case in galaxies, then of course there are no Alfvén waves and the equilibration mechanism is different, perhaps leading to smaller values of a, as shown in recent work by Schekochihin, Cowley, Maron & Malyshkin (2002).

The idea that a is significant is given support from three very different numerical studies. The first (Brandenburg 2001) considers flow in a periodic domain, forced by a helical body force on a small scale. There is eventual growth of significant large-scale fields, which are force-free and can grow to large size free of dynamical constraints. While increasing R_m leads to more rapid initial growth, the time taken for final equilibration also increases. The α-effect is calculated by solving a short-time initial value problem, and by superposing a uniform mean field and calculating $\boldsymbol{\mathcal{E}}$ directly. Both methods (see Figure 1.3) yield a significant dependence on R_m in the α-quenching formula, with $a \geq 1$. Brandenburg also finds that the turbulent diffusivity is quenched, but that the dependence on R_m is rather weaker, as suggested above. The remaining studies were carried out by Cattaneo & Hughes (1996) and Cattaneo, Hughes & Thelen (2002). In the first, a kinematic flow is forced that has the form of the so-called CP-flow of Galloway & Proctor (1992). A fully three-dimensional calculation is undertaken, starting from this velocity field with an imposed uniform field in the z-direction.. Only that part of the α-effect which derives from fully three-dimensional, that is nonlinearly driven, flows is evaluated by direct calculation of $\boldsymbol{\mathcal{E}}$ and the results show that $a \sim 1$ for the quenching properties. The magnitude of the turbulent fluctuations, however, scarcely changes with the imposed field. This last result was predicted previously by Cattaneo & Hughes (1996). In the paper of Cattaneo *et al.* the CP flow is again employed, but now solutions are sought in a long periodic box in the z-direction (the original flow being independent of z). The length of the box is chosen as 8 times the period of the most rapidly growing mode; the latter then plays the role of fluctuating field, while the mode with the same period as the box plays the role of the large-scale field. Two different case are considered. In one the initial condition has comparable energy in the small and large scales, while in the other the large-scale energy is initially much greater. The final

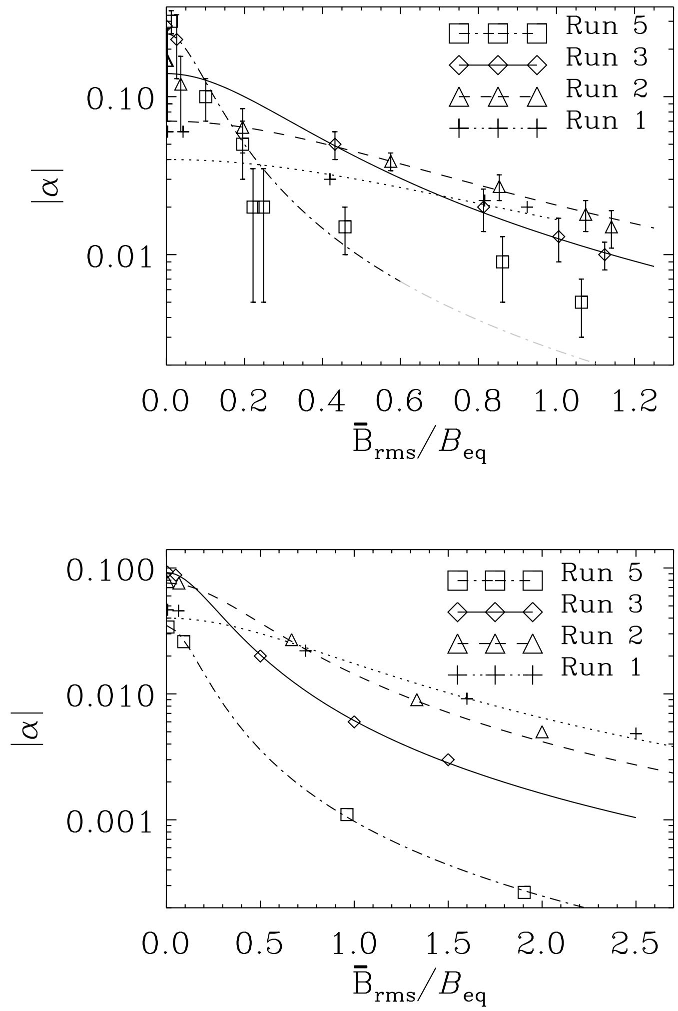

Fig. 1.3. Graphs of various runs from the paper of Brandenburg (2001), showing the reduction of α with increasing $\overline{\mathbf{B}}$. The first figure shows the result of solving a short-time initial value problem, and the second the value calculated from imposing a uniform field. The results are very similar. The lower curves correspond to greater values of R_m.

state appears to depend on these initial conditions. In the first case the nonlinear interactions between different wavenumbers force rapid growth of the large-scale field, although its natural growth rate is much less than that of the small-scale field, but growth stops when the large-scale field has much

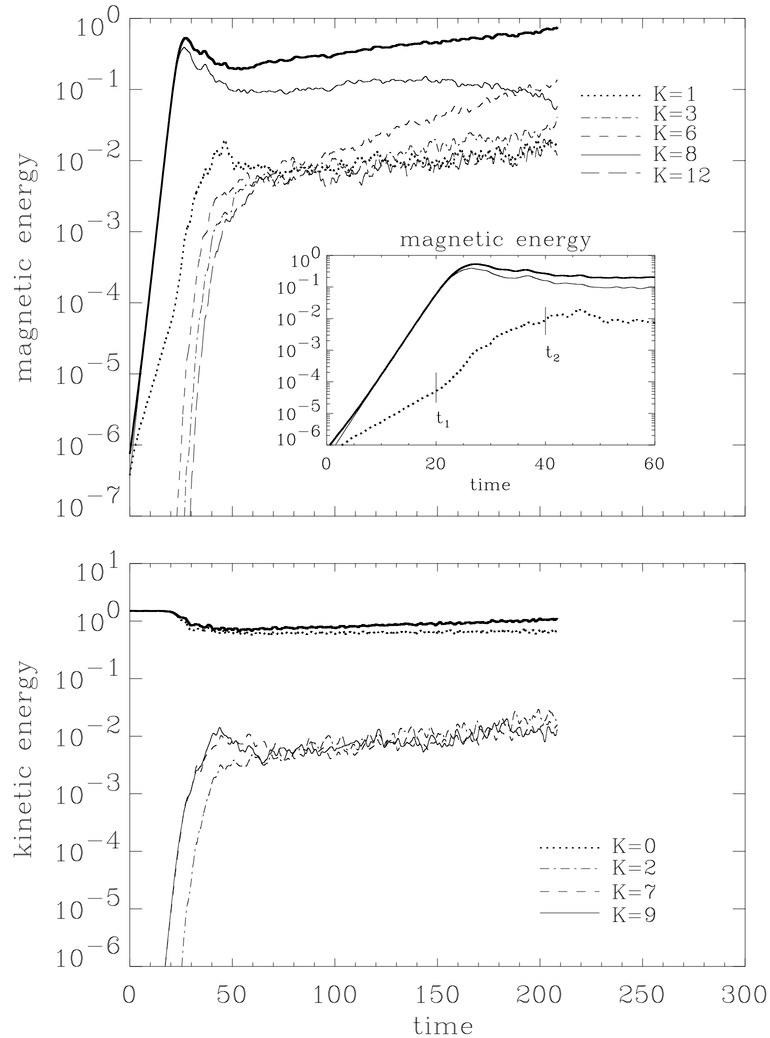

Fig. 1.4. Graphs of runs from the paper of Cattaneo *et al.* (2002). The different lines refer to modes of different wavenumber. The initial energy of the largest scale ($K = 1$) and the most unstable modes ($K = 8$) are comparable. Growth of the $K = 1$ mode is accelerated above its kinematic rate between times t_1 and t_2.

lower energy than the short lengthscale mode. There seems to be a further adjustment on a much longer timescale. In the second case the large scale quickly equilibrates, leaving the other scales at lower values. Results are shown in Figures 1.4, 1.5. It turns out that the evolution of the large scale field can be discussed in terms of an α-effect. This is verified by looking at similar short box calculations and evaluating the α-effect as in the earlier

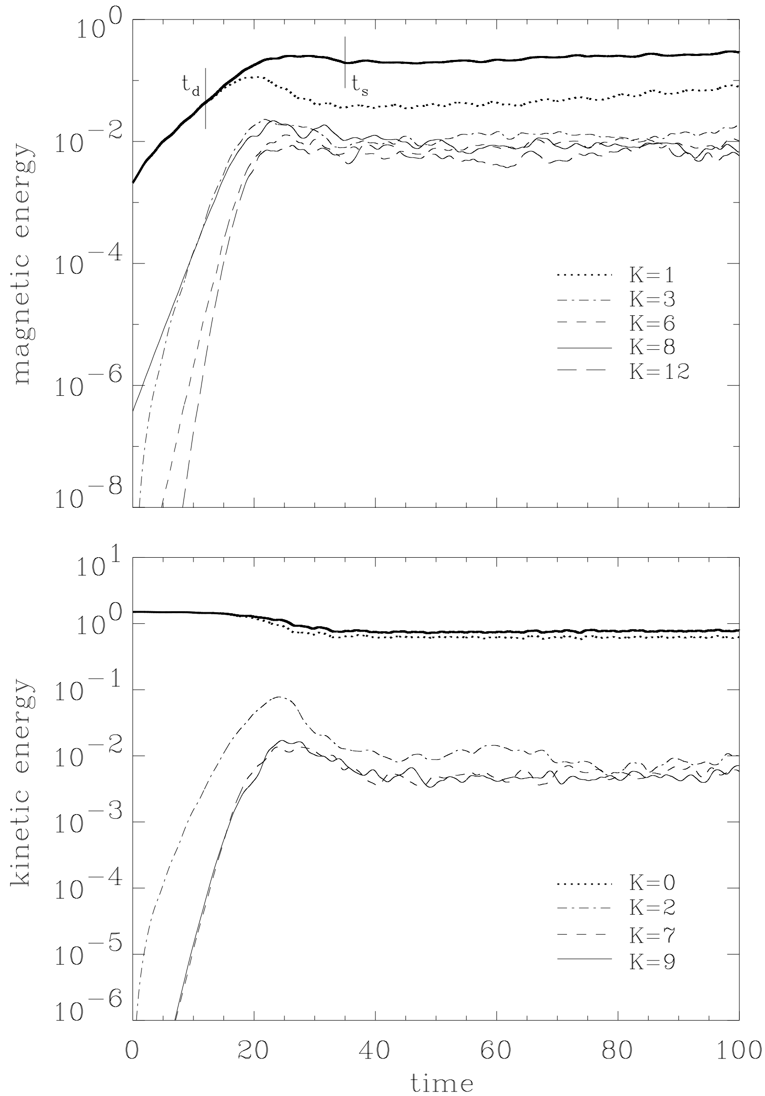

Fig. 1.5. As for the previous figure except that the energy in the largest scale is initially much greater than that in the other scales. The magnetic field becomes dynamically active at time t_d, and nonlinear saturation occurs at time t_s.

paper described above. The two methods give very similar results, justifying the interpretation. It is again found that the process of α-quenching depends strongly on R_m, as indeed does the initial value of α for weak imposed fields. (see Figure 1.6). From the results (Figure 1.7) we can see that α falls to diffusive values while the mean fields are well below equipartition values.

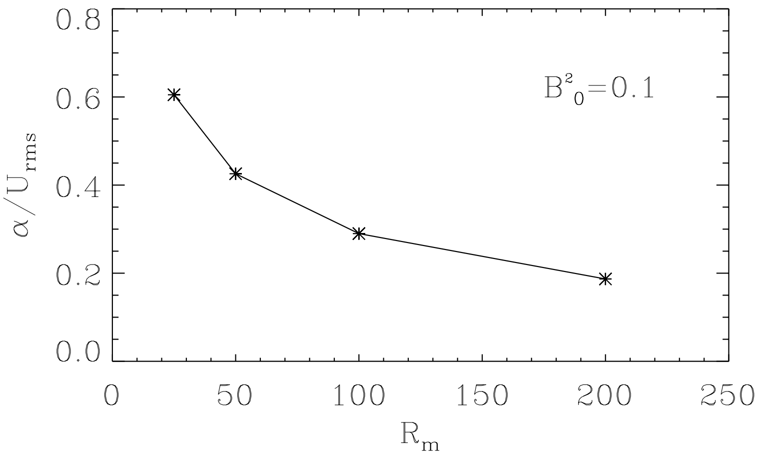

Fig. 1.6. The behaviour of the α coefficient for fixed $\overline{\mathbf{B}}$ as a function of R_m, in the calculation of Cattaneo *et al.* with comparable initial large and small-scale field. It can be seen that there is a strong reduction as R_m increases.

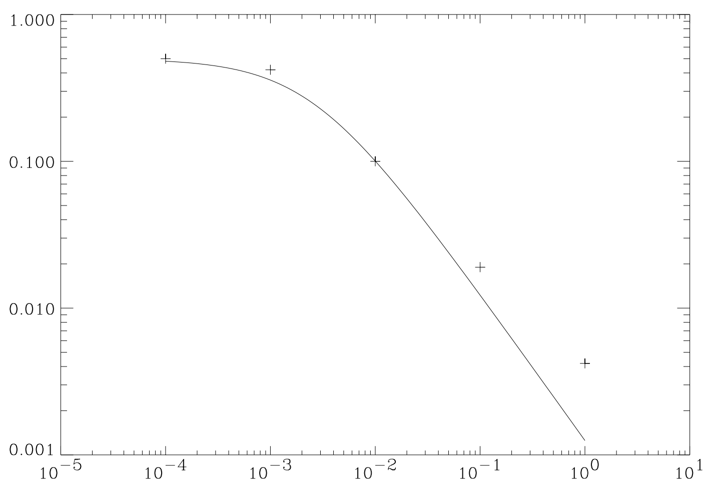

Fig. 1.7. The behaviour of the α coefficient (y-axis) for $R_m = 100$ as a function of $\overline{\mathbf{B}}^2$ (x-axis), in the same calculation as for the previous figure. It can be seen that there is a strong reduction in α when $R_m\overline{\mathbf{B}}^2$ is of order unity.

The conclusion of these studies, that α-quenching is very strong at large values of R_m, of course makes it difficult to see how large-scale fields could arise on other than irrelevant diffusive timescales. A possible chink in the reasoning has been identified by Blackman & Field (2000), who argue that the results depend strongly on the constraint that *small-scale* helicity (and not just total helicity) is conserved. Such conservation is natural in model experiments with periodic boundary conditions, but it may be that with more realistic boundary conditions the separate conservation of small-scale and large-scale magnetic helicity will be destroyed, allowing a decrease of small-scale helicity, which may affect the quenching process. Calculations so far have been inconclusive. Indeed Brandenburg & Dobler (2001), who carried out model calculations with forced helical flows with non-periodic boundary conditions, reported that the crucial value of a was still $\geq 1/2$, with peak fields actually reduced over the periodic case. The results were somewhat model dependent, however, in that while an imposed vertical field boundary condition lowered the mean field over the periodic case, the use of a "potential field" condition could actually increase it. However in the latter case the time taken to reach these larger values increased with R_m. In my view calculations with more realistic boundary conditions are worth pursuing further; nonetheless there are powerful local arguments, not dependent on any conservation law, which support the idea that the dynamo properties of a turbulent flow (which depend very subtly on its Lagrangian structure) are going to be strongly affected when the Lorentz forces become significant on the smallest scales.

1.5 Conclusion

In this short review I have tried to put forward the current state of play regarding the important question of the effects on and by large scale fields of small-scale MHD turbulence. The difficulty in the past has been the misleading prejudices induced by the study of models with small R_m on the smallest scales. Not only do these give a wrong picture of the nature of the α-effect, but they fail to take into account the fact that at large R_m the small-scale flow is likely to be a dynamo in its own right, with effects on dynamo generation for mean fields that are only now becoming apparent. There are many questions that need to be answered before a satisfactory theory can emerge that will account for the observed large-scale solar fields. An important aspect that has yet to receive full attention is the inhomogeneity of the process. This will inevitably lead to large scale fields on dynamic timescales, as foreshadowed by the calculations of Cattaneo *et al.* (2002).

Acknowledgements I am grateful for discussions over many years with my colleagues Nigel Weiss, Fausto Cattaneo and David Hughes.

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