An Introduction to Planetary Dynamos

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Outline

(1) Observed properties of planetary magnetic fields

(2) Energy sources for planetary dynamos

(3) Conditions needed for thermal convection

(4) Conditions needed for compositional convection

(5) Energy and entropy balance: ohmic heating

(6) Dynamical regimes in planetary cores

Terrestrial Planets

Earth, Ganymede Mercury:- Active Dynamos

Mars, Moon:- Extinct Dynamos

Venus, other moons of Jupiter: - No Current Dynamo

- All have, or have had, liquid iron cores

- Different behaviour reflects different thermal history, composition

Nature and magnitude of the magnetic fields

Earth: mainly dipolar, inclination to rotation axis currently $11.5°$, field at CMB $\sim 8 \times 10^{-4}$T. Core radius 3480 km

Mercury: form of field not known. Strength at CMB $\sim 1.4 \times 10^{-6}$T. Rotates only once every 57 days. Core radius 1900 km

Ganymede: approx dipolar, inclination $\sim 10°$. Strength at CMB $\sim 2.5 \times 10^{-4}$T. Rotates once every 7 days. Core radius 480 km

Extinct Martian field is deduced from strong crustal magnetism, which suggests Mars had a dipole field in the past. Lunar rocks also have remanent magnetism

Gas Giants

Jupiter and Saturn both have strong magnetic fields

High pressure $\rightarrow$ electrical conductivity

Metallic hydrogen state

Both mainly dipolar: Jupiter's field is about 17×10^{-4}T at the magnetic core boundary, and the dipole inclination is 9.6°

Saturn's field is 2.5×10^{-4}T and is aligned with its rotation axis to within 1°, despite Cowling's theorem!

Uranus and Neptune also have fields of magnitude around 10^{-4}T but they are not dipole dominated.

Strong quadrupolar components, and no alignment with rotation axis

What is needed for a dynamo?

$$\frac{\partial \mathbf{B}}{\partial t} = \eta \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

Induction must overcome diffusion

Earth's core magnetic diffusivity $\eta \sim 2\,\mathrm{m}^2\mathrm{s}^{-1}$

Earth's core velocity $U_* \sim 2 \times 10^{-4}\,\mathrm{ms}^{-1}$

Earth's core size $\quad \ell \sim \quad 3.5 \times 10^6\,\mathrm{m}$

Magnetic Reynolds number $R_m = U_* \ell / \eta \sim 350$

U_* from secular variation studies 'Westward Drift'

Ohmic decay time $\ell^2/\pi^2\eta \sim 20,000$ years

Other Planets

Jupiter: Magnetic diffusivity $\eta \sim 1\,\mathrm{m^2s^{-1}}$. Typical velocity $U_* \sim 10^{-3}\,\mathrm{ms^{-1}}$. Core Size $\quad \ell \sim \quad 5 \times 10^7\,\mathrm{m}$

$R_m = U_*\ell/\eta \sim 5 \times 10^4$

U_* from differences in magnetic field: Voyager/Galileo

Uranus, Neptune magnetic diffusivity $5 \times 10^2\,\mathrm{m^2s^{-1}}$?

U_* must be greater than Earth value to get R_m large enough

Ganymede has core radius 0.1 of Earth's, and similar conductivity, so it must also have a larger core velocity than Earth

Major Problems in Planetary Dynamo theory

(i) Why is Mercury's field so weak?

(ii) Why does Venus not have a magnetic field?

(iii) How did the Earth's dynamo work before the inner core formed?

(iv) Why did the dynamos of Mars and the Moon fail?

(v) How does Ganymede maintain a dynamo when its core is so small?

(vi) Why is Saturn's field so axisymmetric?

(vii) Why are the fields of Uranus and Neptune non-dipolar?

Energy Sources for Planetary Dynamos

- Are all planetary magnetic fields dynamo driven?

Crustal magnetism (remanent magnetism), as on Mars, produces small scale stationary fields only

Many mechanisms for producing currents: battery effects, thermoelectric effects, etc. None can produce enough field to explain observed field strengths, except possibly the weak field of Mercury

- Dynamo energy source: Precession, Tidal interactions, Thermal Convection, Compositional Convection

• Tides and precession derive their energy from the Earth's core rotation

• Rotational/Gravitational energy

$$\sim \Omega^2 \ell / g \sim 10^{-3}$$

so precessional dynamos must have less dissipation for a given field strength

• Most dynamos probably convection driven, but tides and/or precession could possibly drive a dynamo in a stably stratified liquid core

• High frequency inertial waves excited: these must be converted to slow large scale flows

Compositional convection

As the inner core grows, dense iron freezes at the ICB

This releases lighter buoyant material which rises, stirring the fluid outer core

This process liberates about 0.5 TW of gravitational energy

Latent heat is also released

Cooling may also induce phase changes at the outer core boundary. This could release heavier material which falls through the fluid core

'Helium rain-out' in Saturn

Convection driven dynamo requirements

- Fluid, electrically conducting core
- For thermal convection, fluid core must be superadiabatic
- For compositional convection there must be an active phase change in the conducting region
- The rate of working of buoyancy forces must be large enough to balance ohmic dissipation of the magnetic field

Conditions for fluid cores

Melting temperature rises with pressure (solid inner core) but is depressed by impurities

As a terrestrial planet cools, inner core grows. Remaining liquid outer core has progressively higher impurity content

Thin shell dynamo in Mercury?

Why is Ganymede's core still liquid?

Melting properties, and other physical properties such as thermal conductivity determined from high pressure physics experiments

'ab initio' quantum calculations now being developed

• Temperature gradient must be superadiabatic

The adiabatic temperature gradient is

$$T_{ad}^{-1}\left(\frac{dT}{dr}\right)_{ad} = -g\alpha/c_p,$$

Here α is the coefficient of thermal expansion and c_p the specific heat. The heat flux carried down this gradient by conduction/radiation is

$$F_{ad} = -\kappa\rho c_p\left(\frac{dT}{dr}\right)_{ad}$$

If actual heat flux is F, then the conduction gradient is defined by

$$F = -\kappa\rho c_p\left(\frac{dT}{dr}\right)_{cond}$$

Convection occurs if

$$Ra = g\alpha d^4 \left[\left(\frac{dT}{dr} \right)_{cond} - \left(\frac{dT}{dr} \right)_{ad} \right] / \kappa\nu > Ra_{crit}$$

where Ra_{crit} is some number such as $27\pi^4/4$

In practice $\kappa\nu Ra_{crit}/g\alpha d^4$ is a very small temperature gradient, so $F > F_{ad}$, the Schwarzschild criterion, governs whether convection occurs

In the Earth, F and F_{ad} are of similar magnitude

Wiedemann-Franz law $\kappa \rho c_p = 0.02T/\eta$

High electrical conductivity (low η) implies high thermal conductivity, making F_{ad} large

Stevenson's paradox: high electrical conductivity is bad for dynamos, because it makes F_{ad} larger than F thus stopping convection!

- What determines F in terrestrial core?

Mantle convection: poorly understood and varies from planet to planet, e.g. Venus appears to have no plate tectonics

This could reduce F and hence make Venus's core subadiabatic

Radioactivity: significant heat is generated in Earth's mantle. Did radioactive elements (particularly potassium) get into the core? Highly controversial question

- What determines F in outer planets?

Opacity of the fluid, particularly in the outer layers. Uranus has anomalously low heat flux: stably stratified layer in the interior causing a blockage?

If convection is occurring, is it producing enough energy to sustain the magnetic field against ohmic dissipation?

Energy Balance (Earth)

$$\mathcal{Q}^S = \mathcal{Q}_{\text{CMB}} - \mathcal{Q}_{\text{ICB}} - \mathcal{Q}^L - \mathcal{Q}^G \ (-\mathcal{Q}^R)$$

Possible values, depending on inner core age and uncertain physical properties: Roberts et al. 2003

Cooling $\mathcal{Q}^S \sim 2.3$ TW

Heat flowing into ICB $\mathcal{Q}_{\text{ICB}} \sim 0.3$ TW

Latent heat released at ICB $\mathcal{Q}^L \sim 4.0$ TW

Gravitational energy released at ICB $\mathcal{Q}^G \sim 0.5$ TW

Core Radioactivity $\mathcal{Q}^R \sim 0$ TW ??

Gives $\mathcal{Q}_{\text{CMB}} \sim 7.1$ TW

Entropy Balance (Earth)

$$\mathcal{Q}^D \qquad = \frac{T_D}{T_{\rm CMB}}[(\mathcal{Q}_{\rm ICB} + \mathcal{Q}^L)(1 - \frac{T_{\rm CMB}}{T_{\rm ICB}}) + (\mathcal{Q}^S + \mathcal{Q}^R)(1 - \frac{T_{\rm CMB}}{T_M}) + \mathcal{Q}^G - \Sigma T_{\rm CMB}]$$

$T_{CMB} \sim 4,000$ K, $\quad T_{ICB} \sim 5,100$ K

T_D is average temperature where dissipation occurs, T_M is mean temperature. Σ is entropy generated by thermal conduction along adiabat

This gives $\mathcal{Q}^D \sim 1.3$ TW

- Ohmic dissipation related to currents, $\mu\mathbf{j} = \nabla \times \mathbf{B}$,

$$\mathcal{Q}^D = \mu\eta \int \mathbf{j}^2 dv$$

Early Earth dynamo

Cooling rate calculations $\rightarrow$ solid inner core started growing 1-2 Gyrs ago

Paleomagnetic evidence that Earth's field in existence >3 Gyr ago. Gravitational energy only available during inner core era

Removing the inner core removes compositional convection and latent heat release. Core subadiabatic?

Ohmic dissipation gives an estimate for the typical current density in the core, 0.05 Am^{-2}. Assuming a 0.005T field (50 gauss) the dissipation length $B/|curl B|$ is around 70km for the Earth

Dynamical Regime in Planetary Interiors

$$\frac{E}{qPr}\frac{D\mathbf{u}}{Dt} + \hat{\mathbf{z}} \times \mathbf{u} = -\nabla p + \mathbf{j} \times \mathbf{B} + E\nabla^2\mathbf{u} + qRaT\mathbf{r}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla^2\mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

$$\frac{\partial T}{\partial t} = q\nabla^2 T - \mathbf{u} \cdot \nabla T$$

$$\nabla \cdot \mathbf{B} = \nabla \cdot \mathbf{U} = 0$$

Dimensionless parameters

Ekman number $E = \nu/2\Omega\ell^2 \quad (10^{-15})$

Roberts number $q = \kappa/\eta \quad (10^{-5})$

Modified Rayleigh number $Ra = g\alpha\beta\ell^2/2\Omega\kappa \quad$ (Large)

Prandtl number $Pr = \nu/\kappa \quad (10^{-1})$

η magnetic diffusivity, ν kinematic viscosity,

κ is the thermal diffusivity, ℓ is core radius

MAC balance: Earth, Jupiter, Saturn

$$\hat{\mathbf{z}} \times \mathbf{u} = -\nabla p + \mathbf{j} \times \mathbf{B} + qRaT\mathbf{r}$$

$$\frac{\partial \mathbf{B}}{\partial t} = +\nabla \times (\mathbf{u} \times \mathbf{B})$$

$$\frac{\partial T}{\partial t} = -\mathbf{u} \cdot \nabla T, \quad F_{conv} = \frac{1}{S} \int_S \rho c_p u_r T \, dS$$

In MAC balance, inertia is negligible, and all dissipation neglected

MAC waves

Linearise about a uniform magnetic field $\mathbf{B_0}$ and constant temperature gradient β, waves $\exp i(\mathbf{k}\cdot\mathbf{x}+\omega t)$

$$\omega_{MAC} = \omega_{MC}(1+\omega_A^2/\omega_M^2)^{1/2}$$

where $\omega_{MC} = \omega_M^2/\omega_C$, and $\omega_A^2 = g\alpha\beta$

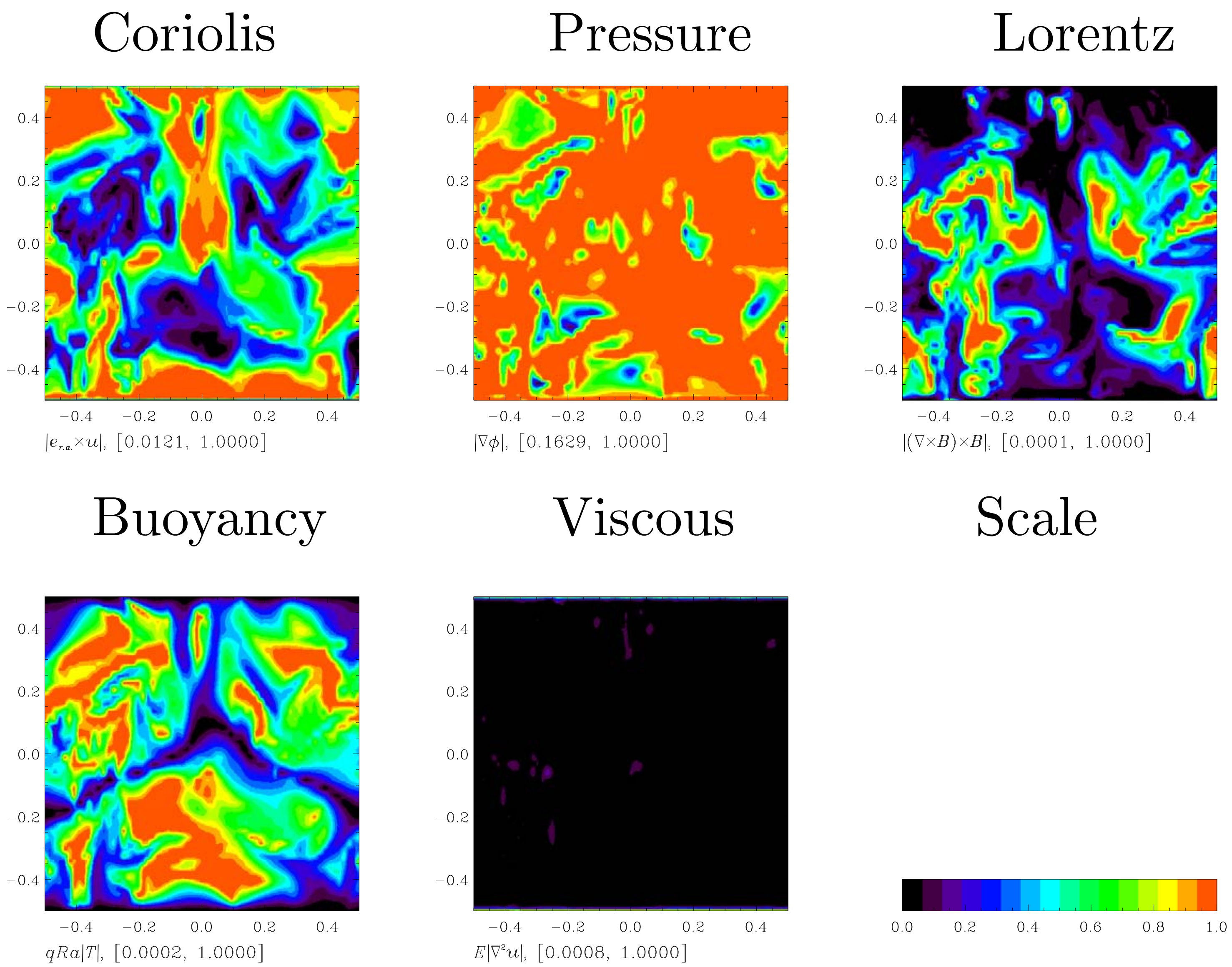
Alfven frequency being $\omega_M = (\mathbf{B_0}\cdot\mathbf{k})/(\mu_0\rho)^{1/2}$,

Inertia wave frequency $\omega_C = 2(\mathbf{\Omega}\cdot\mathbf{k})/|\mathbf{k}|$

$$\omega_{MC} = \frac{(\mathbf{B_0}\cdot\mathbf{k})^2|\mathbf{k}|}{2(\mathbf{\Omega}\cdot\mathbf{k})(\mu_0\rho)}$$

thousand year timescale

Force balance



Typical velocity and field estimates

- From the dynamical balance

$$2\Omega U_* \sim g\alpha\Delta T$$

where ΔT is a typical temperature fluctuation from the adiabatic reference value

- The heat flux equation gives

$$F \sim \rho c_p U_* \Delta T$$

Eliminating ΔT

$$U_{mac} \sim \left[\frac{g\alpha F}{\rho c_p \Omega}\right]^{\frac{1}{2}}$$

This gives

Earth: $U_{mac} \sim 2 \times 10^{-4}\ \mathrm{ms}^{-1}$

Jupiter $U_{mac} \sim 2 \times 10^{-3}\ \mathrm{ms}^{-1}$

• These estimates are in reasonable agreement with the observed values. Suggests these planets are in the MAC regime. Are all planets in this regime?

Other dynamical regimes

- There is a consistency condition, because the velocity

$$U_{mac} \sim \left[\frac{g\alpha F}{\rho c_p \Omega}\right]^{\frac{1}{2}}$$

must be large enough to give a magnetic Reynolds number big enough for magnetic field to grow

$$R_m = \frac{U_* \ell}{\eta} >\sim 100$$

In the case of Neptune this is marginal, for Uranus $R_m \sim 25$ and for Ganymede $R_m \sim 5$

Magnetic field keeps the velocity low in MAC balance. If fields are temporally or spatially intermittent, higher velocities could be achieved. Magnetic field adjusts velocity so dynamo operates near critical

Mixing length regime

• Here the dominant dynamical balance is buoyancy and inertia

$$2U_*^2/\ell \sim g\alpha\Delta T$$

where ℓ is the mixing length, typically the core size or the density scale height. As before, the heat flux equation gives

$$F \sim \rho c_p U_* \Delta T$$

Eliminating ΔT

$$U_{ml} \sim \left[\frac{g\alpha F\ell}{\rho c_p}\right]^{\frac{1}{3}} = U_{mac}^{2/3}(\Omega\ell)^{1/3}$$

(Stevenson has a coefficient 0.3 in this formula).

Since $\omega\ell$ is a high velocity, $U_{ml} > U_{mac}$. The mixing length regime is believed to operate in stars.

Since the ratio of inertia to Coriolis is the Rossby number $Ro = U_* / \ell\Omega$, the MAC regime is a low Rossby number regime, whereas mixing length regime is high Rossby number.

Tall thin column regime

In rapidly rotating nonlinear non-magnetic convection, tall thin columns are found. This regime has been discussed by Ingersoll & Pollard, Aubert et al. The balance is between Inertia, Coriolis, and Buoyancy, but the width of the columns is not ℓ but a smaller length-scale δ

This gives

$$U_*^2/\delta^2 \sim g\alpha\Delta T/\delta \sim \Omega U_*/\ell$$

with the usual $F \sim \rho c_p U_* \Delta T$, eliminating ΔT and δ

$$U_{ttc} \sim \left[(\Omega\ell)^{1/5} \frac{g\alpha F}{\rho c_p \Omega} \right]^{\frac{2}{5}} = U_{mac}^{4/5} (\Omega\ell)^{1/5}$$

This is still faster than the MAC regime, but is not as large as the mixing length regime

These tall thin columns are not the same as the linear theory Busse rolls, which have thickness $E^{1/3}$ and so depend on viscosity. In Earth conditions $\delta \sim 10km$

Possible scenario: when the MAC velocity is too low to sustain a magnetic field, the velocity rises to U_{ttc}. Then field is generated, and velocity falls to the critical value for dynamo action

Conclusions

• Behaviour of planetary dynamos depends on the physical conditions in the planet's core, such as heat flux and thermal conductivity

Involves modelling of planetary cores using high pressure physics, mantle convection

• Given the uncertainties in these subjects, we need dynamo theory to give a deeper understanding of these physical conditions

• The Earth seems to be in the MAC dynamical regime. Other planets may be in different dynamical regimes, and so may behave very differently from the geodynamo