Effect of shear flows on dynamo action

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Small and large scale dynamos

- Dynamo action due to flow can be divided into two types:
- *Small-scale dynamo (fluctuation dynamo)*: in sufficiently vigorous flows with sufficiently complex structure magnetic energy enhanced by stretching. This process dominates cancellation due to folding and diffusion. Broken mirror-symmetry not required.
- *Large-scale dynamo ("mean-field" dynamo)*: this works even when flows are very weak,provided there is a sufficiently large outer scale. Small-scale flow interacts with small scale induced magnetic field to produce large scale emf's parallel to the large scale magnetic field. Chirality is essential for the main effect (the 'alpha-effect').

Mean field dynamos and shear

$$\frac{\partial A}{\partial t} = \mathcal{E}[B,x] + \eta \frac{\partial^2 A}{\partial x^2}$$
$$\frac{\partial B}{\partial t} = \Omega \frac{\partial A}{\partial x} + \eta \frac{\partial^2 B}{\partial x^2}$$

$$\mathcal{E}[B,x] = \alpha B - G \frac{\partial B}{\partial x} + \ldots$$

- In classical mean-field dynamo theory azimuthal shear produces toroidal field from poloidal field while α-effect produces poloidal from toroidal field - the $\alpha\Omega$ dynamo. Growth rate of dynamo depends on product of α-effect and shear. In simple 1D Parker model with wavenumber k, growth rate

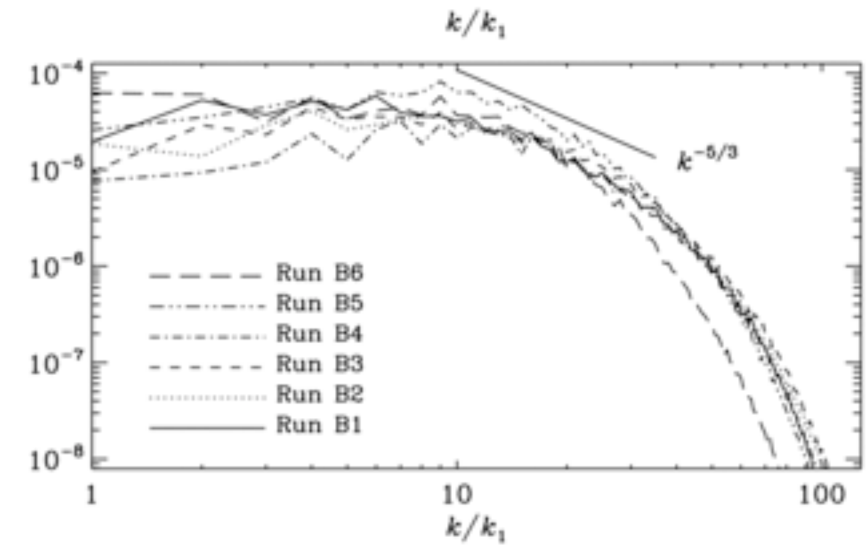
$$s = \sqrt{\alpha\Omega k/2} - \eta k^2; \quad s_{\max} \propto (\alpha\Omega)^{\frac{2}{3}}$$

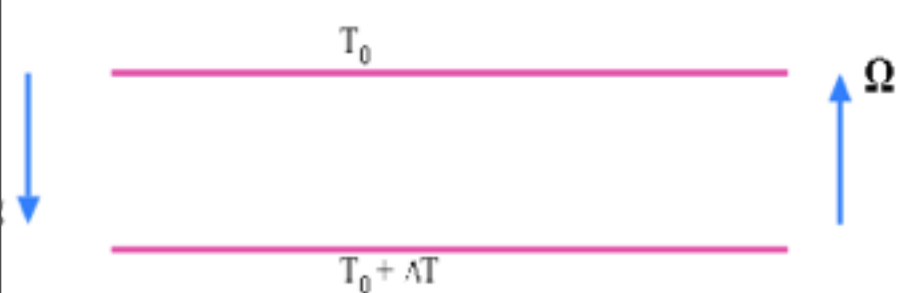
- But not at all clear that the mean field ansatz works at all well for problems with large *Rm* on small scale, e.g results of Hughes & Cattaneo 2006

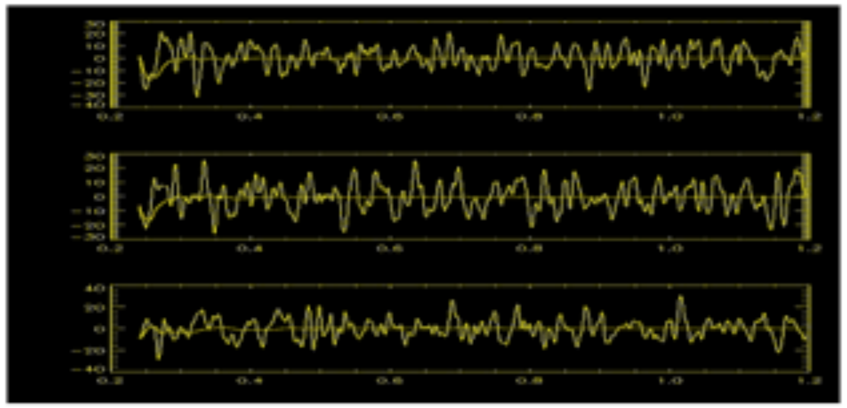
EMFs due to rotating convection, without shear

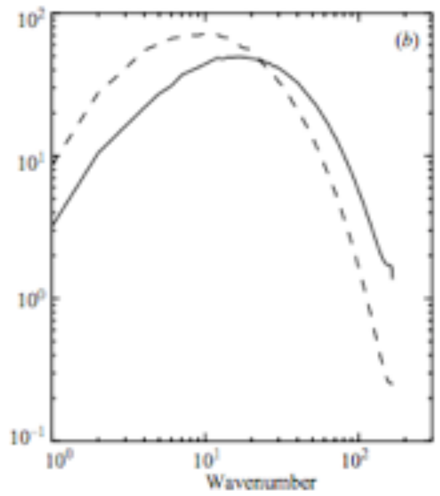
•Boussinesq convection in a rotating layer with aspect ratio λ (Jones & Roberts 2000 Stellmach & Hansen 2004, Cattaneo & Hughes 2006, Kapyla *et al.* 2010,...)

$$\begin{aligned} \sigma^{-1}\frac{\partial \mathbf{u}}{\partial t} + Ta^{\frac{1}{2}}\hat{\mathbf{z}}\times\mathbf{u} &= -\nabla p + Ra\,\theta\hat{\mathbf{z}} + \mathbf{B}\cdot\nabla\mathbf{B} + \nabla^2\mathbf{u} \\ \frac{\partial\theta}{\partial t} &= \mathbf{u}\cdot\hat{\mathbf{z}} + \nabla^2\theta \\ \frac{\partial \mathbf{B}}{\partial t} &= \nabla\times(\mathbf{u}\times\mathbf{B}) + \zeta\nabla^2\mathbf{B} \\ \nabla\cdot\mathbf{u} &= 0 \end{aligned}$$









•Even with vigorous convection plus helicity can get very small α-effect if aspect ratio large, depending on the Taylor number

•Motivates looking at effects of fluctuations in mean field coefficients and interaction with shear

Fluctuating α-effect with shear

• Proctor (2007) and Richardson & Proctor (2011, 2012) investigated effect of rapid fluctuations in the α-effect in presence of shear

•Both temporal and spatial fluctuations considered - in simple case use 1D Parker model, temporal variation only. Define small parameter ε . Write $\alpha=\alpha_0+\varepsilon^{-1}\alpha'(\tau)$, $\tau=t/\varepsilon$. Induces fluctuating fields $A'(\tau)/\varepsilon$, $B'(\tau)$. Leads to additional emf $\langle\alpha'B'\rangle=-G^2\Omega B_x$ with G depending on statistics of $\alpha'(\tau)$ leading to growth rate for modes $\propto$ $exp(ikx)$ if $\alpha_0=0$:

$$s = G\Omega k - \eta k^2; \quad s_{\max} \propto (G\Omega)^2$$

•So growth rate increases with Ω for fixed k but optimum growth rate increases faster. Similar formula derives from investigation of 'shear current effect' due to anisotropic turbulent magnetic diffusivity

•Assumptions break down when $G\Omega$ is too large (when $G\Omega\sim\varepsilon^{-1/2}$). Otherwise Backus' shear criterion would be violated

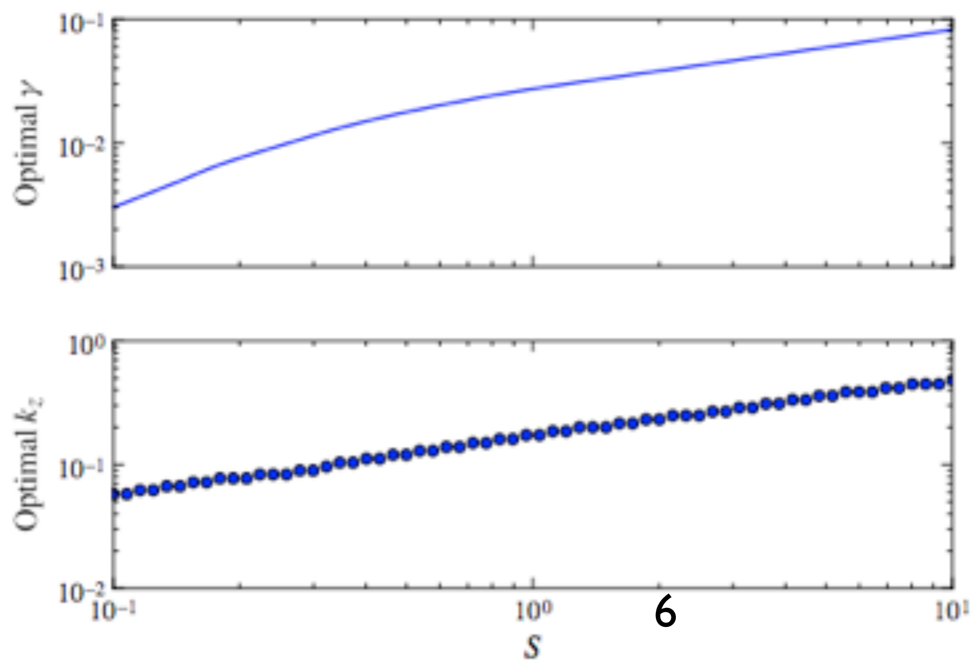
Analytic investigations

- Recent papers apply stochastic methods to models including fluctuations of helicity.

•**Mitra & Brandenburg** have fluctuating α delta-correlated in time.
•Find that mean energy of field can grow at rate proportional to Ω.
•Coupling seen in the Proctor scaling is absent as is likely given the zero correlation time. Mean field, averaged over ensembles, decays.

•**Heinemann et al** consider a dissipative flow forced by delta-correlated stochastic forcing in the presence of shear (coloured noise for the flow). Flow is affected directly by shear.
•They also use a simple 1D model for illustration.
•α-effect calculated by integrating ensemble of wave-packets over flow correlation time.
•Mean field again decays:Mean energy grows, again at rate proportional to Ω.

•**McWilliams** conducts an asymptotic study of dynamo action of forced Kelvin modes. For large shear rates cf. the correlation time of the forcing, growth rate ~ $\Omega^{1/2}$



DNS of shear and forced Turbulence 1

Yousef *et al.* (2008, 2009) considered the effect of an imposed shear on initially non-helically forced turbulence. Forcing has helicity but no net helicity. Shear is imposed in y direction and is linear in x. Box height L_z much larger than other dimensions to allow detection of large scale field. Rotating and non-rotating cases considered.

Result: shear (magnitude proportional to shear S) enhances field growth and leads to dynamo action with large scale features in z. Note that uniform shear has no natural scale in x or z direction

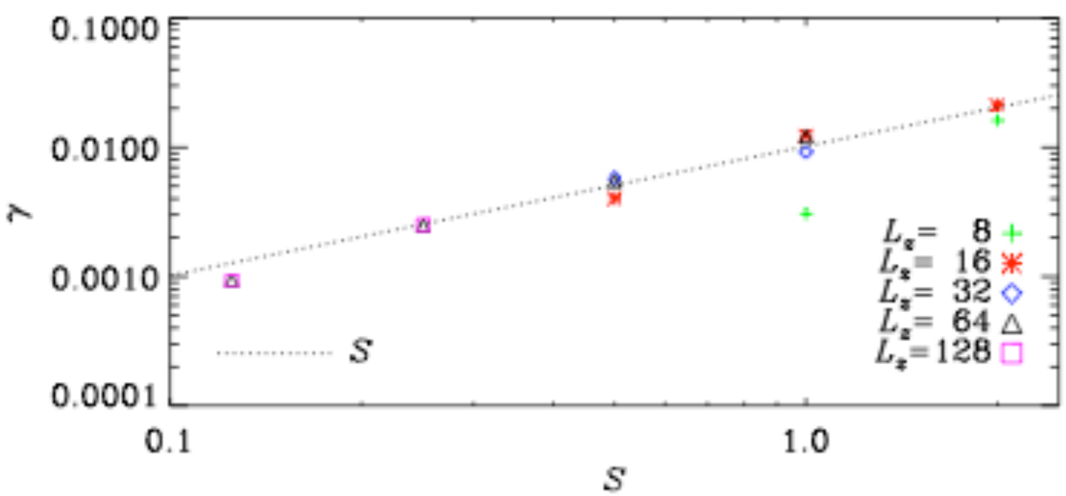


FIG. 2: Growth rates γ of $B_{\rm rms}$ for all runs (Tab. I). The dotted line shows the slope corresponding to $\gamma \propto S$.

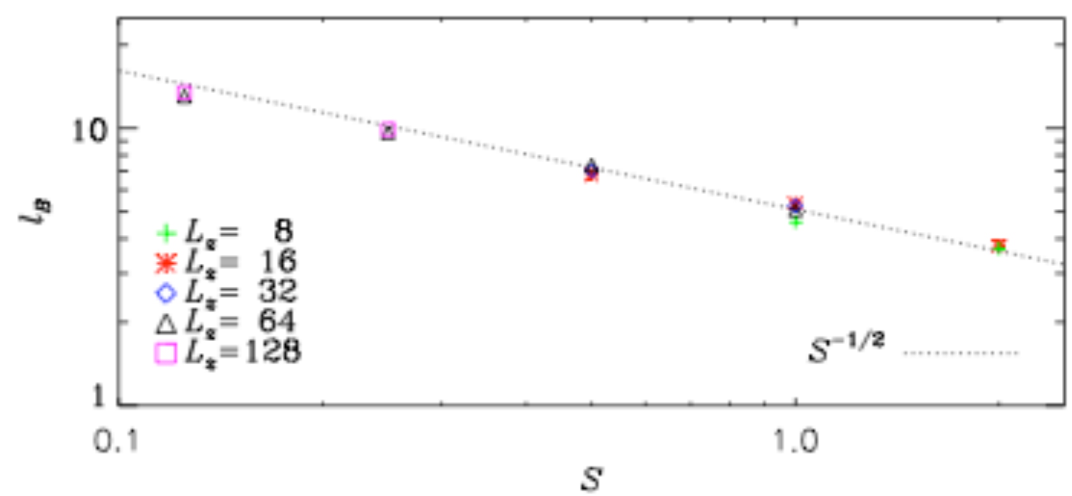


FIG. 4: The characteristic scale of the magnetic field [Eq. (4)] for all runs. The dotted line showes the slope $S^{-1/2}$.

DNS of shear and forced Turbulence 2

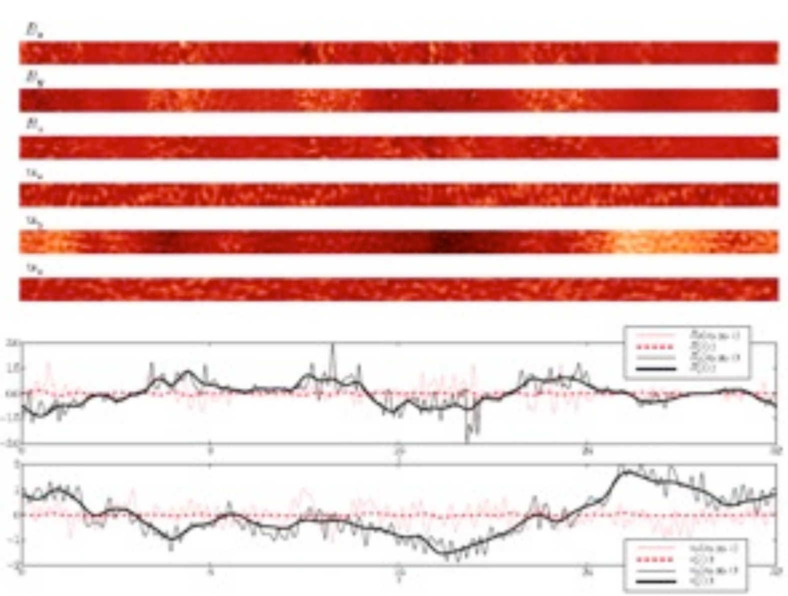
- Note that the two power laws do not identify the form of the induced emf. If we have a dispersion relation of the form below

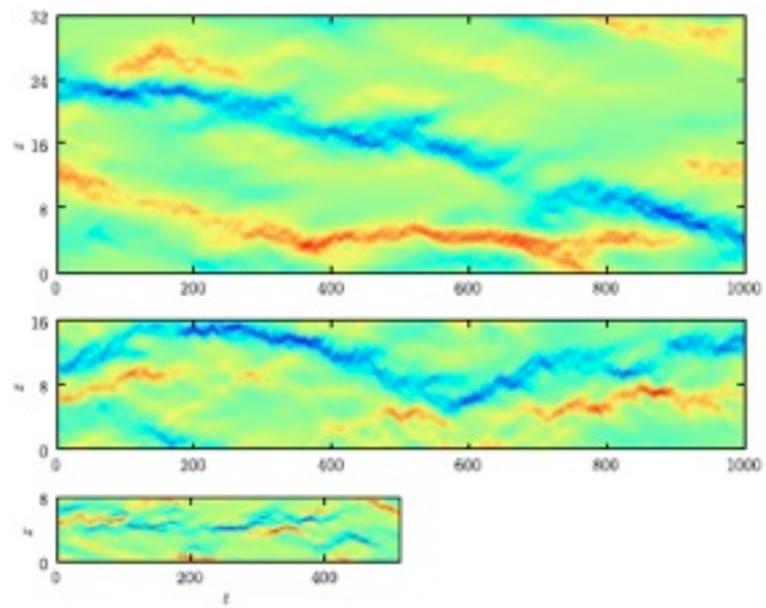
$$s = CS^p k^q - \eta k^2; \quad k_{\max} \propto S^{p/(2-q)}, \quad s_{\max} \propto S^{2p/(2-q)}$$

$$s_{\max} \propto S \Rightarrow 2p = 2 - q, \quad k_{\max} \propto S^{1/2}$$

$$\text{e.g.} \quad p = 1, q = 0; p = 1/2, q = 1; \quad \text{etc.}$$

- In this case results consistent with p=q=4/3
- Field that results is reasonably coherent in space, and also the amplitudes are coherent in time, though the phases are disorganised
- Reasonable to suppose that over *very* long times mean field vanishes
- But recall that even for a periodic mean field dynamo the time averaged field is zero!





- So stochastic models not inconsistent but need more refined interpretation

Can linear scaling persist?

- Richardson & Proctor (2012) investigated one-dimensional model of mean field dynamo with various form of fluctuating alpha-effect.

- Found that if $\Omega T >> 1$ where T is the correlation time then optimum growth rate scales with $\Omega^{2/3}$ while for $\Omega T << 1$ scaling depends on form of fluctuation.

- Also proved rigorously that $\Omega^{2/3}$ power law is maximum possible.

- Proctor (2012) proved much more generally that for periodic dynamos growth rate cannot increase faster than $\Omega^{1/2}$ or for mean field models $\Omega^{2/3}$.

$$\frac{\partial B_x}{\partial t} + \mathbf{U}\cdot\nabla B_x = \mathbf{B}\cdot\nabla u_x + \eta\nabla^2 B_x + Sf'(y)B_y,$$
$$\frac{\partial B_y}{\partial t} + \mathbf{U}\cdot\nabla B_y = \mathbf{B}\cdot\nabla u_y + \eta\nabla^2 B_y,$$
$$\frac{\partial B_z}{\partial t} + \mathbf{U}\cdot\nabla B_z = \mathbf{B}\cdot\nabla u_z + \eta\nabla^2 B_z.$$

$$\frac{1}{2}\frac{\partial}{\partial t}(M_x^2) = \left\langle (B_x^2\frac{\partial u_x}{\partial x} + B_xB_y\frac{\partial u_x}{\partial y} + B_xB_z\frac{\partial u_x}{\partial z})\right\rangle - \eta\left\langle |\nabla B_x|^2\right\rangle + S\left\langle f'(y)B_xB_y\right\rangle,$$

$$\frac{1}{2}\frac{\partial}{\partial t}(M_y^2) = \left\langle (B_yB_x\frac{\partial u_y}{\partial x} + B_y^2\frac{\partial u_y}{\partial y} + B_yB_z\frac{\partial u_y}{\partial z})\right\rangle - \eta\left\langle |\nabla B_y|^2\right\rangle,$$
$$\frac{1}{2}\frac{\partial}{\partial t}(M_z^2) = \left\langle (B_zB_x\frac{\partial u_z}{\partial x} + B_zB_y\frac{\partial u_z}{\partial y} + B_z^2\frac{\partial u_z}{\partial z})\right\rangle - \eta\left\langle |\nabla B_z|^2\right\rangle.$$

$$\frac{\partial M_x}{\partial t} < \gamma_{xx}M_x + \gamma_{xy}M_y + \gamma_{xz}M_z + SM_y,$$
$$\frac{\partial M_y}{\partial t} < \gamma_{yx}M_x + \gamma_{yy}M_y + \gamma_{yz}M_z,$$
$$\frac{\partial M_z}{\partial t} < \gamma_{zx}M_x + \gamma_{zy}M_y + \gamma_{zz}M_z,$$

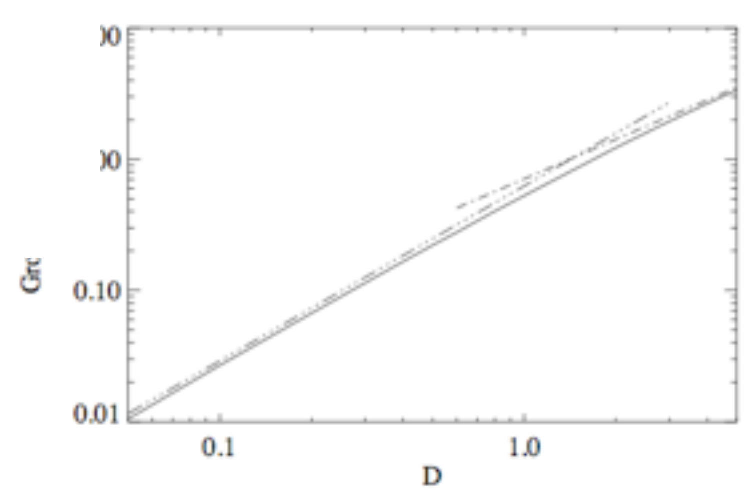


Figure 2. Upper panel: growth rate as a function of D for stochastic switching between α and $-\alpha$, with line showing linear asymptote as $D \to \infty$. Lower panel: The same data on a log-log plot, with lines showing the two power laws $D^{\frac{4}{3}}$ and D.

Convective dynamos and shear

Work with D.W.Hughes

- Convection in a rotating fluid layer can act as an efficient small-scale dynamo. Effects of rotation on the nature of the dynamo are very small so local stretching most important dynamo mechanism

- Even in the presence of (moderate) rotation with a well defined helicity distribution there is no evidence of a 'mean field' dynamo. Very little net emf is produced by the small scale flow

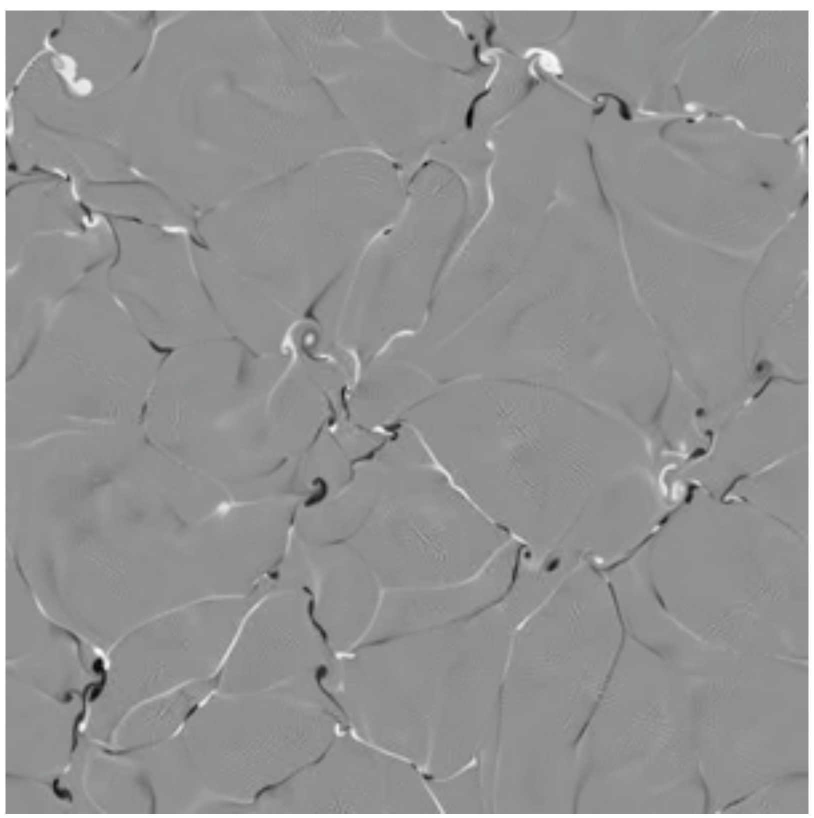
- While there are certainly circumstances in which the convective flow can act as a mean field dynamo (Childress & Soward 1972, Jones & Roberts 2000, Stellmach & Hansen 2004... Kapyla *et al.* 2010, etc etc), these were in special parameter ranges, typically for large Taylor numbers not too far from onset. In general for large enough domains any correlation that might lead to significant mean emf is very weak except possibly for very large rotation rates. Complete survey not yet attempted.

Convective dynamos wthout rotation (Cattaneo)

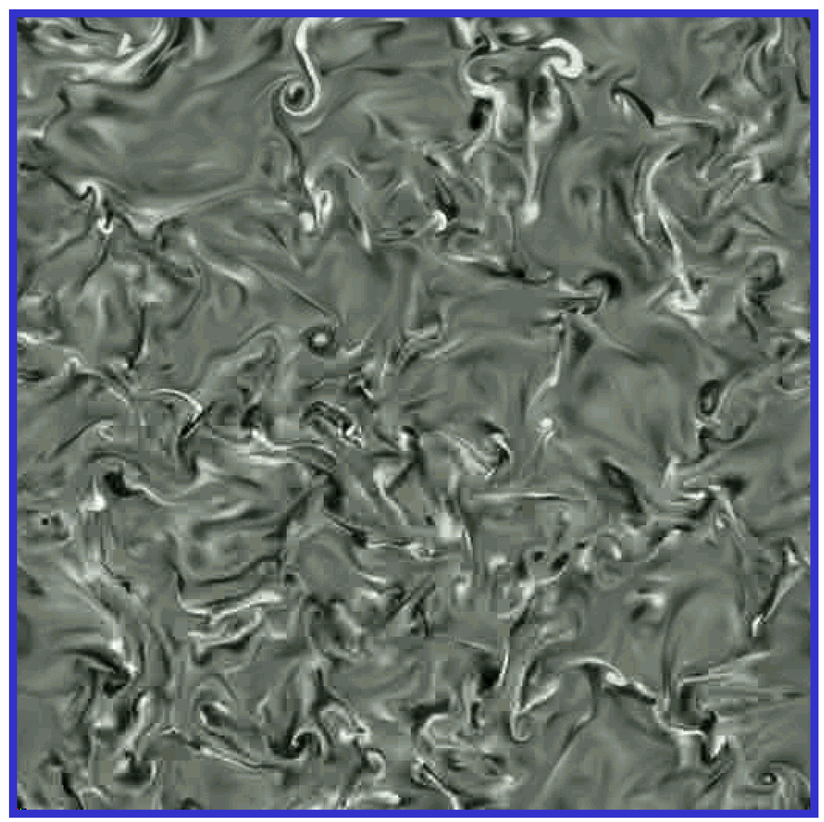
- Without rotation at sufficiently large *Ra* get dynamo action.

This is of *small-scale* type - no large scale field

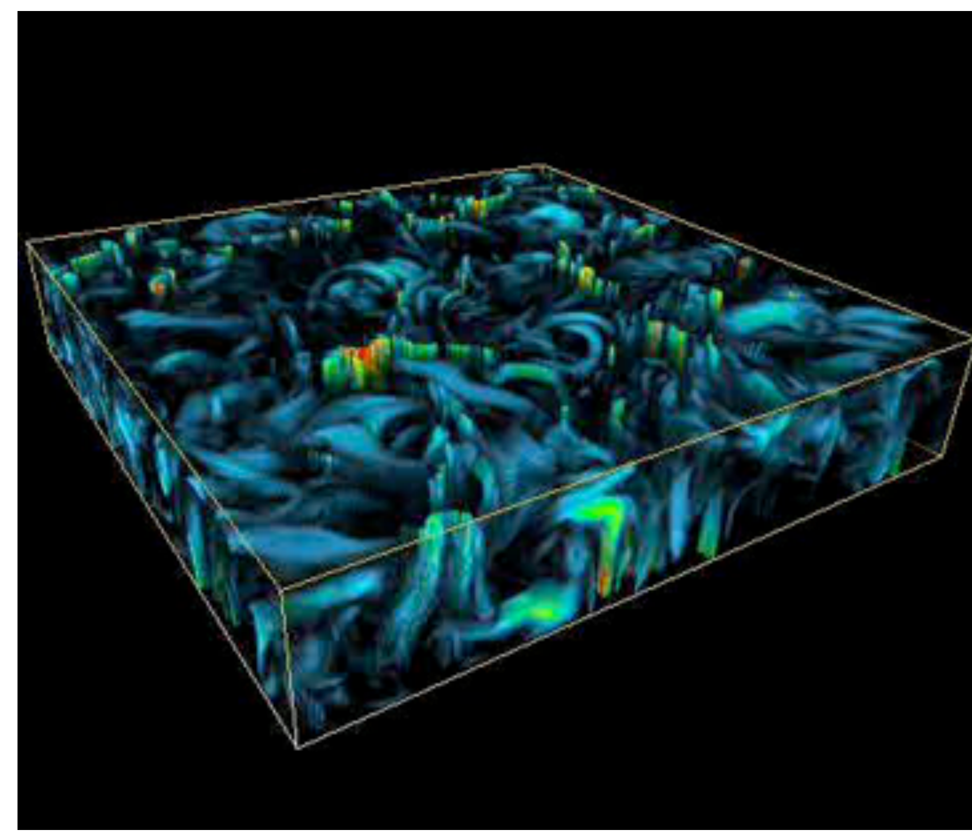
$Ra = 50{,}000$: $\sigma = 1$, $\zeta = 0.2$



Top



Middle

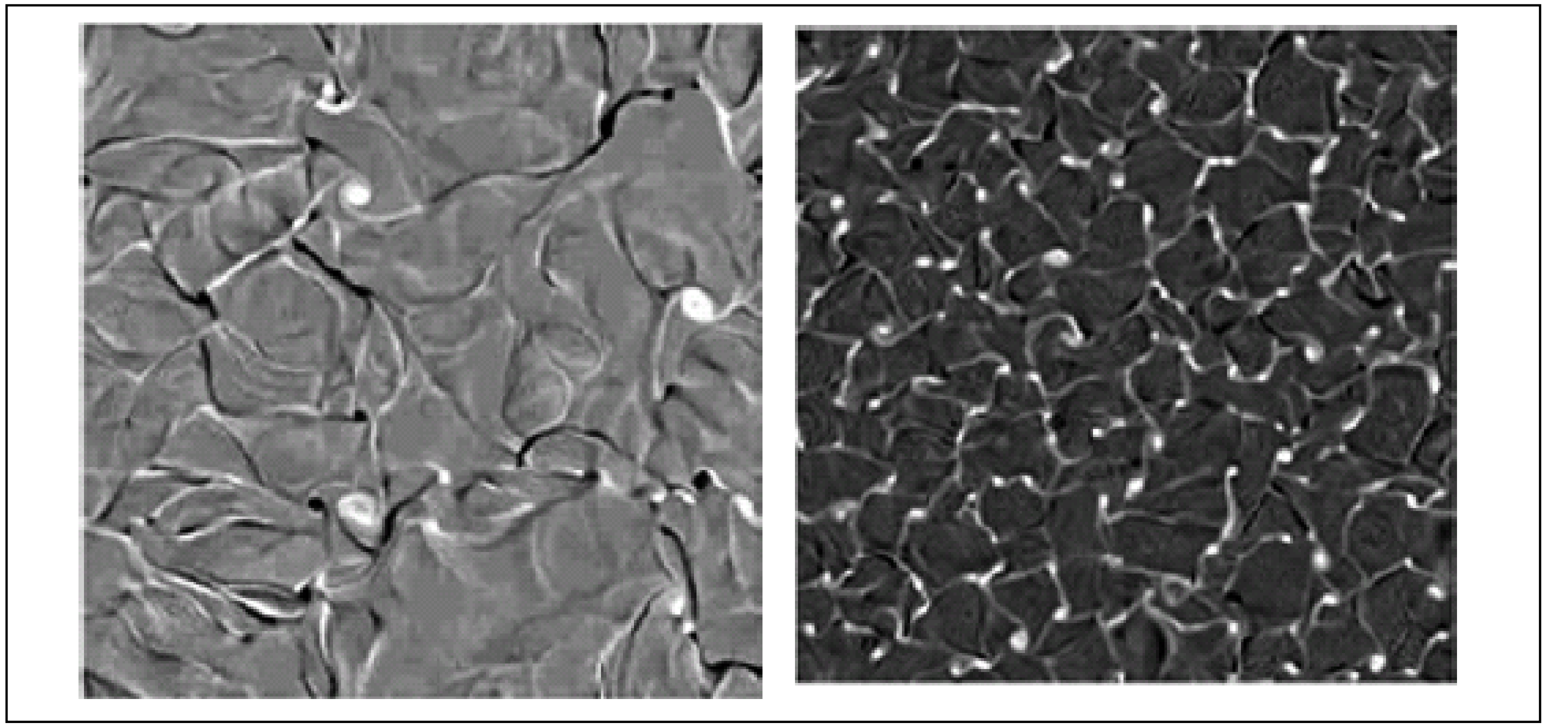


Rendering

Convective dynamos at moderate Taylor number (H+C 2006)

- In rotating case if *Ra* is large enough then dynamo action is possible, but there is still no sign of large scale features

- $Ta = 4\Omega^2 d^4/\nu^2 = 5 \times 10^5$, $\sigma = \nu/\kappa = 1$, $\zeta = \eta/\kappa = 0.2$, λ=5,10,20.
- Critical *Ra* for onset of convection = 59,008. *Ra* for onset of dynamo action ≈ 170,000



$Ta = 0$, $Ra = 5 \times 10^5$ $Ta = 5 \times 10^5$, $Ra = 10^6$

Vorticity plot (λ=5)shows moderate changes in convective structure as *Ta* is increased

Dynamo properties

- When *Ra* is large enough for dynamo action then the dynamo is of *small-scale* type (magnetic field scales no larger than convection scales).

- However there is vigorous *helicity* (correlation between velocity and vorticity) suggesting flow might also work as a *large -scale* (mean-field) dynamo. Attempt to measure mean-field effect by imposing uniform horizontal field and trying to evaluate emf by averaging over half the layer

- In fact no significant emf when field imposed. Calculation of the emf is controversial, but in any case the spectra of the growing fields show no large scale features in these parameter ranges.

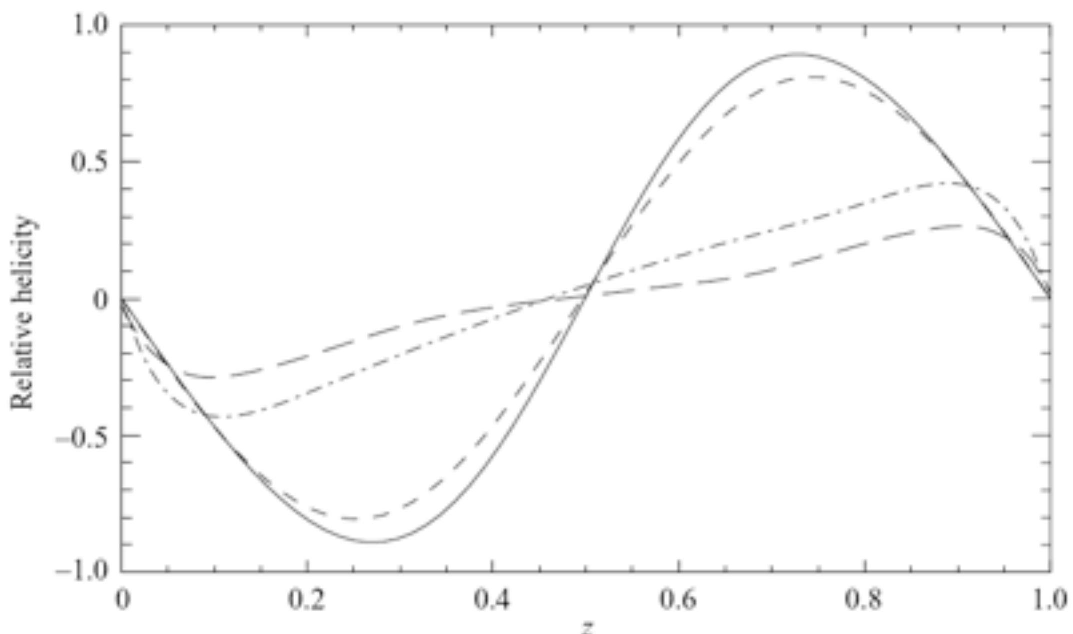
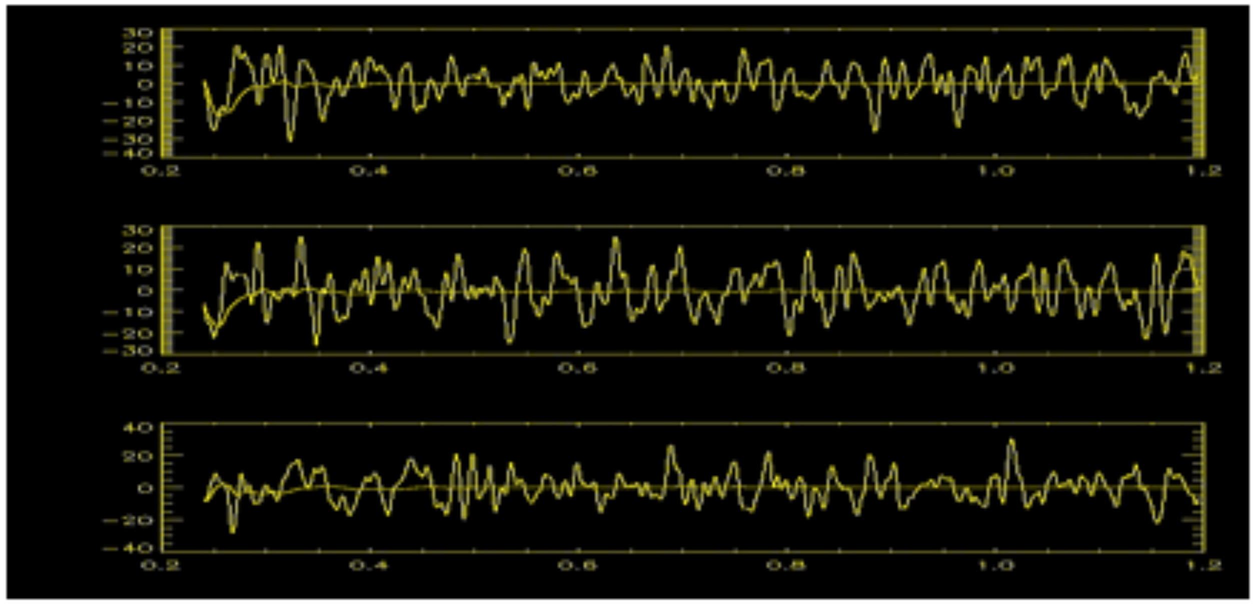


FIGURE 2. Snapshots of $h(z)$ (the horizontally averaged relative flow helicity) for the four cases shown in figure 1: $\lambda = 5$, $Ta = 5\times10^5$, and $Ra = 6.2\times10^4$, $Ra = 7\times10^4$, $Ra = 1.5\times10^5$, and $Ra = 5\times10^5$. The helicity decreases with increasing Ra. Exact antisymmetry about the mid-plane ($z = 0.5$) is recovered by time averaging.



Why is there no mean emf?

•Mean field effect works if:

1. Motions are helical - leading to twisting magnetic field lines
2. Average angle of twist < 90^{o}
3. Coherent twist at all locations/times

•When magnetic Reynolds number is large then angles of twist can very widely: large boxes lead to lack of coherence - paradoxically emf greater in *small boxes*

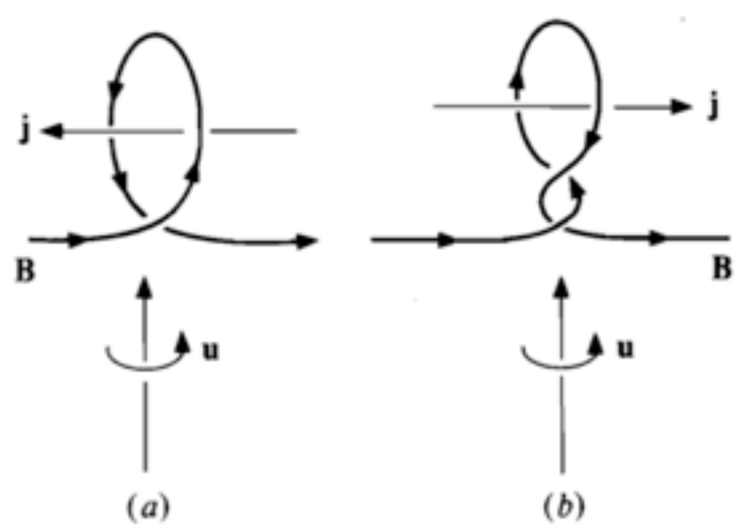


Fig. 7.2 Field distortion by a localised helical disturbance (a 'cyclonic event' in the terminology of Parker, 1970). In (*a*) the loop is twisted through an angle $\pi/2$ and the associated current is anti-parallel to **B**; in (*b*) the twist is $3\pi/2$, and the associated current is parallel to **B**.

Are matters improved when there is a coherent shear imposed on the flow?

Adding shear

- Can a coherent shear flow lead to a change in the nature of the dynamo?

- As before consider Boussinesq convection in a rotating layer with aspect ratio λ, but now with an added imposed sinusoidal shear (just added to flow)

$$\mathbf{U}_0 = U_0 \cos \frac{2\pi y}{\lambda} \hat{\mathbf{x}}$$

- Choose λ=5,10,20; *Ra*=150,000; *Ta*=500,000; σ=1; ζ=0.2. This is not a dynamo without shear

- Solve coupled induction, heat and momentum equations for various values of $S = 2\pi U_0 \ell / \lambda u_{\mathrm{rms}}$ Here $S \approx U_0/600$ for λ=10, *f(y)=cos(y)*

$$(\partial_t - \sigma\nabla^2)\boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} + U_0\left(f(y)\partial_x\boldsymbol{u} + f'(y)u_y\hat{\boldsymbol{x}}\right) + \sigma Ta^{1/2}\hat{\boldsymbol{z}} \times \boldsymbol{u} = -\nabla p + \sigma Ra\,\theta\hat{\boldsymbol{z}}$$

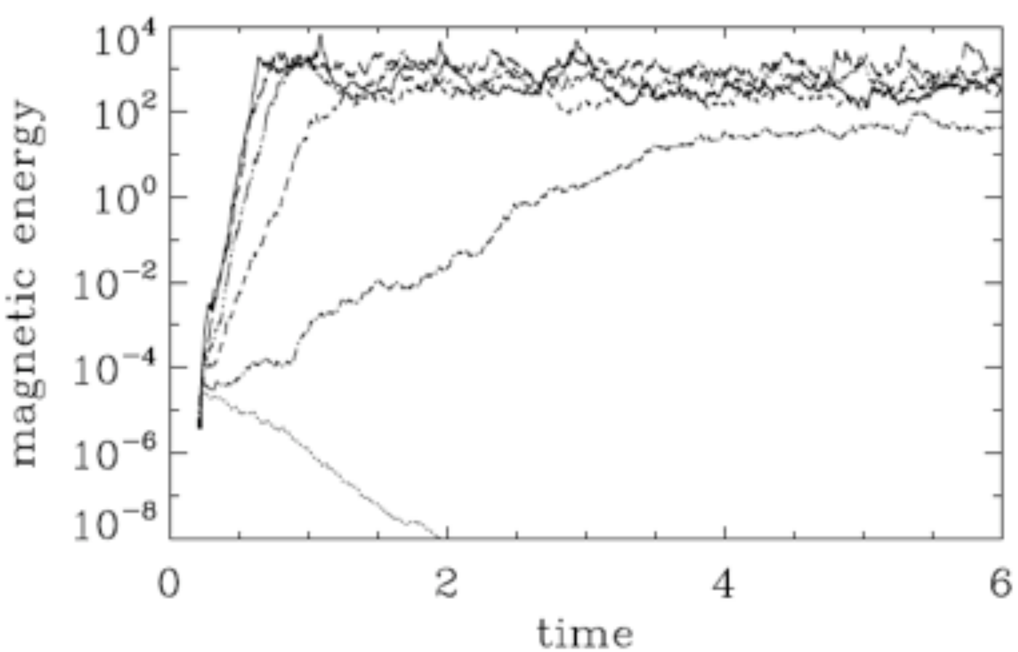
$$(\partial_t - \zeta\nabla^2)\boldsymbol{B} + \boldsymbol{u}\cdot\nabla\boldsymbol{B} + U_0 f(y)\partial_x\boldsymbol{B} = \boldsymbol{B}\cdot\nabla\boldsymbol{u} + U_0 f'(y)B_y\hat{\boldsymbol{x}} \,,$$

$$(\partial_t - \nabla^2)\theta + \boldsymbol{u}\cdot\nabla\theta + U_0 f(y)\partial_x\theta = \boldsymbol{u}\cdot\hat{\boldsymbol{z}} \,,$$

$$\nabla\cdot\boldsymbol{B} = \nabla\cdot\boldsymbol{u} = 0,$$

Results (*Hughes & P, PRL 2009, JFM 2012, submitted*)

- System acts as a dynamo if S is large enough
- As long as flow not too disrupted by shear, growth rate increases monotonically with S



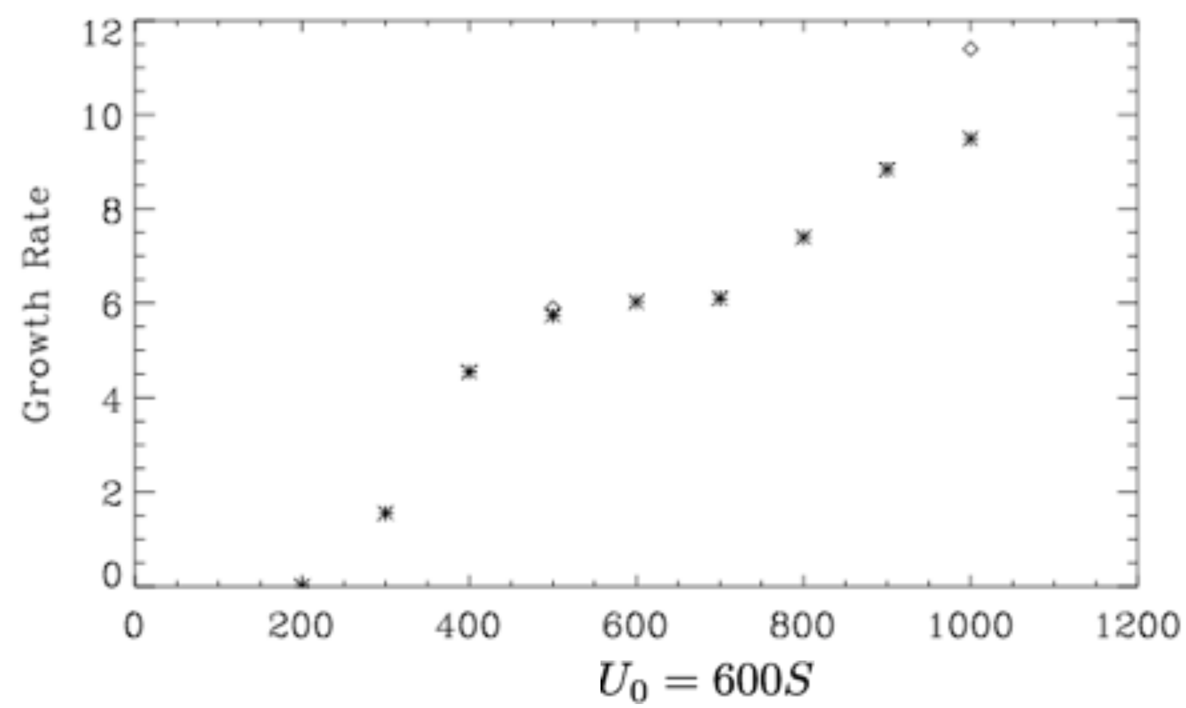
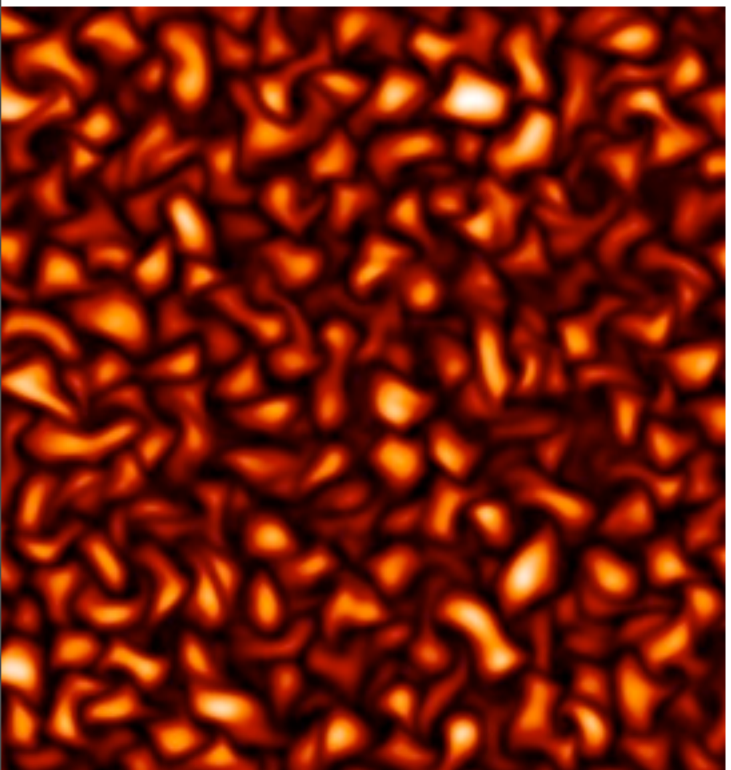
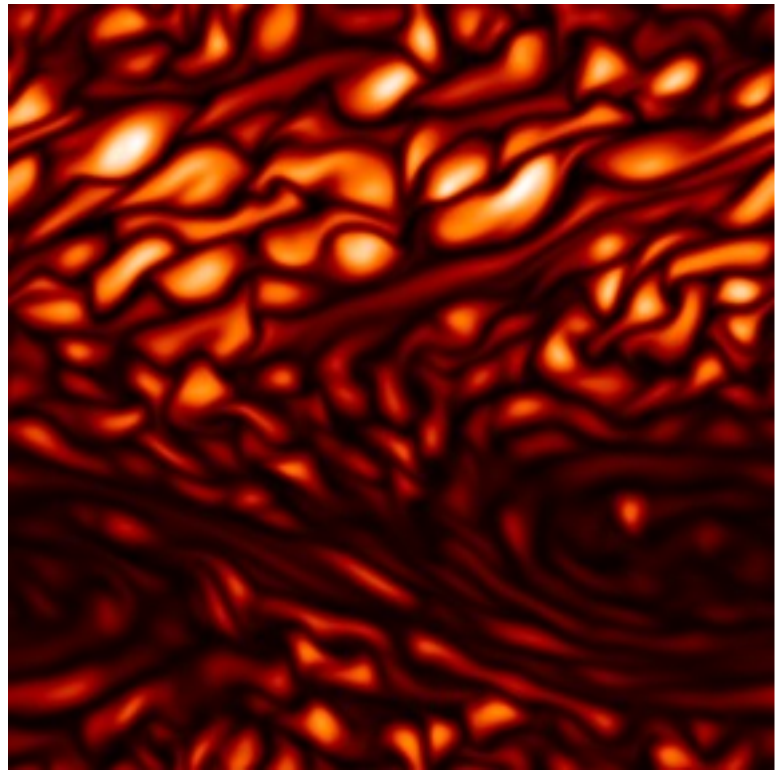


FIG. 1: Magnetic energy evolution for a range of S. In terms of increasing linear growth rate, $S = 1/3$ (not a dynamo), 2/3, 5/3, 5, 20/3, 10/3.

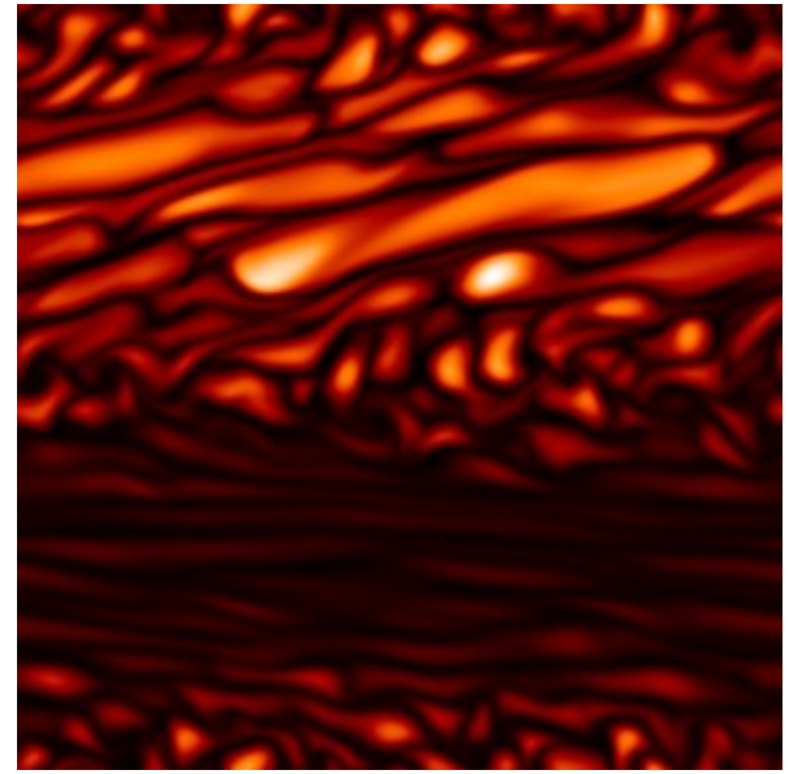
Flow patterns (temp.)



S=0

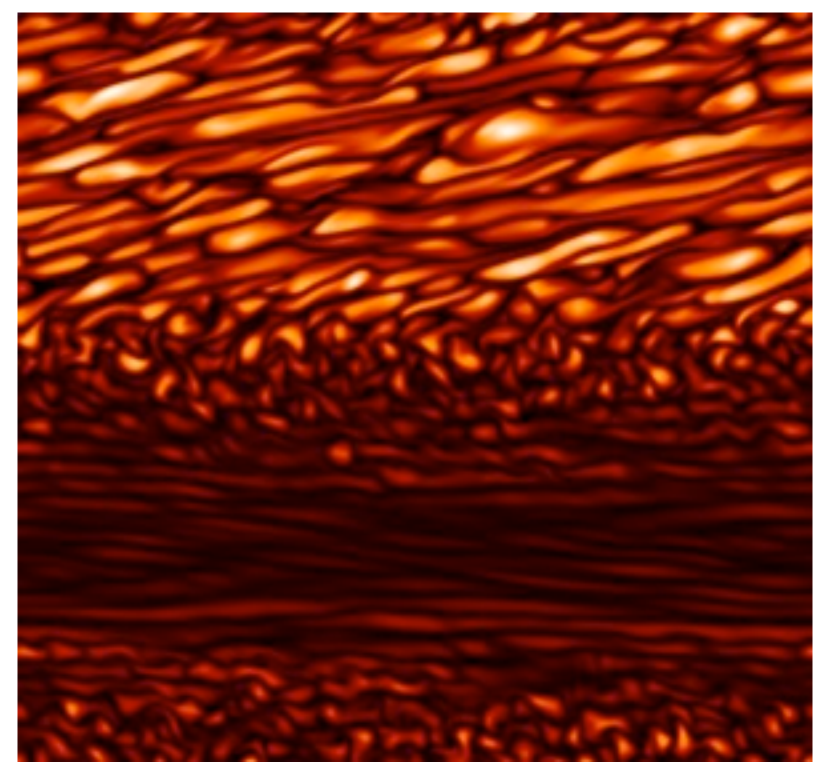


S=5/3



S=10/3

Temperature patterns near top of layer
for $\lambda=10$ and different values of S
Bottom picture shows 20 by 20 box



Effect of shear on the dynamo?

(i) large spatial scale of the shear leads to an enhanced α through greater spatial correlation of the small-scale motions;

(ii) even though mean α remains small there may nonetheless be an effective $\alpha\omega$ dynamo when the shear is significant;

(iii) anisotropy induced by shear may lead to a significant *shear current effect*;

(iv) shear may interact with temporal fluctuations in α to produce an effective mean field dynamo

(v) none of the above!

What kind of dynamo?

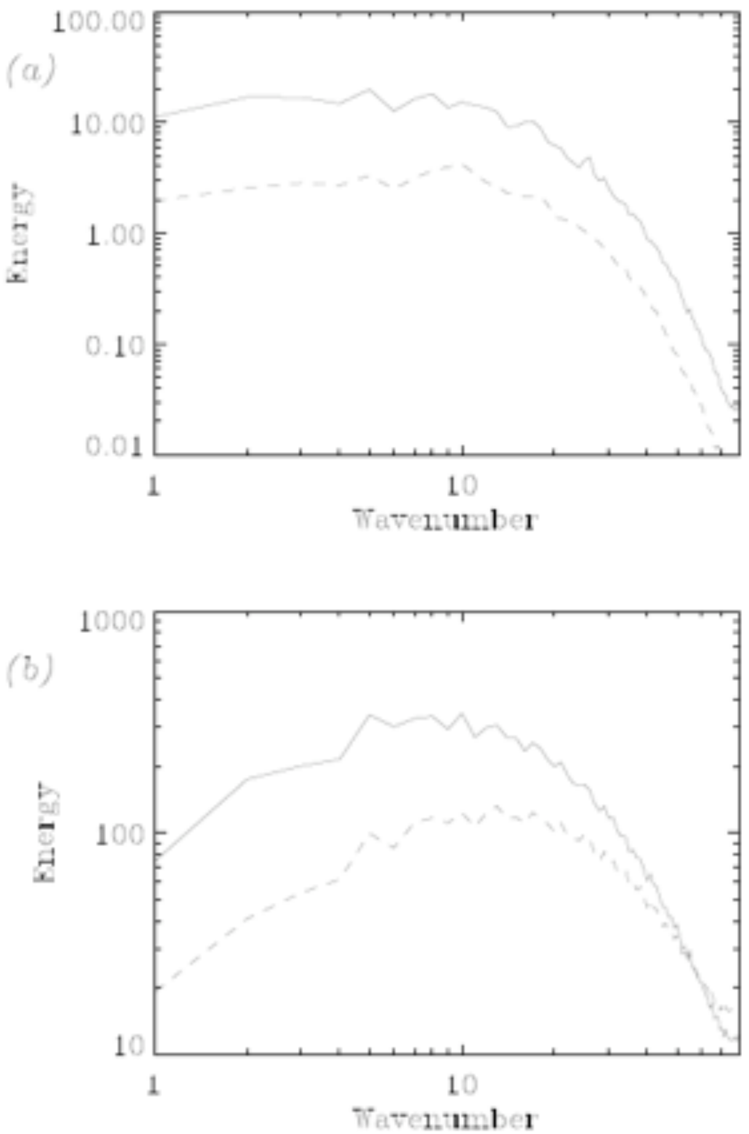


FIG. 3: Horizontal power spectra for the magnetic field in both the kinematic (dashed) and dynamic (solid) regimes. In (*a*) $S = 5/3$, $Ra = 150\,000$; in (*b*) $S = 0$, $Ra = 1\,000\,000$; in both cases $Ta = 500\,000$. The spectra were computed over the interior region of the domain ($0.06 < z < 0.94$). The arbitrary amplitudes of the kinematic spectra have been scaled so as to be on the same plot.

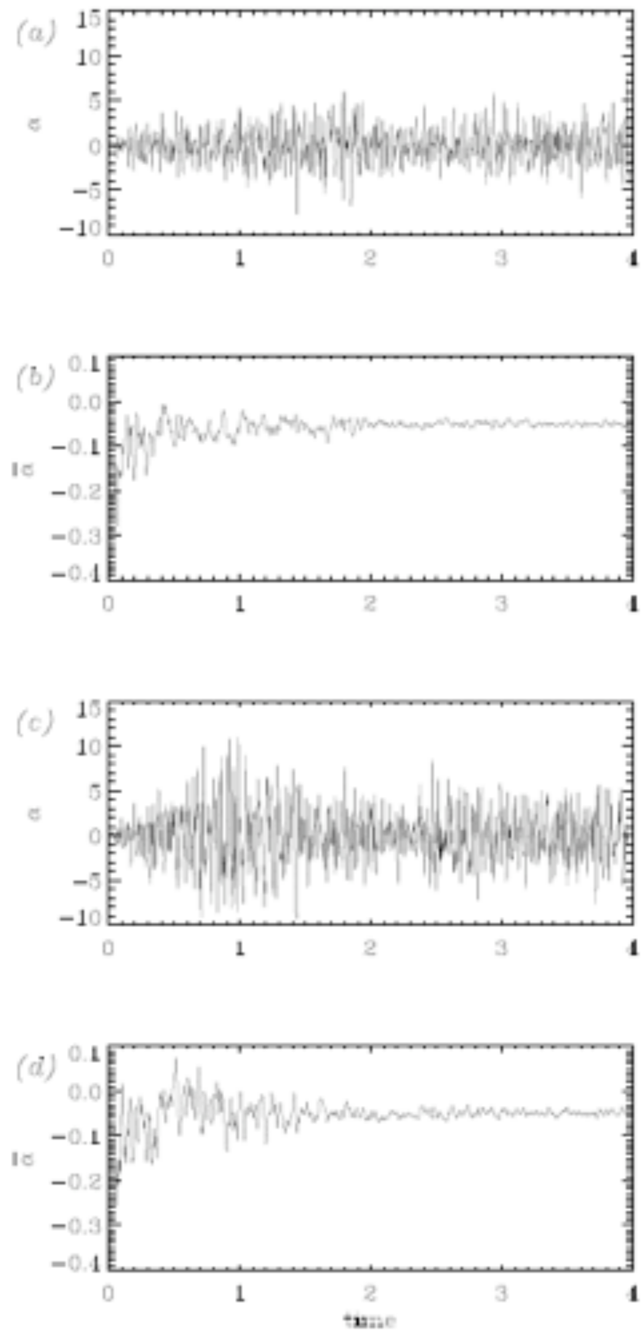


FIG. 4: (a) Longitudinal α-effect versus time for $S = 0$; (b) $\overline{\alpha}$, the cumulative temporal average of α, for $S = 0$; (c) α for $S = 1/3$; (d) $\overline{\alpha}$ for $S = 1/3$.

How does the shear enhance dynamo action?

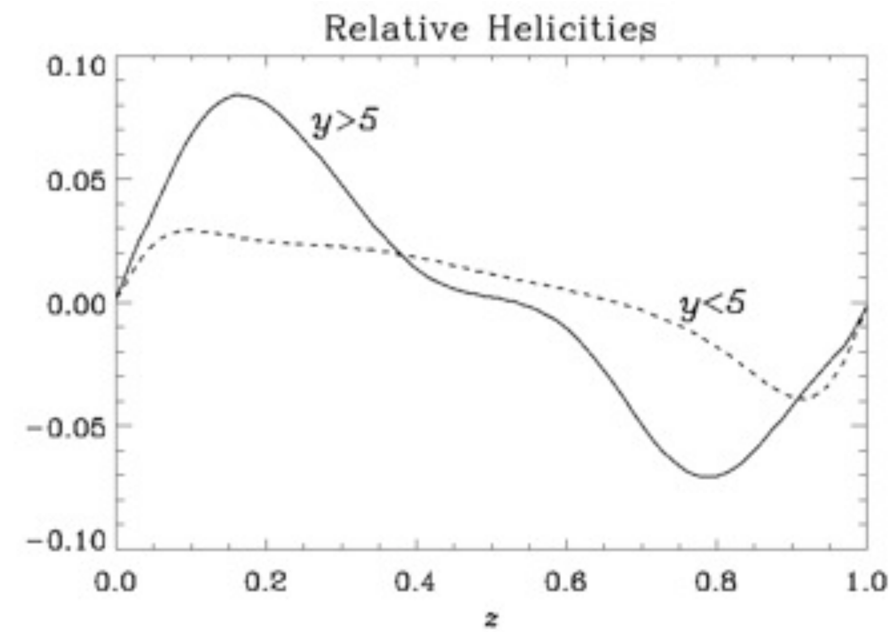
- no evidence of greater coherence due to the shear
- linear rate of increase of growth rate suggests fluctuating alpha or shear current effect, or if optimal scale is selected then the Yousef mechanism
- standard 'mean-field' dynamo apparently gives wrong behaviour

Which scales of flow are responsible for growth?

Properties of velocity fields

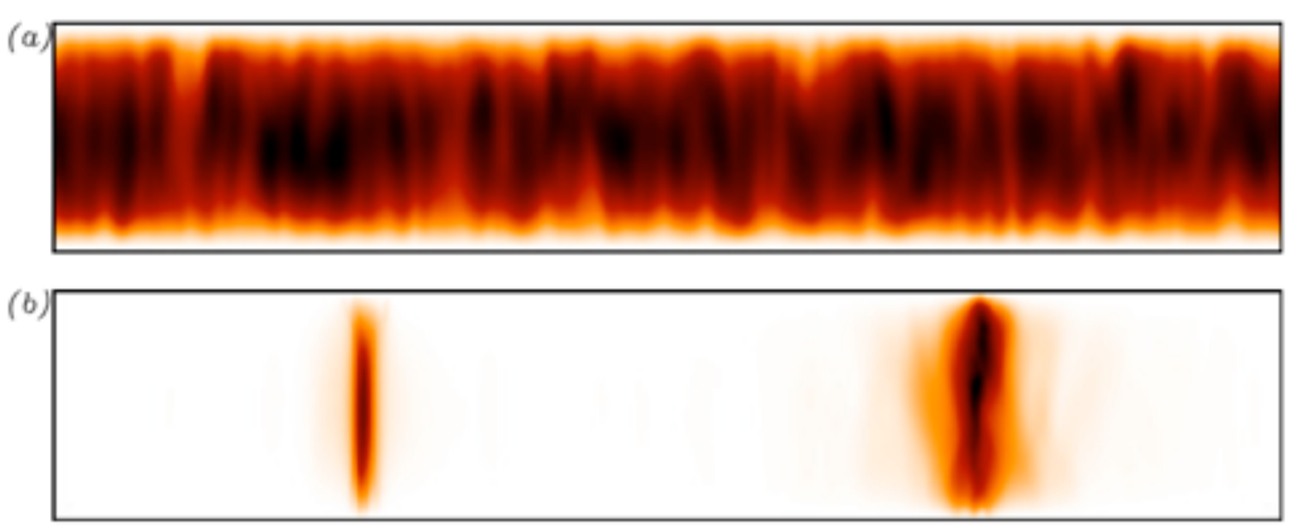
- Strongly asymmetric velocity field structure for intermediate shear rates. Vorticity has form

$$Ta^{1/2} + \frac{2\pi U_0}{\lambda} \sin \frac{2\pi y}{\lambda}.$$



- Symmetric properties (in *y*) when U_0 is very small or very large.

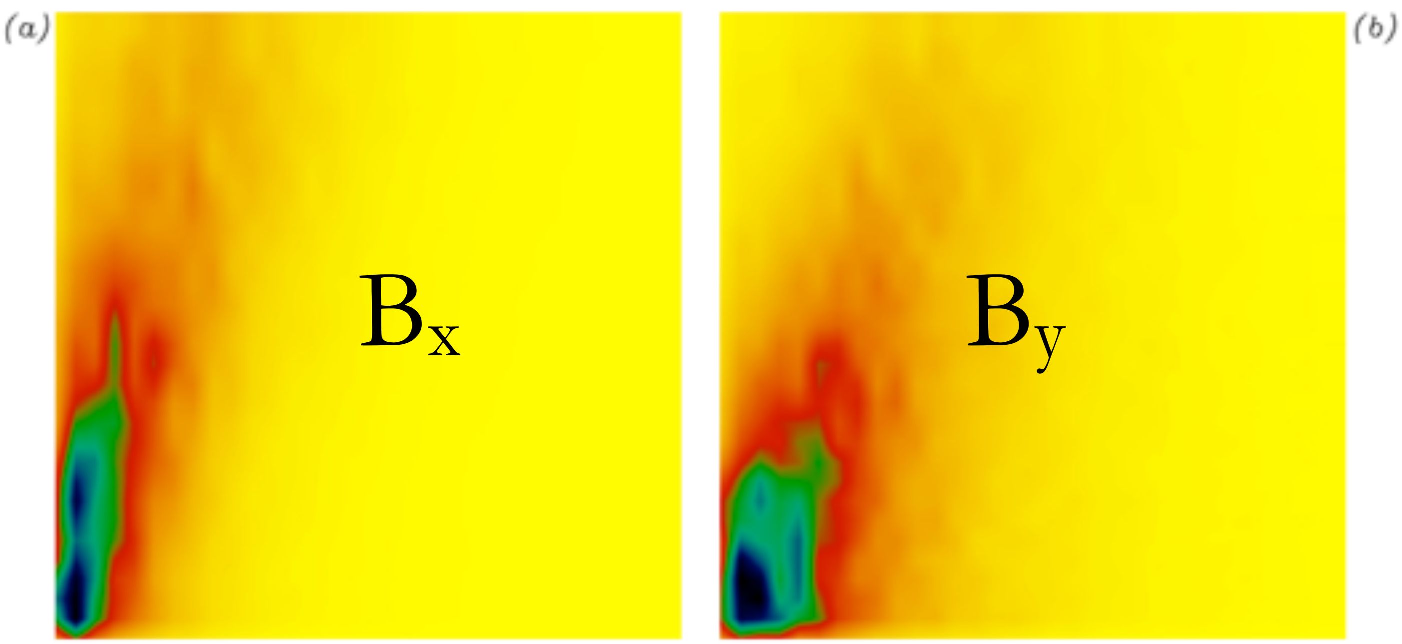
- Typically flow dominated by horizontal flows save near the turning points of the shear

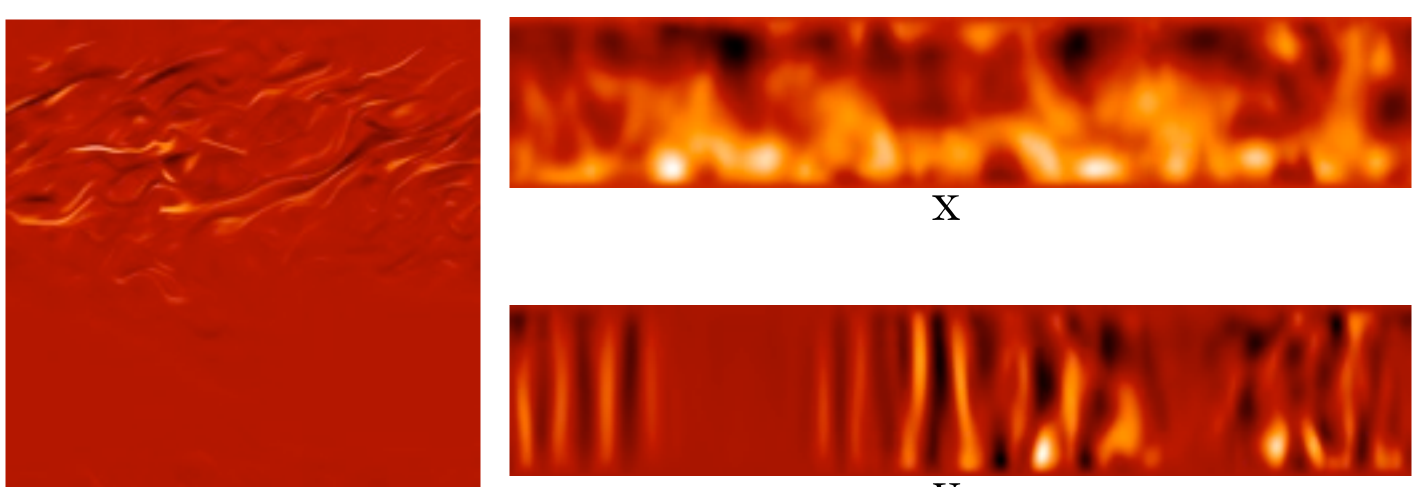


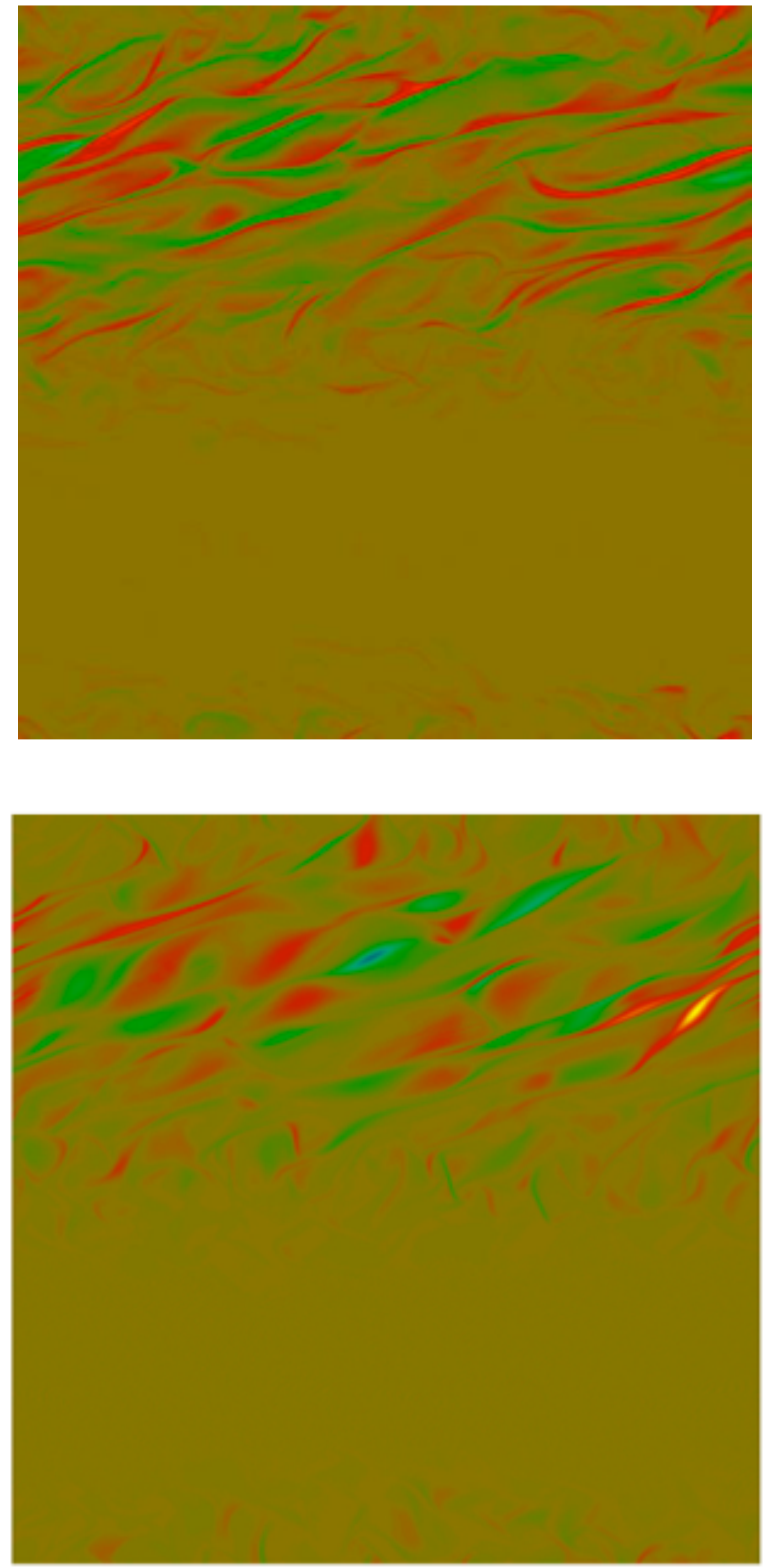
- Helicity also asymmetric

Nature of dynamo fields

- Strongly asymmetric field structure
- Underlying vorticity less in top half (in *y*) of layer







B_x

U_0=1000

B_y

How does the dynamo work?

- To decide what properties of the flow fields contribute to dynamo action, consider *filtered* flows

- So far have only considered kinematic field growth

1. Solve non-magnetic convection problem at high resolution
2. Construct filtered velocity field in real time
3. Solve induction equation for that velocity field at high resolution

- Two kinds of filtration

1. Remove modes with wavenumber $k > n$ **(SWC)**
2. Retain shear but otherwise remove modes with $k < m$ **(LWC)**

Effect of SWC on dynamo

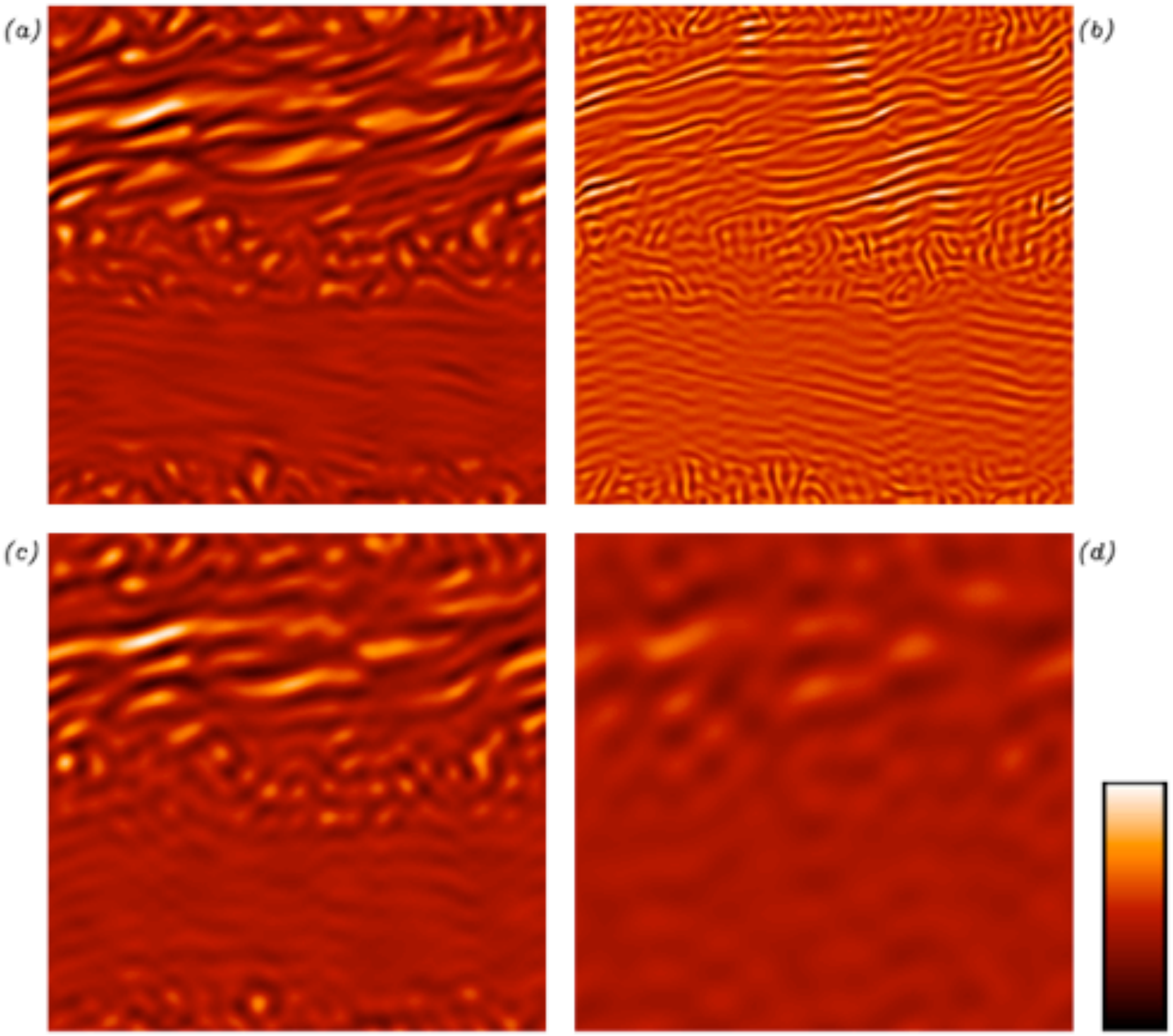


FIGURE 10. Density plots of the vertical velocity close to the upper boundary for (*a*) SWC30, (*c*) SWC20 and (*d*) SWC10. The residual small-scale flow removed by the filtration SWC30 is shown in (*b*). Plots (*a*), (*b*), and (*c*) are on the same scale; the residual velocity in (*b*) is of much smaller amplitude and is scaled independently. White denotes upward velocity, black falling velocity.

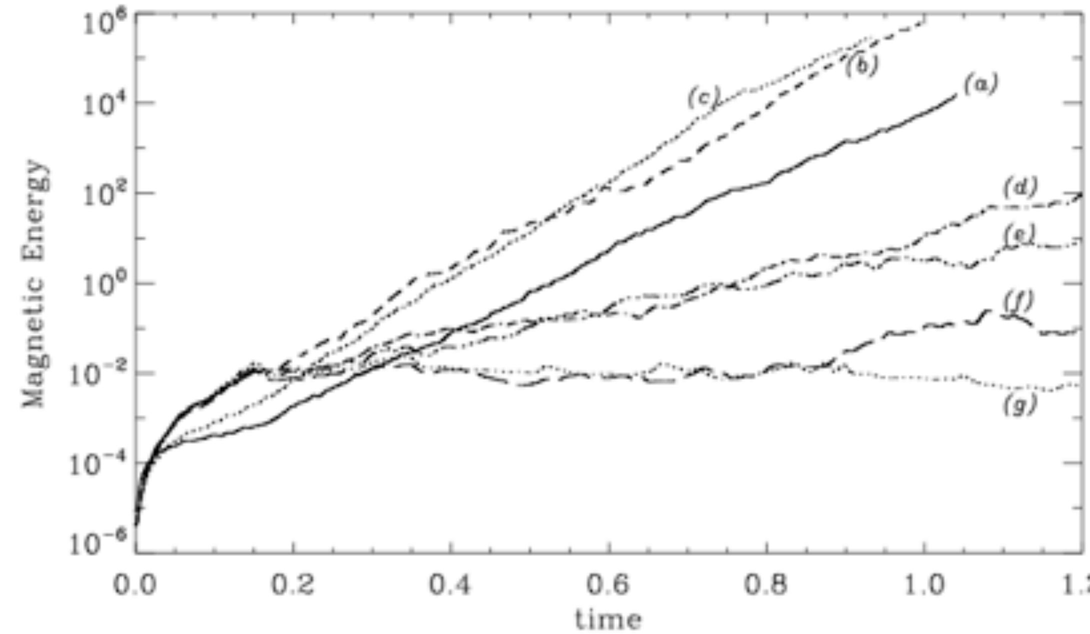


FIGURE 11. Magnetic energy versus time, with $U_0 = 1000$ and for various shortwave cutoffs SWCn: (*a*) all modes retained, (*b*) $n = 20$, (*c*) $n = 10$, (*d*) $n = 5$, (*e*) $n = 4$, (*f*) $n = 3$, (*g*) $n = 2$.

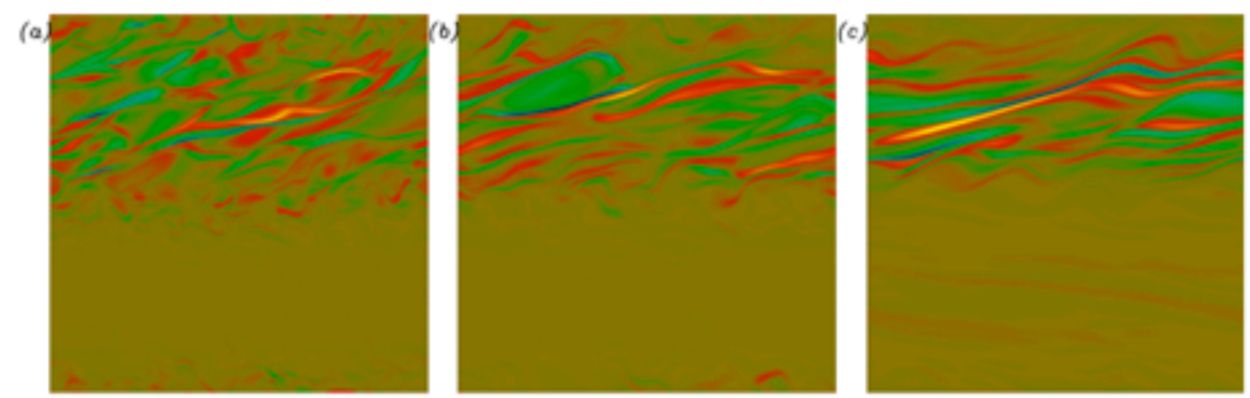
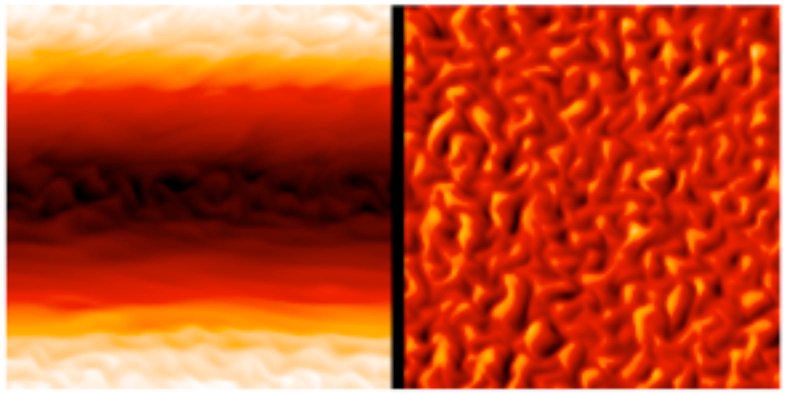
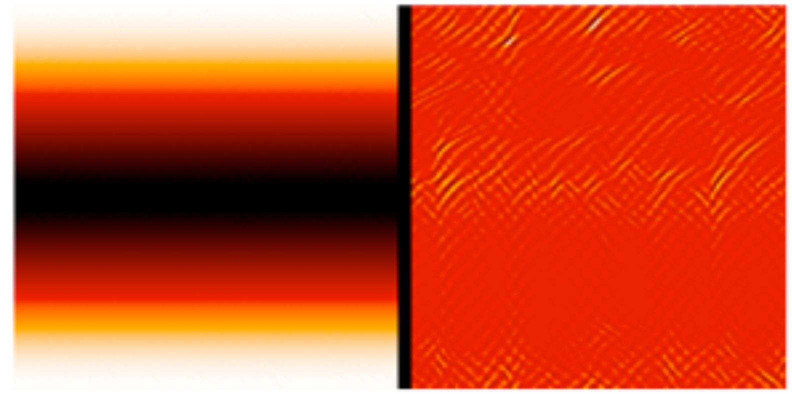


FIGURE 12. Density plots of B_z at the upper boundary, for various shortwave cutoffs SWCn: (*a*) $n = 20$, (*b*) $n = 10$, (*c*) $n = 5$. Colour table as in figure 7.

Effect of LWC on flow





U_x,U_y; unfiltered

U_x,U_y; modes >10 retained

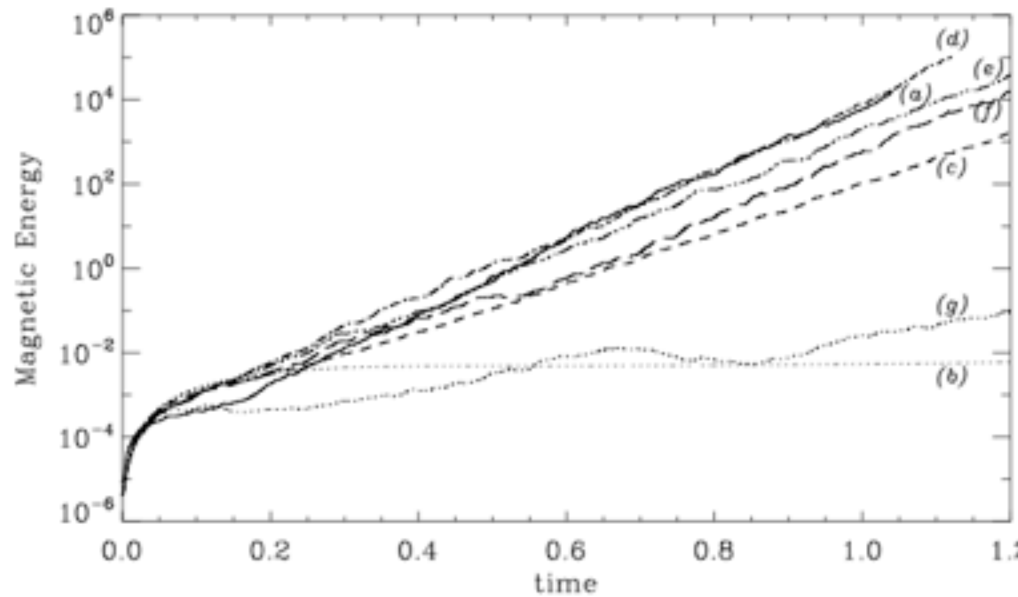


FIGURE 13. Magnetic energy versus time for various long-wave cutoffs LWCn: (*a*) all modes retained, (*b*) $n = 20$, (*c*) $n = 10$, (*d*) $n = 5$, (*e*) $n = 4$, (*f*) $n = 3$, (*g*) $n = 2$.

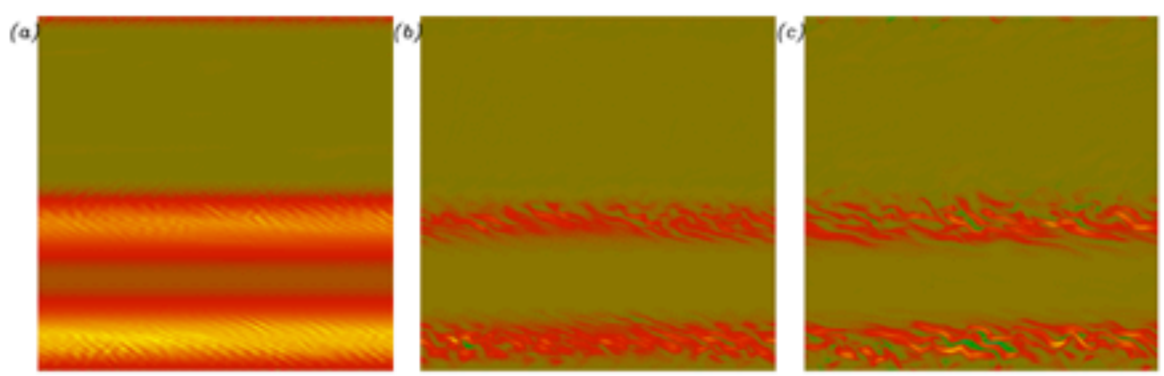
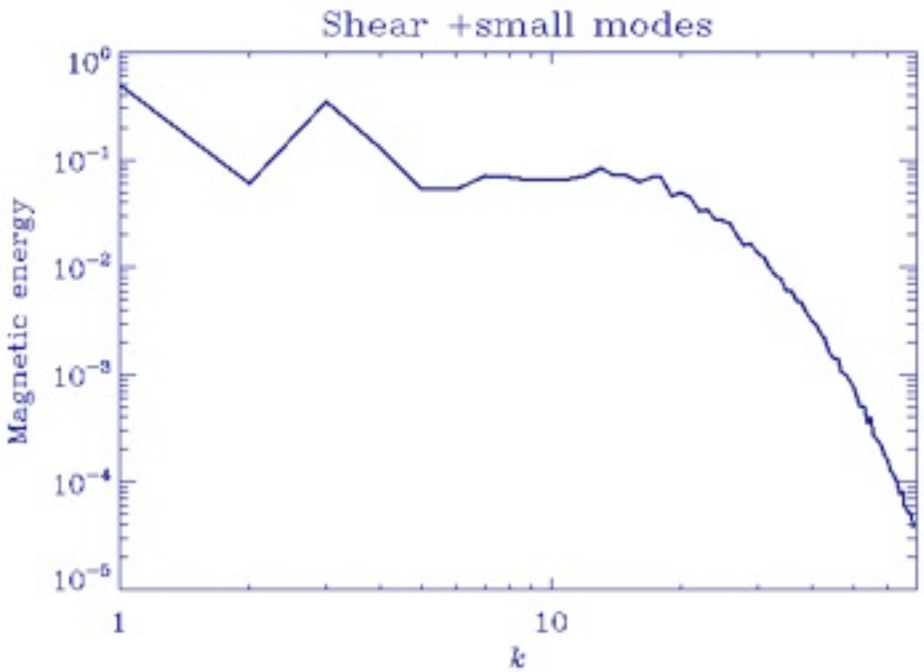
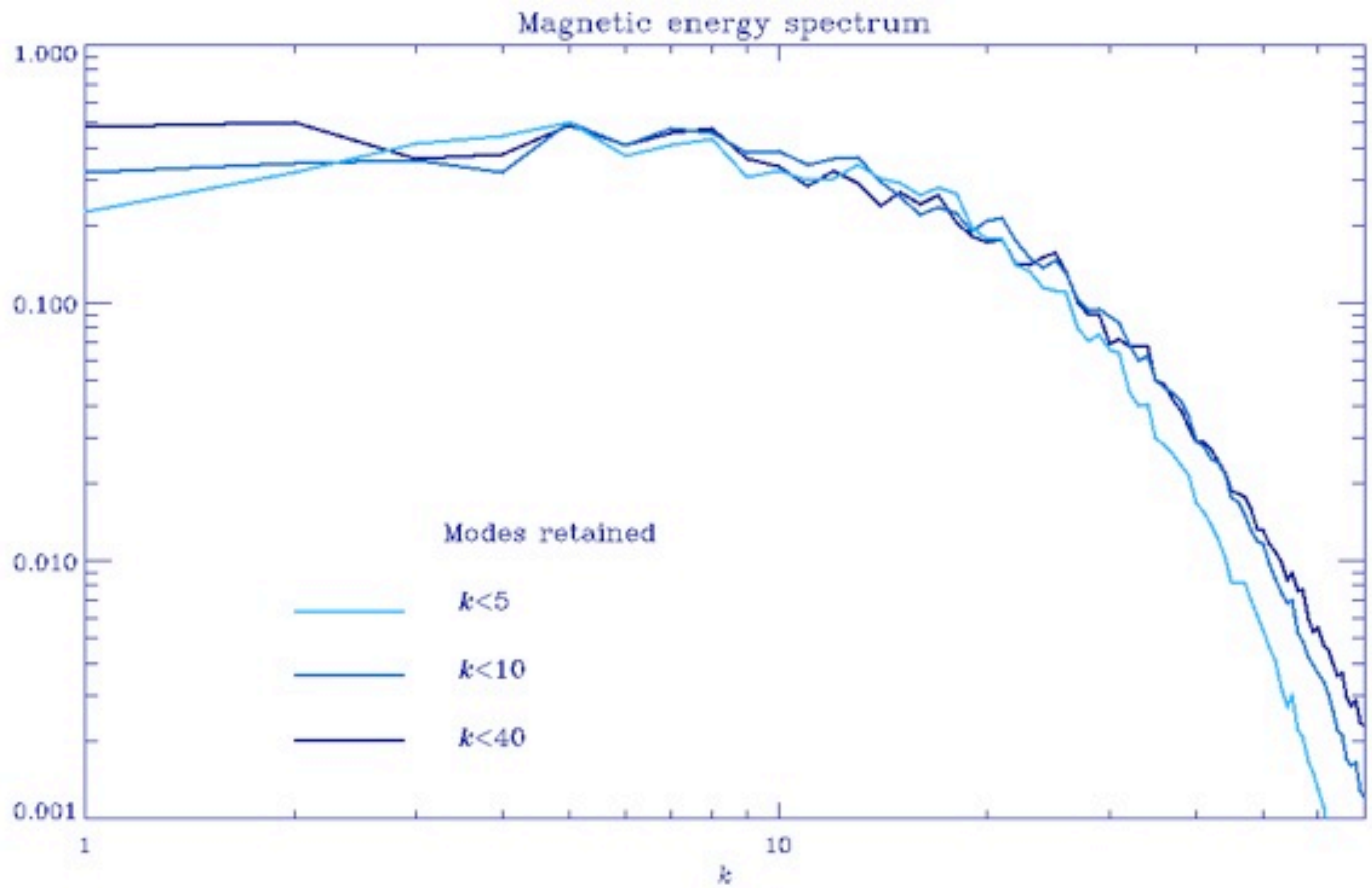


FIGURE 14. Density plots of B_x at the upper boundary, for various long-wave cutoffs LWCn: (*a*) $n = 20$, (*b*) $n = 10$, (*c*) $n = 5$. Colour table as in figure 7.

Magnetic spectra for truncated flows



Wavenumbers <10 removed, shear retained

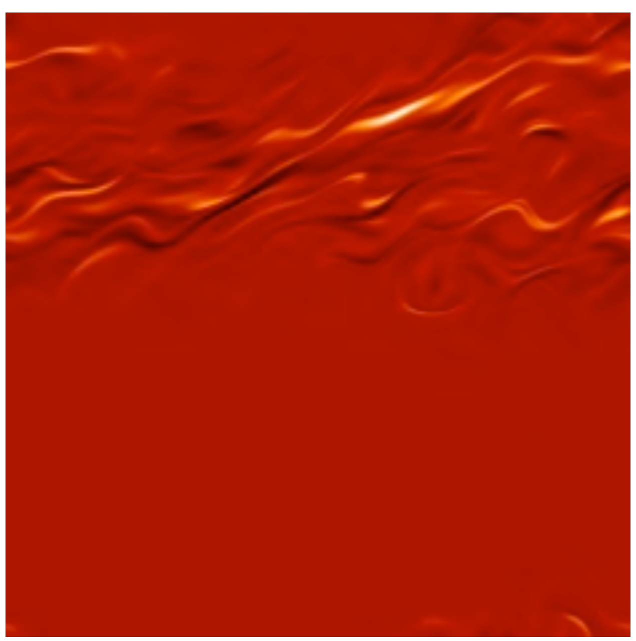


High wavenumbers removed

Magnetic field structure

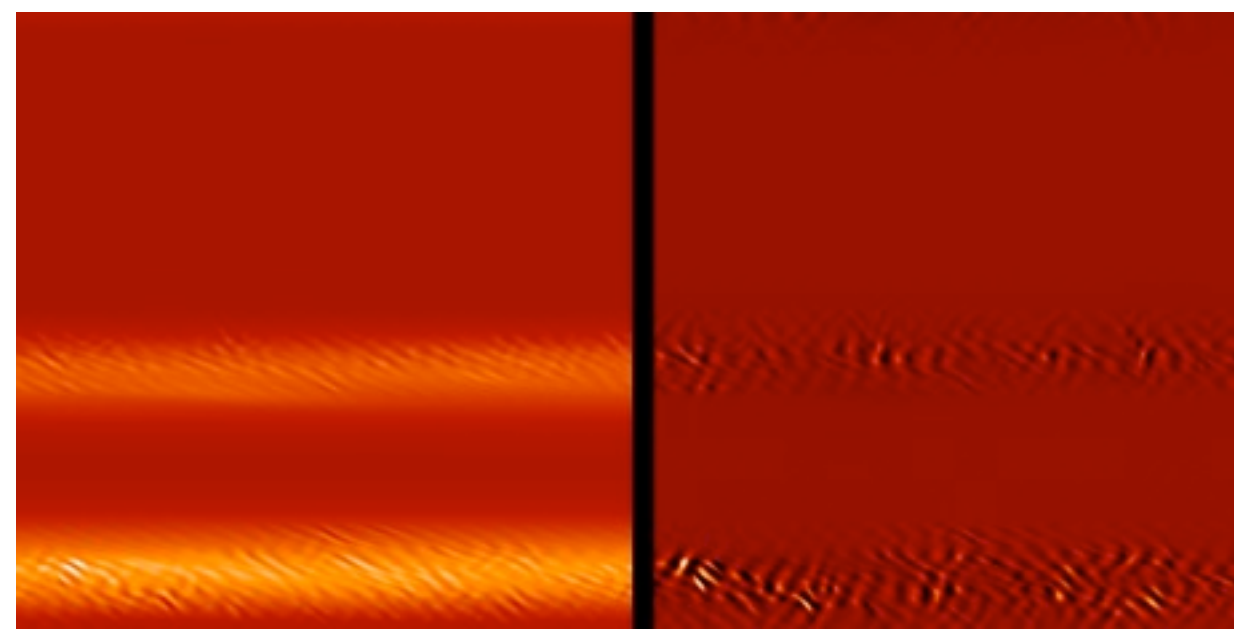
- Form of magnetic field does not change much when large scales removed

- But removal of large scales (to make two-scale dynamo) has drastic effect



B_x - unfiltered

B_x- $k>5$ removed



B_x $k<20$ removed B_y

Conclusion

•Forced flows with uniform shear in long boxes shown to produce dynamo action. Mechanism not explicable in terms of usual dynamo approximations. Qu - is uniform shear special in any way?

•In rotating convection large scale shear promotes dynamo action, with large scale features and inhomogeneity of field structure

•Nature of mechanism still unclear, but certainly not due to any increased coherence of small scale flow

•Dynamo properties of filtered flows show that growth rates do not depend much on limits on spectrum of convective flow - plausible characterization of long wavelength filtered fields as two-scale dynamos, but form of eigenfunction not like that for full flow, and simple alpha-effect model not applicable.