Instabilities inside a precessing cylinder

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MOTIVATIONS

Fluid motions inside flying engines

→ Deviation of the trajectories

Precession of planets

→ Complex motion of liquid core
→ Generation of magnetic field?
A precessing cylinder

- Cylinder of water rotating at $\omega_1$
- Platform rotating at $\omega_2$
- Angle of precession: $\alpha$

Schematic of a precessing cylinder
Experimental set-up

• Transverse PIV measurements with a laser sheet

• Visualizations with Kalliroscope particles
Transverse velocity field: $\alpha = 1\degree$

(without solid body rotation)

$Re = 4000$

$\omega = 1.4$

Forcing of a Kelvin mode (inertial wave)
Equation de Navier-Stokes

- Vecteur rotation: \[ \Omega(t) = \Omega \left( u_z + \varepsilon \delta(t) \right) \]

\[ \Omega = \omega_1 + \omega_2 \cos \alpha, \]

\[ \varepsilon = \frac{\omega_2 \sin \alpha}{\omega_1 + \omega_2 \cos \alpha}, \]

av\c{c} \[ \delta(t) = \cos(\omega_1 t)u_x - \sin(\omega_1 t)u_y, \]

- Navier-Stokes:

\[ \frac{\partial V}{\partial T} + (V \cdot \nabla) V + 2\Omega \times V + \Omega \times (\Omega \times R) + \frac{d\Omega}{dt} \times R = -\frac{1}{\rho} \nabla P + \nu \Delta V. \]

Coriolis centrifique acc\c{c}l\e{}r\'ation

- Adimensionn\e{}\e{}e:

\[ \frac{\partial v}{\partial t} + 2u_z \times v + \nabla p + 2\omega r \cos (\omega t + \theta) u_z = -\varepsilon (v \cdot \nabla) v - 2\varepsilon \delta \times v + \varepsilon q + \frac{1}{Re} \Delta v, \]
Linear theory

Dimensionless Navier-Stokes equations:
\[
\frac{\partial v}{\partial t} + 2u_z \times v + \nabla p = -2\omega \cos(\omega t + \theta)u_z
\]

(in the cylinder frame of reference, small \(\alpha\))

\[
v_z = \omega_2 \sin \alpha R \left( \sum_{i=1}^{\infty} a_i \frac{-k_i J_1(\delta_i r)}{\omega} \cos(k_i z) \sin(\omega t + \theta) - 2r \sin(\omega t + \theta) \right)
\]

- general solution without forcing
- particular solution

Boundary conditions lead to:
\[
a_i = \frac{4\omega^2}{(\omega - 2)(k_i^2 + 1)k_i J_1(\delta_i) \cos(k_i H / 2R)}
\]

with \(\delta_i^2 = \frac{4 - \omega^2}{\omega^2} k_i^2\)

- amplitude of the modes diverges when \(H = \lambda/2, 3\lambda/2\ldots\)
- strong viscous smoothing of higher modes

Amplitude of the 1\textsuperscript{st}, 2\textsuperscript{nd}, 3\textsuperscript{rd}, 4\textsuperscript{th} and 5\textsuperscript{th} modes
Visualisation of modes (vorticity): $\alpha=1^\circ$, $Re=2000$

Mode 3 ($\omega=0.4$)

Mode 2 ($\omega=0.5$)

Mode 5 ($\omega=0.2$)

Mode 1 ($\omega=1.75$)
Amplitude of the first mode

- good agreement outside of resonances
- large scatter
- strong smoothing of resonances
Geostrophic mode at the resonance: $\alpha=1^\circ$, $Re=2000$

1\textsuperscript{st} Resonance of mode 1 ($\omega=0.9$)

2\textsuperscript{nd} Resonance of mode 2 ($\omega=1.3$)

- appearance of a geostrophic mode ($m=0$)
- in agreement with Kobine (1996)
- due to nonlinear self-coupling of a mode?
Non stationary: 1\textsuperscript{st} resonance of mode 2 (\(\alpha=1^\circ\), Re=2000)

- Coherent structures rotating in the frame of reference of the rotating platform
- Triadic resonance of lower order Kelvin modes?
Non stationary: 1\textsuperscript{st} resonance of mode 2  ($\alpha=1^\circ$, Re=2000, $\Delta t=T_1/10$)
Kalliroscope visualisations

1st Resonance of mode 2

\[ H = \frac{1}{2\lambda} \]
\[ \omega = 0.56 \quad \alpha = 1^\circ \]
\[ \text{Re} = 20000 \]

2nd Resonance of mode 1

\[ H = \frac{3}{2\lambda} \]
\[ \omega = 1.73 \quad \alpha = 1^\circ \]
\[ \text{Re} = 20000 \]

3rd Resonance of mode 1

\[ H = \frac{5}{2\lambda} \]
\[ \omega = 1.89 \quad \alpha = 1^\circ \]
\[ \text{Re} = 20000 \]

- Non stationary patterns and possible intermittency
- Wavelengths close to those of the theoretical resonant modes
CONCLUSION

- PIV measurements on a precessing cylinder,

- Forcing of Kelvin modes in quantitative agreement with linear theory,

- Strong geostrophic modes ($m=0$) at resonances,

- Breakdown of the flow for high Re or high angles of precession