ENERGY TRANSFERS FOR LARGE EDDY SIMULATIONS OF MAGNETOHYDRODYNAMIC TURBULENCE

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Abstract Our goal is to use Large Eddy Simulation (LES) to simulate forced magnetohydrodynamic (MHD) turbulence at Reynolds numbers (kinetic and magnetic) higher than those used in direct numerical simulation (DNS). As a first step we test several subgrid scale models against DNS by comparing the energy transfers between the filtered and subgrid scales. For an eddy diffusivity model we find that the effect of the forcing type, helical or not, is crucial, suggesting that helicity should be taken into account in MHD LES models.

LES FORMALISM OF MHD EQUATIONS

We distinguish the filtered and subgrid scales by applying a filtering operator with a given filter size Δ. Thus the filtered velocity $\bar{u}_i$ and induction $\bar{b}_i$, expressed in Alfven-speed unit, obey to the the following filtered equations

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_i\bar{u}_j - \bar{b}_i\bar{b}_j) = \frac{\partial}{\partial x_j}\left(\nu\frac{\partial \bar{u}_i}{\partial x_j} - \tau^u_{ij} + \tau^b_{ij}\right) - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} \tag{1}$$

$$\frac{\partial \bar{b}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{b}_i\bar{u}_j - \bar{b}_j\bar{u}_i) = \frac{\partial}{\partial x_j}\left(\eta\frac{\partial \bar{b}_i}{\partial x_j} - \tau^{ub}_{ij}\right) \tag{2}$$

where ν and η are the kinematic viscosity and magnetic diffusivity and $\bar{p}$ is the filtered total pressure (including the magnetic pressure). In these equations $\tau^u_{ij} = \overline{u_i u_j} - \bar{u}_i\bar{u}_j$, $\tau^b_{ij} = \overline{b_i b_j} - \bar{b}_j\bar{b}_i$, and $\tau^{ub}_{ij} = \overline{b_i u_j} - \bar{b}_i\bar{u}_j - \left(\overline{u_i b_j} - \bar{u}_i\bar{b}_j\right)$ are subgrid-scale (SGS) tensors. In a LES these tensors cannot explicitly be determined but instead they are estimated via SGS models assuming relationships with the resolved quantities $\bar{u}_i$ and $\bar{b}_i$. From (1) and (2) the equations for the filtered kinetic and magnetic energies $E^{\bar{u}} = 1/2\bar{u}_i\bar{u}_i$ and $E^{\bar{b}} = 1/2\bar{b}_i\bar{b}_i$ take the following form

$$\frac{\partial E^{\bar{u}}}{\partial t} + \frac{\partial \bar{u}_j E^{\bar{u}}}{\partial x_j} = D^{\bar{u}} - \epsilon^{\bar{u}} + T^u_{sgs,u} + T^u_{sgs,b} + T_E \tag{3}$$

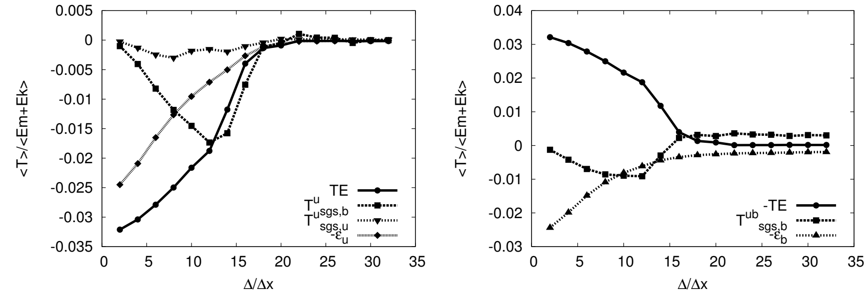
$$\frac{\partial E^{\bar{b}}}{\partial t} + \frac{\partial \bar{u}_j E^{\bar{b}}}{\partial x_j} = D^{\bar{b}} - \epsilon^{\bar{b}} + T^{ub}_{sgs,b} - T_E, \tag{4}$$

where we distinguish the diffusion terms $D^{\bar{u}}$ and $D^{\bar{b}}$ from the molecular dissipation terms $\epsilon^{\bar{u}}$ and $\epsilon^{\bar{b}}$. On the right hand side the other terms correspond to energies transfers. The quantity T_E is the transfer between the filtered kinetic and magnetic energies. The other quantities $T^u_{sgs,u}$, $T^u_{sgs,b}$ and $T^{ub}_{sgs,b}$ are energy transfers between the filtered and subgrid scales. They are defined by $T^u_{sgs,u} = \bar{S}_{ij}\tau^u_{ij}$, $T^u_{sgs,b} = -\bar{S}_{ij}\tau^b_{ij}$ and $T^{ub}_{sgs,b} = \tau^{ub}_{ij}\bar{J}_{ij}$ where $\bar{S}_{ij}$ is the filtered strain-rate tensor and $\bar{J}_{ij}$ the filtered magnetic rotation-rate tensor. Of course performant SGS models should be able to account correctly for these transfers.

ENERGY TRANSFERS OBTAINED BY FILTERING AT SIZE Δ

We solve the (unfiltered) equations of MHD turbulence using a pseudo-spectral code. Using 128^3 grid points the Reynolds number based on the Taylor microscale is about 25 and the magnetic Prandtl number $\mathrm{Pm} = \eta/\nu$ is taken equal to unity. Two isotropic flow forcing will be compared, one being pointwise helical, the other one being pointwise non-helical. For both forcings dynamo action occurs and a statistically stationary saturated regime is reached. With the helical forcing a large scale magnetic field is obtained in agreement with [2].

The energy transfers T_E, $T^u_{sgs,u}$, $T^u_{sgs,b}$ and $T^{ub}_{sgs,b}$ are calculated by filtering the DNS data with a filter size Δ. In Fig. 1(a) and Fig. 1(b) these quantities are plotted versus $\Delta/\Delta x$ where Δx is the spatial resolution of the DNS (here $\Delta x = 2\pi/128$). The results are shown for the helical forcing only. As expected, the dynamo action leads to a large transfer from kinetic to magnetic energy, as shown by the negative value of T_E. In Fig. 1(a) we see that the transfer of kinetic energy from filtered to subgrid scales is direct and mainly due to the Lorentz forces $T^u_{sgs,b}$. In Fig. 1(b) we see that the transfer of magnetic energy from filtered to subgrid scales scales ($T^{ub}_{sgs,b}$) is inverse for $\Delta/\Delta x > 16$ in agreement with an α-effect of the mean-field theory [2]. Such transfer is challenging to obtain from SGS model as it would correspond to a negative dissipation term.



(a) (b)

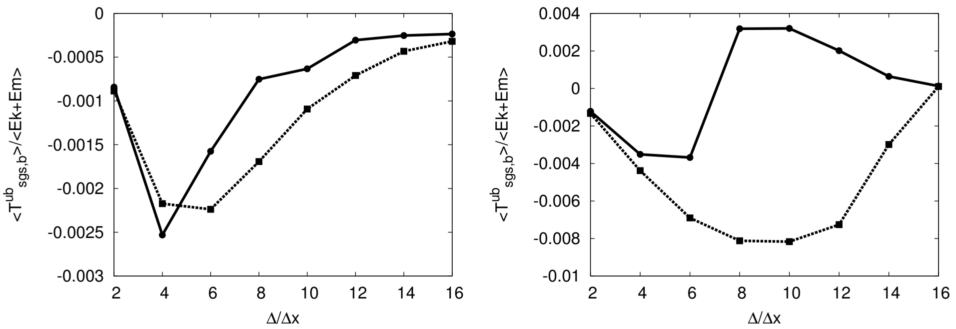
Figure 1. Energy transfers versus the filtering size Δ/Δ_x involved in (a) the kinetic energy equation (3), and (b) the magnetic energy equation (4), for a helical forcing.

A PRIORI ENERGY TRANSFERS USING SGS MODELS

From the previous DNS the energy transfers are now calculated using a classic SGS model which is based on eddy magnetic diffusivity [1]. The energy transfer $T^{ub}_{sgs,b}$ is again plotted versus $\Delta/\Delta x$ for the non-helical (Fig. 2(a)) and helical (Fig. 2(b)) forcing. In each case the energy transfer $T^{ub}_{sgs,b}$ obtained by filtering (shown in the previous section) is also plotted for comparison.

In the non-helical case (Fig. 2(a)), the SGS model leads to a transfer rather similar to the filtered DNS. Its magnitude is however a bit smaller. Therefore running a LES with such model might lead to a lack of magnetic dissipation and then either to numerical instability or spurious large scale magnetic energy.

In the helical case (Fig. 2(b)), the SGS model leads to a transfer in strong disagreement with the DNS filtered results. In particular for $6 < \Delta/\Delta_x < 16$ spurious energy is transfered to large scales.



(a) (b)

Figure 2. Energy transfer $T^{ub}_{sgs,b}$ versus $\Delta/\Delta x$ for (a) non-helical and (b) helical forcing. The solid line corresponds to the eddy magnetic diffusivity model [1] and the dotted line to the filtered DNS.

Other energy transfers will be shown during the conference. Higher resolution DNS (256^3 and 512^3 grid points) will be achieved in order to extend the study on a broader range of scales. The difference between large and small scales dynamo will be also discussed in terms of LES. Finally, the transfers obtained by other SGS models (as the dynamical scale similarity model [3]) will be presented and again compared with the filtered DNS.

References

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