Large Scale Fields and Small Scale Dynamos

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What is the role of a large-scale magnetic field on a small-scale dynamo? e.g. the emergence in the Sun of a deep-seated field into the supergranular convection.

Just one aspect of one of the crucial issues of astrophysical MHD – the influence of a large-scale field on small scale dynamics; cf. suppression of the _-effect, suppression of turbulent diffusion, _.

In general, it is difficult to separate the contributions to field growth due to amplification of the large-scale field from those due to small-scale dynamo action.

However, for the special case of quasi-two-dimensional flows this can be done unambiguously – for the kinematic dynamo problem.

The Initial State

Evolve the system to a quasi-2D initial state (with $\boldsymbol{u} = \boldsymbol{u}(x,y,t)$, $\boldsymbol{B} = \boldsymbol{B}(x,y,t)$) via a forcing $\boldsymbol{F}(x,y,t)$ from an initial field $\boldsymbol{B}=B_0\boldsymbol{e}_x$:

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U}.\nabla\mathbf{U} = -\nabla p + \mathbf{B}.\nabla\mathbf{B} + \nu\nabla^2\mathbf{U} + \mathbf{F},$$

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{U}.\nabla\mathbf{B} = \mathbf{B}.\nabla\mathbf{U} + \eta\nabla^2\mathbf{B},$$

$$\nabla.\mathbf{U} = \nabla.\mathbf{B} = 0.$$

The forcing function $\boldsymbol{F}$ is chosen so as to drive known fast dynamo flows in the absence of a background field B_0 – subject to stability constraints.

The Perturbation Equations

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla\mathbf{U} + \mathbf{U}.\nabla\mathbf{u} = -\nabla\pi + \mathbf{b}.\nabla\mathbf{B} + \mathbf{B}.\nabla\mathbf{b} + \nu\nabla^2\mathbf{u},$$

$$\frac{\partial \mathbf{b}}{\partial t} + \mathbf{U}.\nabla\mathbf{b} + \mathbf{u}.\nabla\mathbf{B} = \mathbf{B}.\nabla\mathbf{u} + \mathbf{b}.\nabla\mathbf{U} + \eta\nabla^2\mathbf{b},$$

$$\nabla.\mathbf{u} = \nabla.\mathbf{b} = 0.$$

Seek solutions of the form:

$$\mathbf{u}(x, y, z, t) = \tilde{\mathbf{u}}(x, y, t)\mathrm{e}^{ikz}.$$

The Forcing Function

Here we choose ***F*** to drive the Galloway & Proctor CP flow:

$$\mathbf{u} = \square\ \square \square \mathbf{e}_z + \square \mathbf{e}_z ,$$

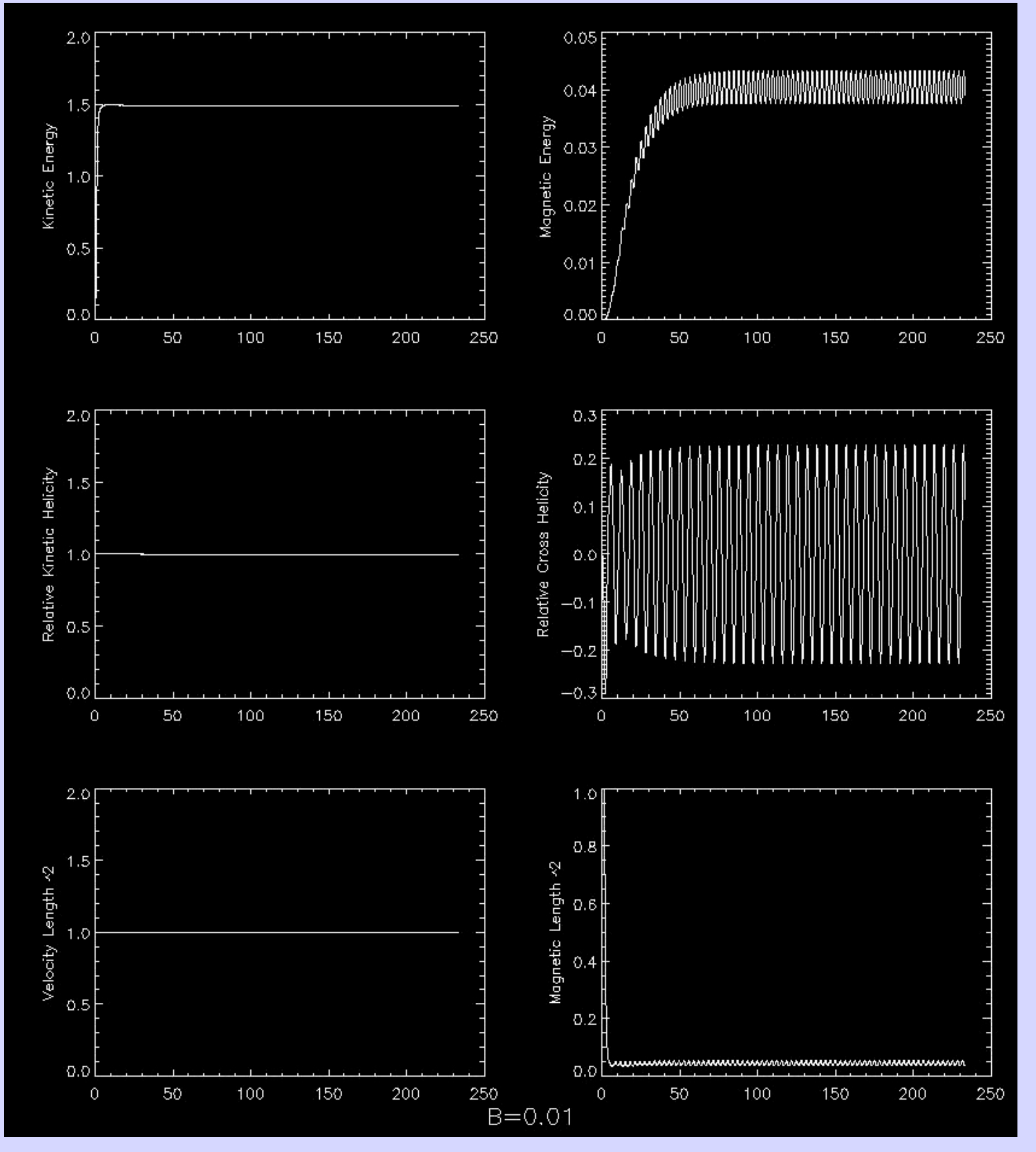
with $$\square = \sqrt{3/2}\left(\cos(x+\cos(t)) + \sin(y+\sin(t))\right).$$

i.e. take

$$\mathbf{F} = (\partial_t\ \square \square \square^2)\mathbf{U}.$$

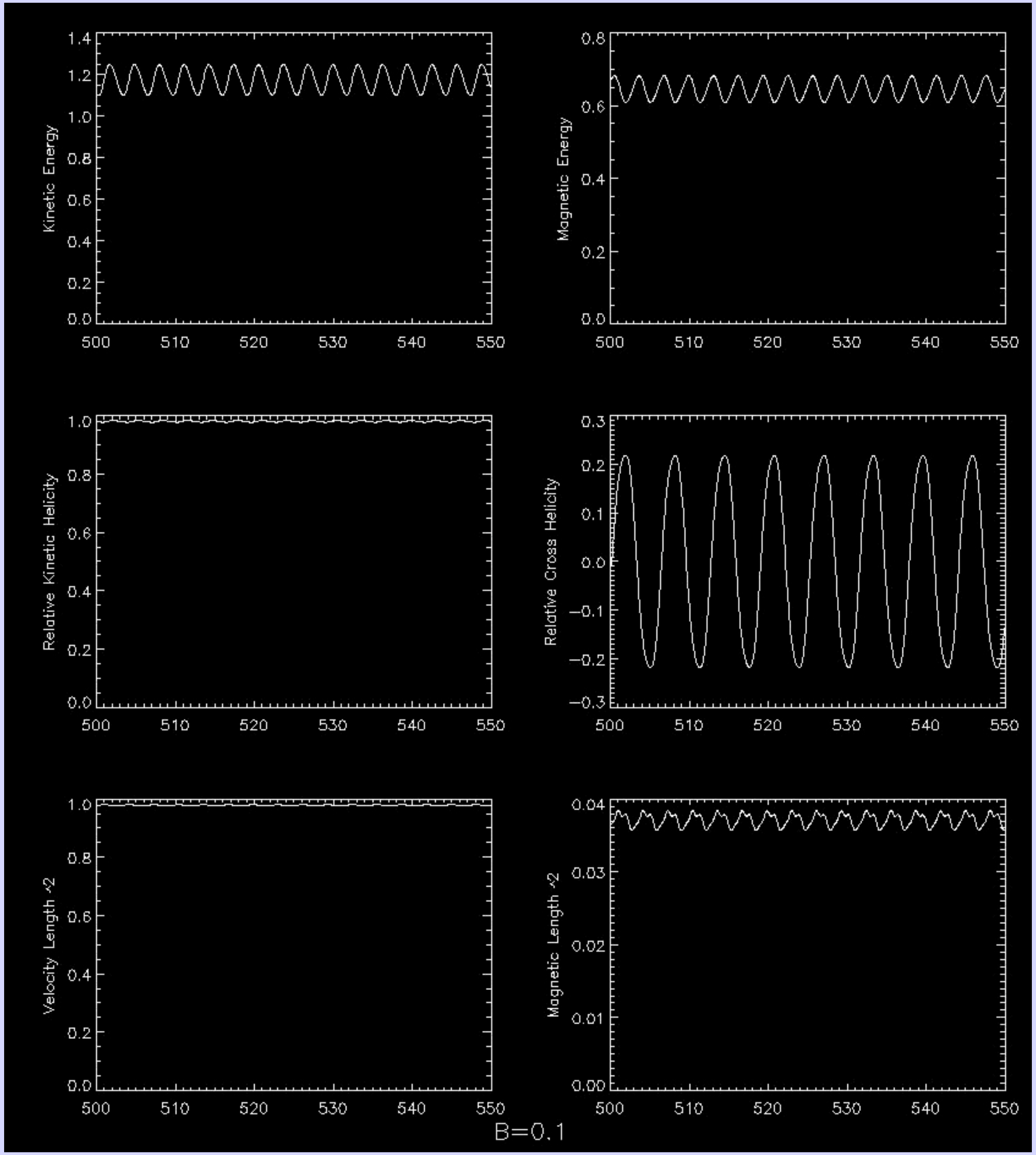
All runs here have _ = 1 (*Re* = 1) and _ = 0.01 (*Rm* =100).

$B_0 = 0.01$



Kinetic Energy
Magnetic Energy
Relative Kinetic Helicity
Relative Cross Helicity
Velocity Length ^2
Magnetic Length ^2
B=0.01

$B_0 = 0.1$



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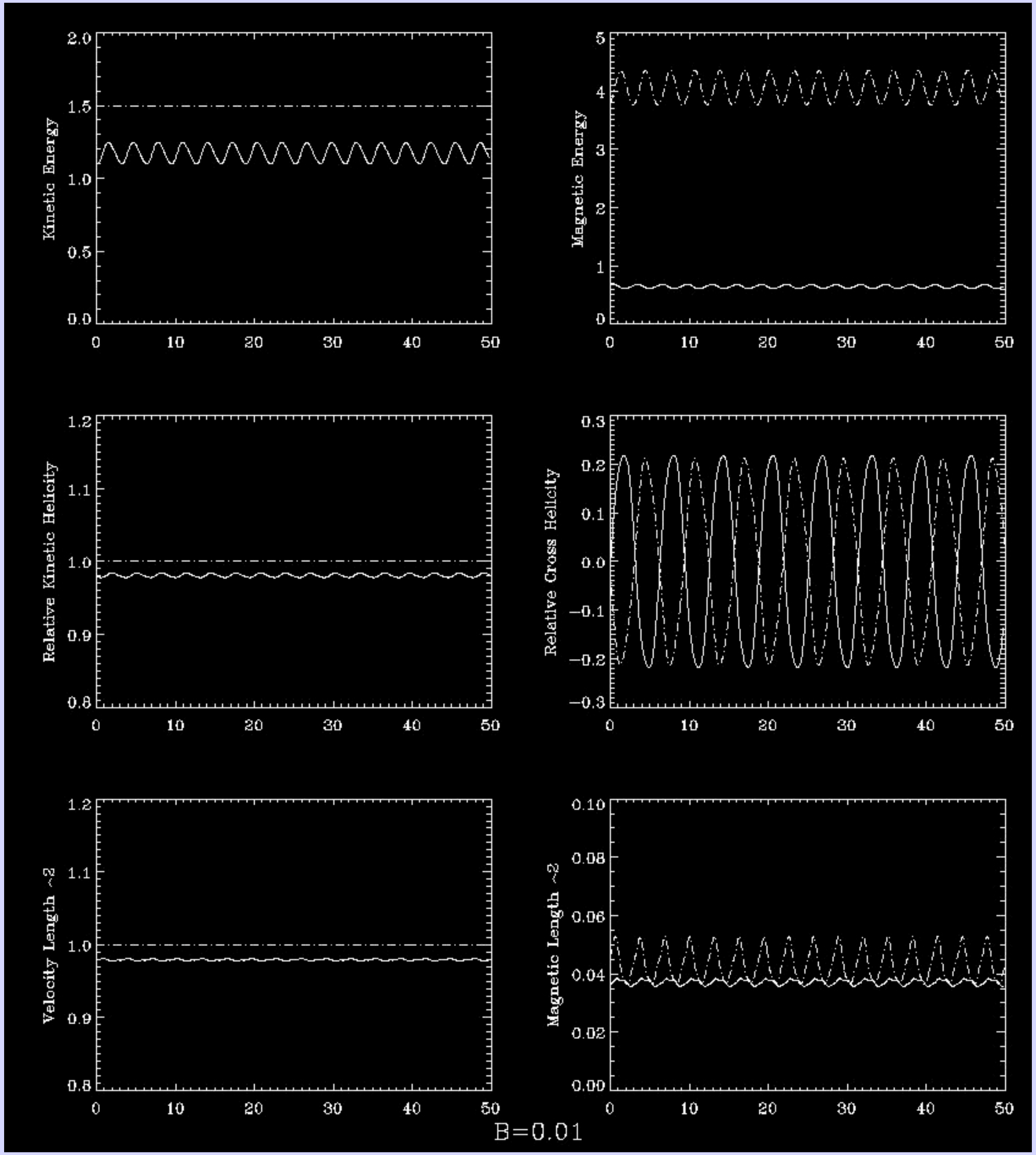
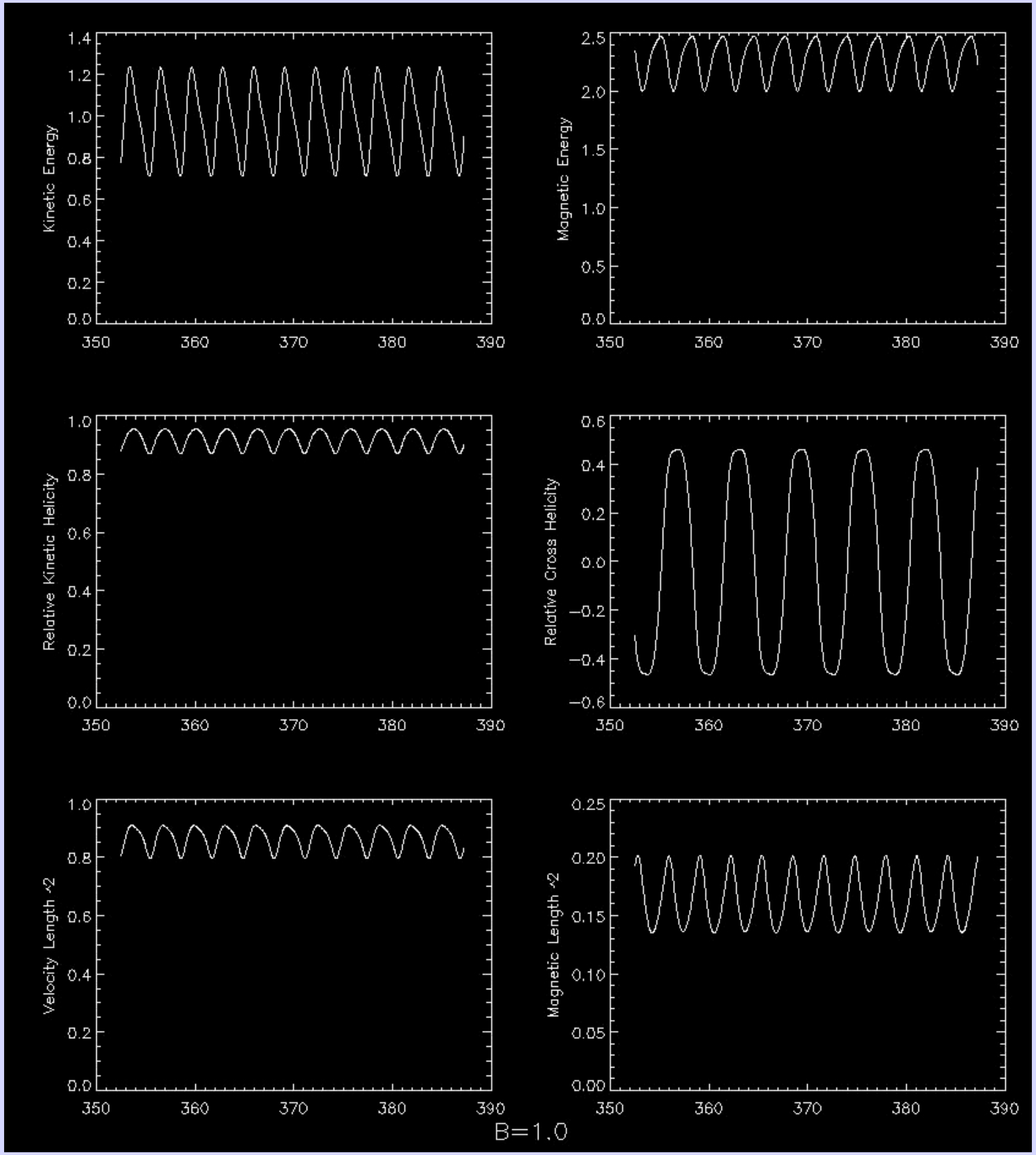
$B_0 = 0.1$

Comparison of dynamic and kinematic states



$B_0 = 1.0$



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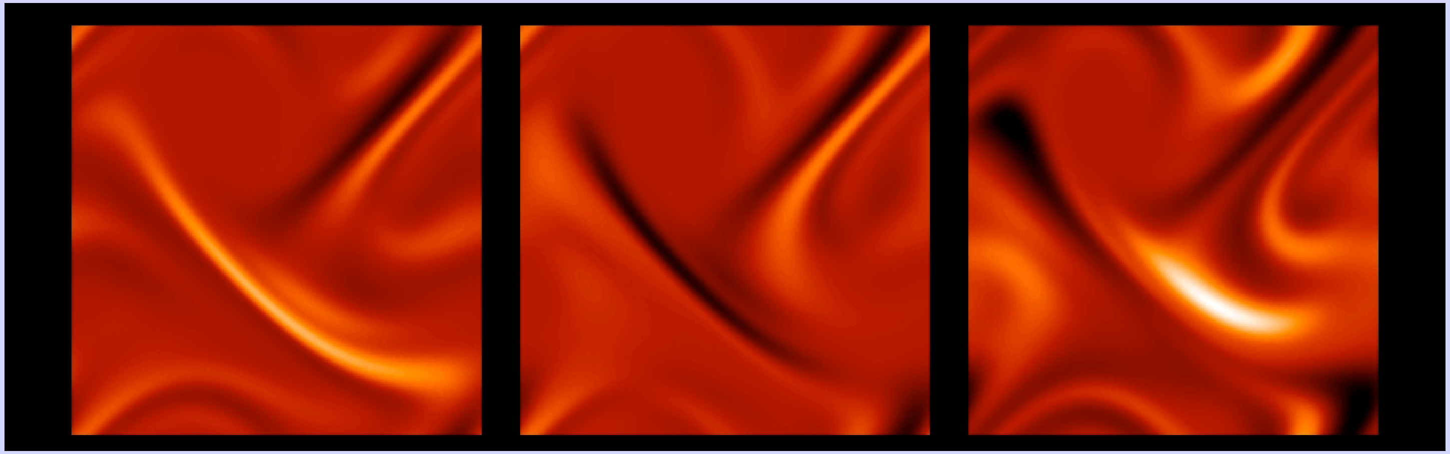
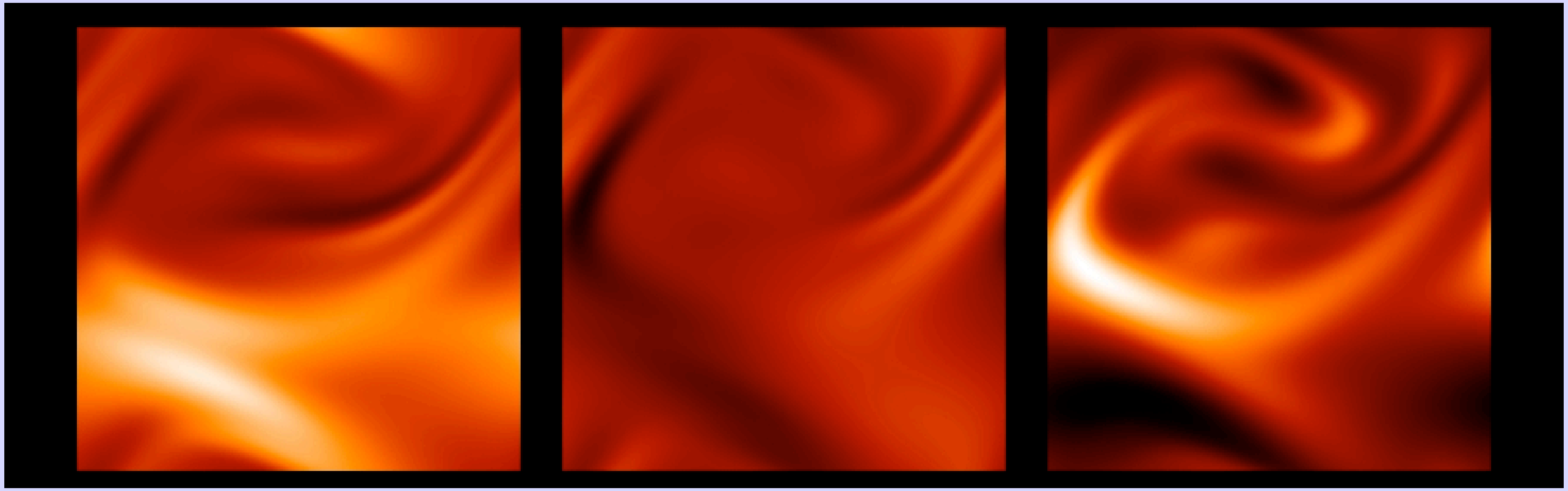
$B_0 = 0.01$

B_x B_y B_z

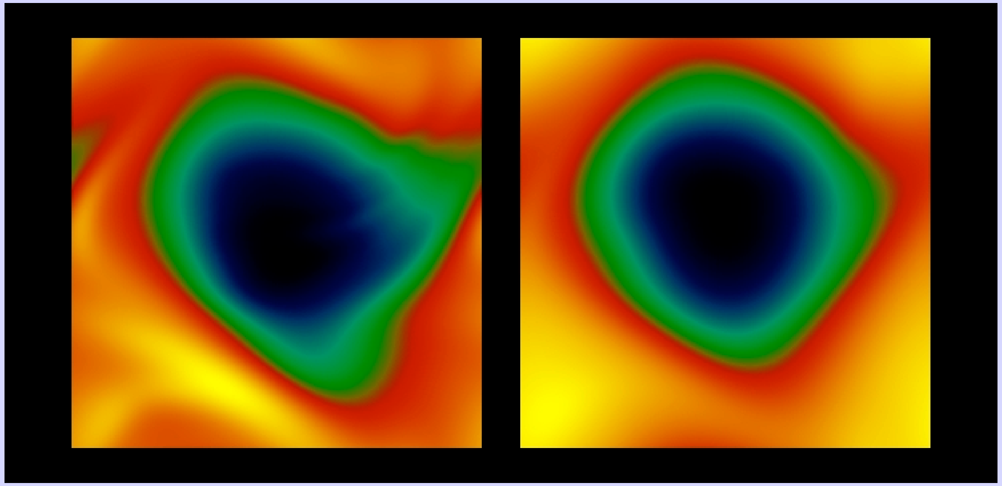
$B_0 = 0.1$



$B_0 = 1$



B_x B_y B_z



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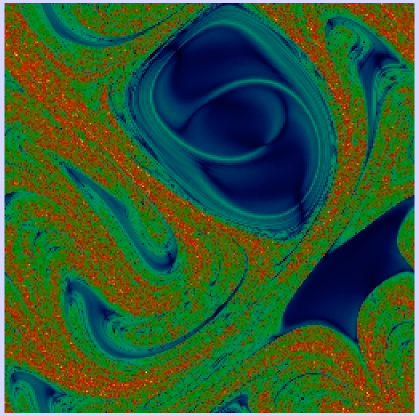
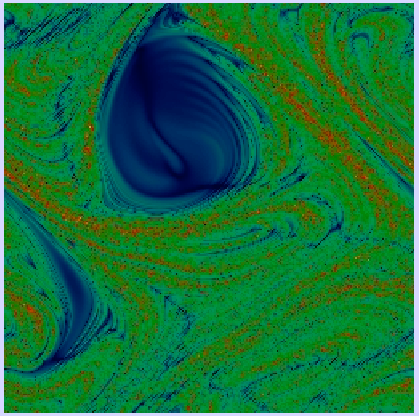
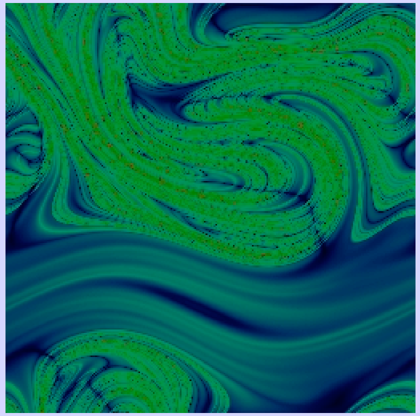
Finite-time Lyapunov Exponents



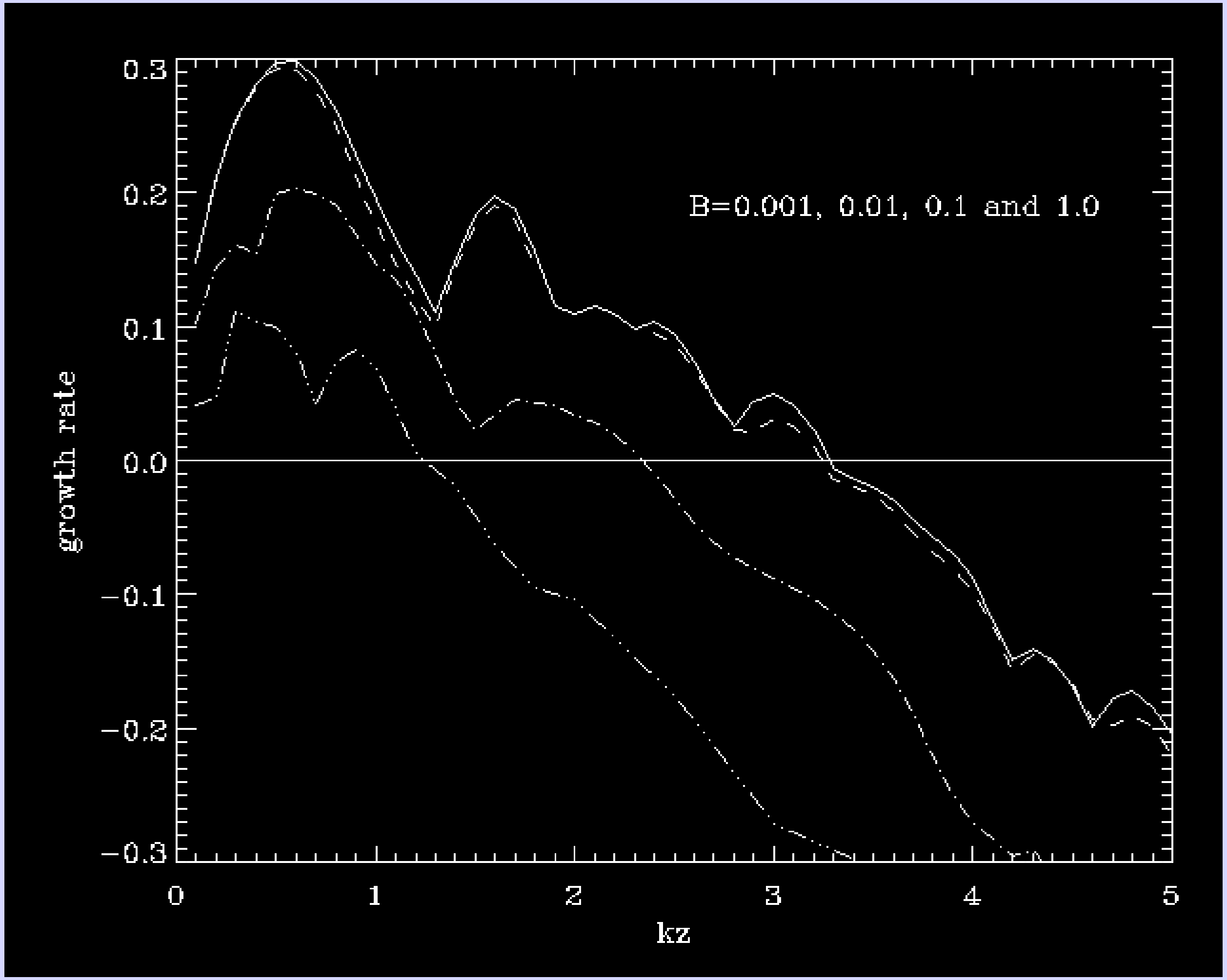
$B_0 = 0.01$



$B_0 = 0.1$



$B_0 = 1.0$


B=0.001, 0.01, 0.1 and 1.0
growth rate
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
0
1
2
3
4
5
kz

Conclusions and Future Work

1. For this flow, at least, the introduction of a large-scale field suppresses the growth rate of the dynamo.

2. Suppression of the chaos of the underlying flow.

3. Range of unstable wavenumbers reduced.

4. Noticeable effect when $B_0^2 \sim U^2/R_m$.

5. Is the dynamo still fast?