DYNAMIC EFFECTS ON THE STRETCHING OF MAGNETIC FIELD BY A PLASMA FLOW

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Successful kinematic dynamos need:

1) Stretching of the magnetic field by the plasma flow.

2) Constructive folding.

In the absence of diffusivity,

$$\frac{D\mathbf{B}}{Dt} = \mathbf{B}\cdot\nabla\mathbf{u}\,, \qquad \frac{1}{2}\frac{DB^2}{Dt} = \mathbf{B}\cdot\nabla\mathbf{u}(\mathbf{B}) = \mathbf{B}\cdot S(\mathbf{B}),$$

D/Dt is the lagrangian derivative: $\dfrac{D}{Dt} = \dfrac{\partial}{\partial t} + \mathbf{u}\cdot\nabla$

$S = (1/2)(\nabla\mathbf{u} + (\nabla\mathbf{u})^{+})$ is the strain matrix $(1/2)(\partial_i u_j + \partial_j u_i)$.

What is the evolution of the rate of growth of magnetic energy

$$\Phi = \mathbf{B} \cdot S(\mathbf{B})?$$

May optimal growth be maintained for long?

May $\mathbf{B}$ keep pointing in the direction of an eigenvector of S with positive eigenvalue?

Optimistic analogy:

The vorticity in the Navier-Stokes equations satisfies also the induction equation and has been found to have a tendency to align with the intermediate vector of the strain matrix.

Some of these simulations seem to show that the vorticity points first to the largest eigenvector, and as the turbulence develops, to the second one.

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The Lorentz force imposes a forcing absent in the vorticity case.

Ideal MHD equations:

$$\frac{D\mathbf{u}}{Dt} = \mathbf{B}\cdot\nabla\mathbf{B} - \nabla p_* + \mathbf{g}\,, \qquad \frac{D\mathbf{B}}{Dt} = \mathbf{B}\cdot\nabla\mathbf{u},$$

$p_* = p + (1/2)B^2$ is the total (kinetic plus magnetic) pressure.

$\mathbf{g}$ a possible forcing upon the momentum equation.

$$\begin{aligned}\frac{D\Phi}{Dt} =& |\nabla\mathbf{u}(\mathbf{B})|^2 - |\nabla\mathbf{B}(\mathbf{B})|^2 + \frac{1}{2}\mathbf{B}\cdot\nabla(\mathbf{B}\cdot\nabla B^2) \\ &- \mathbf{B}\cdot P_*''(\mathbf{B}) + \mathbf{B}\cdot G(\mathbf{B}).\end{aligned}$$

$G = (1/2)(\partial_i g_j + \partial_j g_i), \qquad P_*'' = (\partial_{i,j} p_*).$

All the terms depend only on local behavior of the magnitudes, except for P''_*:

$$\Delta p_* = \|T\|_2^2 - \frac{1}{2}|\mathbf{J}|^2 - \|S\|_2^2 + \frac{1}{2}|\omega|^2 + \nabla \cdot \mathbf{g}.$$

$$T = \frac{1}{2}(\partial_i B_j + \partial_j B_i)\,, \quad \mathbf{b} = \mathbf{B}/B\,.$$

The action of P''_* is hard to foresee:

a) It cannot be ignored,

b) It tends to damps instabilities.

The magnitude

$$\Psi = \Phi/B^2 = \mathbf{b}\cdot S(\mathbf{b}),$$

represents, not the rate of increase of magnetic energy as before, but the efficiency of the magnetic field direction to increase magnetic energy by pointing in an appropriate direction.

$$\begin{aligned}\frac{D\Psi}{Dt} =& -2(\mathbf{b}\cdot S(\mathbf{b}))^2 + |\nabla\mathbf{u}(\mathbf{b})|^2 \\ &+ 2(\mathbf{b}\cdot T(\mathbf{b}))^2 - |\nabla\mathbf{B}(\mathbf{b})|^2 \\ &- \mathbf{b}\cdot P_*''(\mathbf{b}) + \mathbf{b}\cdot G(\mathbf{b}) + \frac{1}{2}\mathbf{B}\cdot\nabla\left(\frac{\mathbf{B}\cdot\nabla B^2}{B^2}\right).\end{aligned}$$

Second line: $2(\mathbf{b}\cdot T(\mathbf{b}))^2 - |\nabla\mathbf{B}(\mathbf{b})|^2 = \left(\frac{dB}{ds}\right)^2 - B^2\kappa^2.$

$d/ds =$ arc length along the magnetic field line.

In chaotic flows, this term is usually negative.

First line: when $\mathbf{b}$ is near an eigenvector of S, Ψ =eigenvalue, and

$$-2(\mathbf{b} \cdot S(\mathbf{b}))^2 + |\nabla \mathbf{u}(\mathbf{b})|^2 = -\Psi^2 + \frac{1}{4}|\omega \times \mathbf{b}|^2.$$

Terms decreasing Ψ (efficiency of magnetic field stretching):

1) Size of Ψ: The largest is the field growth, the more rapidly it decreases.

2) Chaotic magnetic field lines (brought about e.g. by a chaotic flow).

Terms increasing Ψ:

$|\omega \times \mathbf{b}|^2$: plasma rotation.

Plus nonlocal effects and terms of zero mean.

The rate of growth of magnetic energy is governed by

a) Nonlocal effects,

b) Fluctuating effects,

c) Local effects:

c1) Local rotation of the plasma may enhance this growth, if the field direction is not collinear with the angular velocity.

c2) The square of the curvature of the field line detracts from the growth rate,

c3) The square of the eigenvalue associated to the eigendirection of the strain matrix where the field points also detracts from the growth rate.

LOCAL DYNAMICS TEND TO QUENCH RAPIDLY FIELD STRETCHING.