dynamo theory – past present and future

Michael Proctor

D A M T P

Astrophysical Fluids and Non-linear Dynamics

Cargèse, 23 September 2010

- The hard man beneath the aimiable exterior!

Macrodynamics
(the "Malkus–Proctor mechanism")

J. Fluid Mech. (1975), *vol.* 67, *part* 3, *pp.* 417–443 417
Printed in Great Britain

The macrodynamics of α-effect dynamos in rotating fluids

By W. V. R. MALKUS AND M. R. E. PROCTOR†

Department of Mathematics, Massachusetts Institute of Technology, Cambridge

Introduced the idea that large scale induced velocity fields could provide equilibration of mean field dynamo models

$$\partial \mathbf{B}/\partial t = \nabla \times (\mathbf{U} \times \mathbf{B}) + \nabla \times (\alpha f \mathbf{B}) + \nabla^2 \mathbf{B},$$
$$E_M(\partial \mathbf{U}/\partial t + \mathbf{U}.\nabla \mathbf{U}) + \nabla p + 2\mathbf{k} \times \mathbf{U} = \nabla \times \mathbf{B} \times \mathbf{B} + E\nabla^2 \mathbf{U},$$
$$\nabla . \mathbf{B} = \nabla . \mathbf{U} = 0,$$

Remarks:

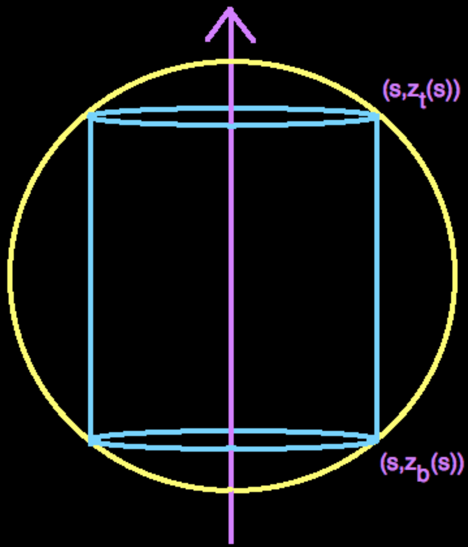
Scaling assumes Elsasser number O(1) – reasonable for Earth $E \ll 1$ and $E_M \ll 1$ so primary balance is magnetostrophic

$$2\mathbf{k} \times \mathbf{U} = -\nabla P + \mathbf{B} \cdot \nabla \mathbf{B}$$

$$\mathbf{U}(s, z_b(s)) \cdot \mathbf{n} = \mathbf{U}(s, z_t(s)) \cdot \mathbf{n} = 0$$

Setting $E, E_M = 0$ gives equations that require a solvability condition – which determines the geostrophic flow V(s) : Taylor's condition

$$T(s) = \int_{z_b(s)}^{z_t(s)} (\mathbf{B} \cdot \nabla \mathbf{B})_\phi \, dz \equiv 0$$



For $E \ll 1$ but nonzero equations have a different character leading to a condition of the form

$$E^\alpha \mathcal{L}[V](s) = T(s), \quad \alpha > 0$$

Results and follow-ups:

As E → 0 does V→ ∞ or T(s)→0? And what is dependence on E?

Appears possible to find solutions where V=O(1) if linear problem sufficiently supercritical. Further work by Soward & Jones, Ierley and Hollerbach confirms. But relations between Taylor type states and the linear problem can be complicated.

Braginsky suggested that perhaps V~ 1/√E, T~√E. Now thought not

α²-DYNAMOS AND TAYLOR'S CONSTRAINT 115

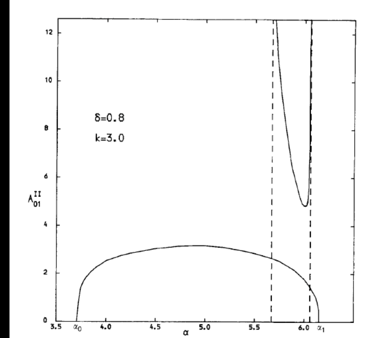
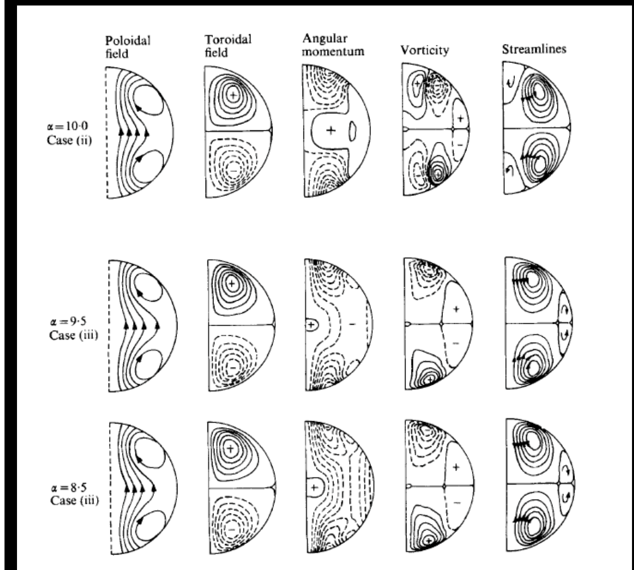


FIGURE 4 As for Figure 3, but with δ=0.8.



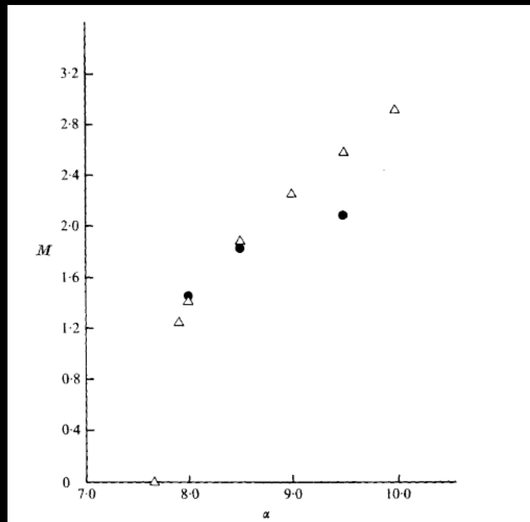


FIGURE 5. M as a function of α. Δ, case (ii) ($E_M^{\frac{1}{2}} = 0{\cdot}2$, $E = 0{\cdot}01$); ●, case (iii) ($E_M^{\frac{1}{2}} = 0{\cdot}5$, $E = 0{\cdot}005$).

Macrodynamics of α-effect dynamos 439

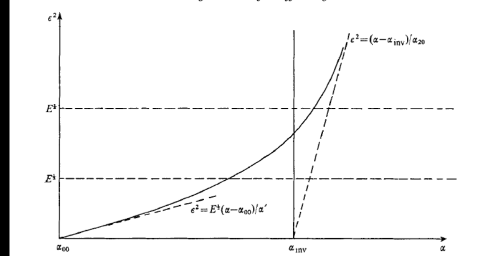


FIGURE 4. The relationship between the magnetic energy ($\propto \epsilon^2$) and $\Delta\alpha \equiv \alpha - \alpha_{00}$ for general f. Here α_{00} is taken as the 'viscous' eigenvalue, with α_{inv} as the 'inviscid' eigenvalue.

Fast dynamos and the Galloway–Proctor flow

Notion of fast dynamo introduced by Zel'dovich – can a dynamo at high Rm grow at a rate independent of diffusion? Principal mechanism is field line stretching at a rate that overcomes negative effects of cancellation of oppositely directed flux. Clearly diffusion is essential if total flux is to increase exponentially.

Many earlier `demonstrations' of fast dynamo action used maps to represent flows – untypical. Numerical calculation of dynamo action in steady ABC flows (3D, helical) suffered from poor resolution so inconclusive.

Vishik proved that exponential stretching of fluid elements necessary for fast dynamo

NATURE · VOL 356 · 23 APRIL 1992

Numerical calculations of fast dynamos in smooth velocity fields with realistic diffusion

D. J. Galloway* & M. R. E. Proctor

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, UK

Solution: combine flow with velocity dependent only on (x,y) with time dependence so as to ensure exponential stretching at least in (x,y) plane. At that time it was thought helicity was useful for fast dynamo action so looked for helical flow. Same idea had by Otani in an earlier abstract but no published paper at that time. So choose flow of form

$$\mathbf{u}(x,y,t)=\nabla\times\psi(x,y,t)\hat{z} + \alpha\psi(x,y,t)\hat{z}$$

$$\psi(x,y,t)=sin(x+sin(\omega t))+cos(y+cos(\omega t)) \text{ (G+P “CP flow”)}$$

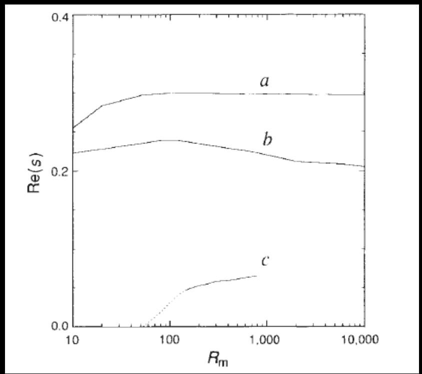
$$\psi(x,y,t)=cos(x)cos^2(t)\text{-}sin(y)sin^2(t) \text{ (Otani “MW+ flow”)}$$

Induction equation separable in z so seek solutions $\propto$exp(ikt)

CP flow like the Roberts flow, stirred (also had the LP flow, shaken not stirred...).

Results suggest that dynamo is fast – apparently confirmed by later computations. Structure of field at high Rm closely related to regions of stretching. Internal structures scale with Rm.

Wavenumber k for maximum growth rate has finite limit – in contrast to that for steady (Roberts) flow.



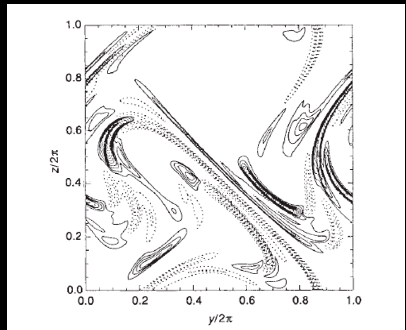
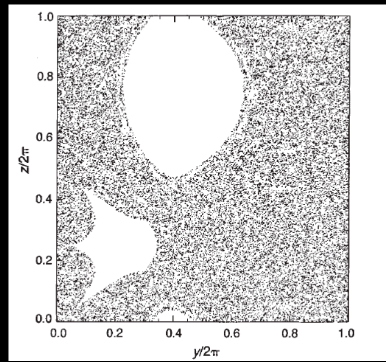
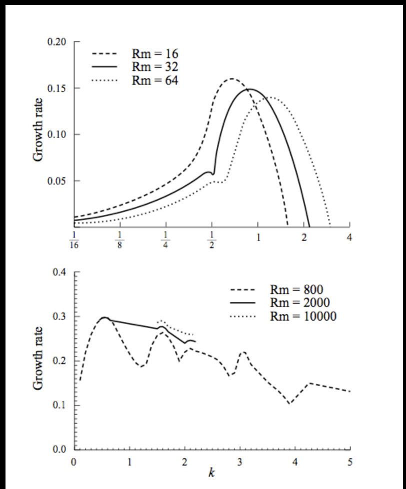
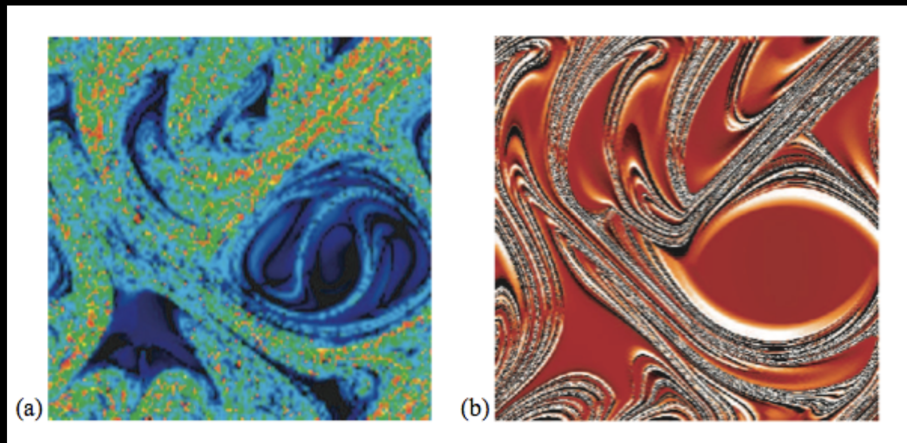


FIG. 3 Contours of B_x at $x=0$: CP flow, $k=0.57$, $R_m=2{,}000$.







How special is the CP flow?

Eulerian flow highly structured – not typical of ‘turbulent’ flows.

Can examine ‘scaling laws’ for a variety of 2D flows (Cattaneo et al) – in particular consider

$$\mathcal{R}_1 = \frac{M}{\Phi^2} \sim R_m^n .$$

where $\quad \Phi = |\langle \mathbf{B} \rangle|, \quad M = \langle |\mathbf{B}|^2 \rangle.$

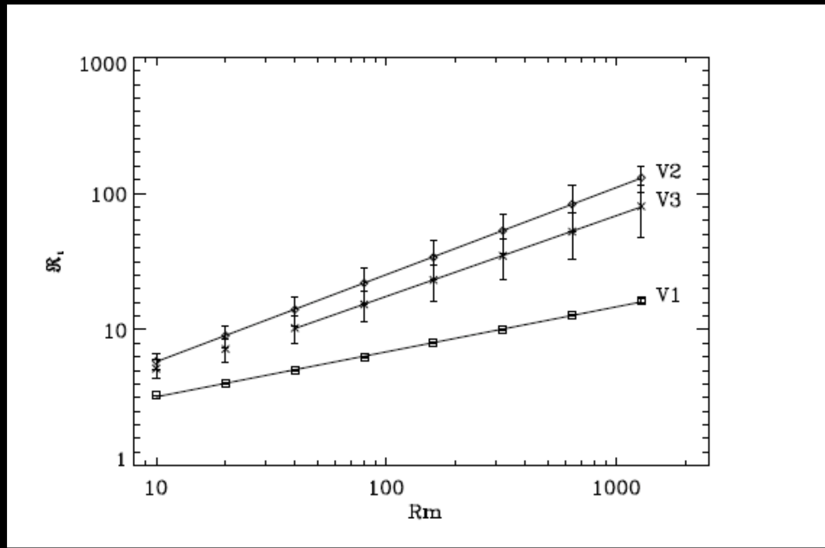
if n~1 and Rm >>1 then flux still small when energy large – so little large scale field generated. But if n<<1 then energy on all scales.

3 different flows:

VOLUME 75, NUMBER 8 PHYSICAL REVIEW LETTERS 21 AUGUST 1995

Fluctuations in Quasi-Two-Dimensional Fast Dynamos

Fausto Cattaneo,[1] Eun-jin Kim,[2] Michael Proctor,[3] and Louis Tao[2]
[1]*Astronomy and Astrophysics Center, University of Chicago, 5640 South Ellis Avenue, Chicago Illinois 60637*
[2]*The Enrico Fermi Institute, University of Chicago, 5640 South Ellis Avenue, Chicago Illinois 60637*
[3]*Department of Applied Mathematics and Theoretical Physics, University of Cambridge Cambridge, CB3 9EW, United Kingdom*
(Received 4 January 1995)



$$\psi = C\left(\sqrt{\tfrac{3}{2}}\left(\cos(y + \alpha\cos\omega t) + \sin(x + \sin\omega t)\right) - \beta\cos\sqrt{3}t\sin 2x\sin 2y\right), \quad C = (1+\beta^2/3)^{-\frac{1}{2}}$$

$$\begin{aligned} \text{V1} &: \quad \alpha = 1, \quad \beta = 0, \quad \omega = 1 \\ \text{V2} &: \quad \alpha = 2, \quad \beta = 1, \quad \omega = \sqrt{3} \\ \text{V3} &: \quad \alpha = 1, \quad \beta = 1, \quad \omega = 1 \end{aligned}$$

Tilted fields in magnetoconvection

As pointed out by Hurlburt, non-Boussinesq convection in a layer with a tilted field has no steady solutions.

Various types of oblique boundary conditions:

1. specified tilt: $B_x/B_z = \tan\phi$

2. vertical field at base, potential field above tending to uniform tilted field at infinity

3. Line-tied condition at both boundaries.

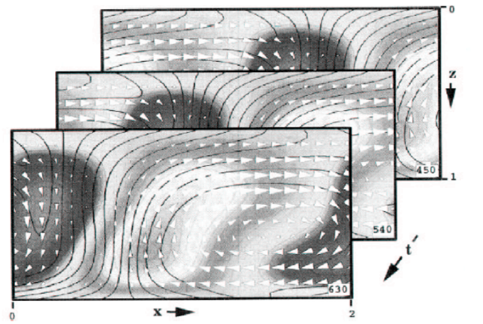
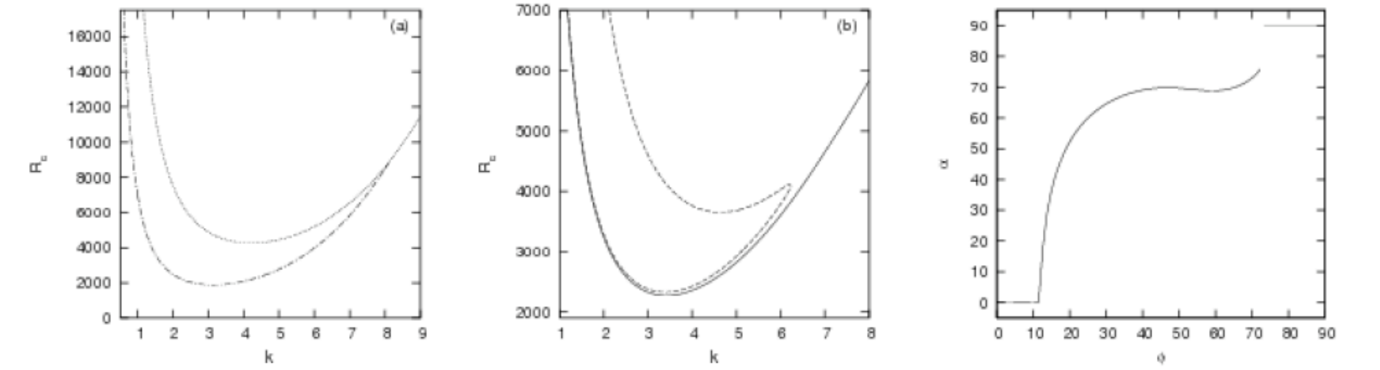


FIGURE 14. Travelling waves in oblique fields (from Hurlburt *et al.* (1996)). the field points up and to the right, but the phase velocity of the wave is to the left for this moderate field strength.

How do conditions determine



For simplicity consider near Boussinesq model with non-uniform thermal conductivity and potential field condition, in case where vertical field leads to oscillation(Thompson 2008)

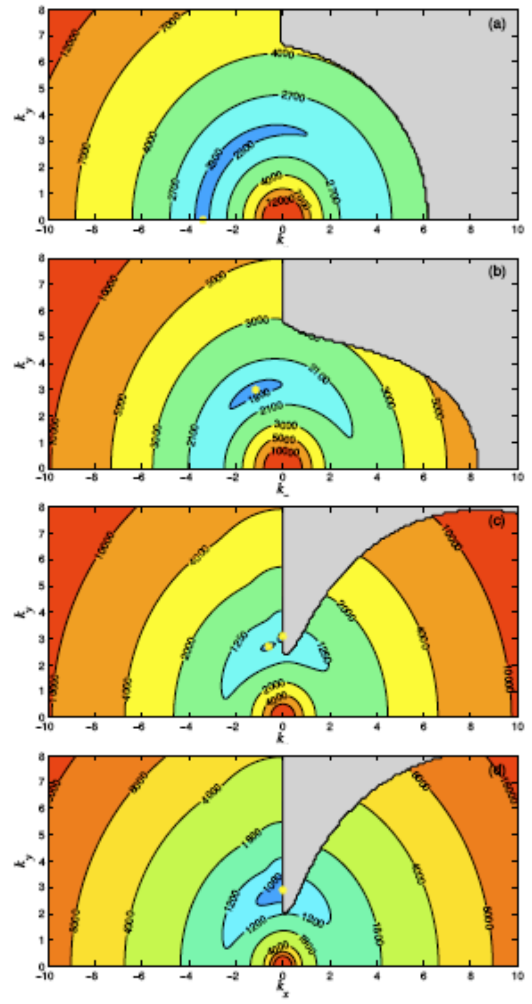


Figure 2.10: *Contour plots showing critical Rayleigh number as a function of wavevector, with positive (negative) k_x representing right-going (left-going) waves. The shaded areas represent regions where no right-going solution exists. The preferred mode (corresponding to the minimum R_c) is indicated by a small asterisk in each plot. The four cases are (a) $\phi = 10°$, (b) $\phi = 40°$, (c) $\phi = 72.5°$, (d) $\phi = 75°$.*

Eigenvalue plot for various cases shows that there can be asymmetrical situations where the spectrum is still symmetrical! How is this

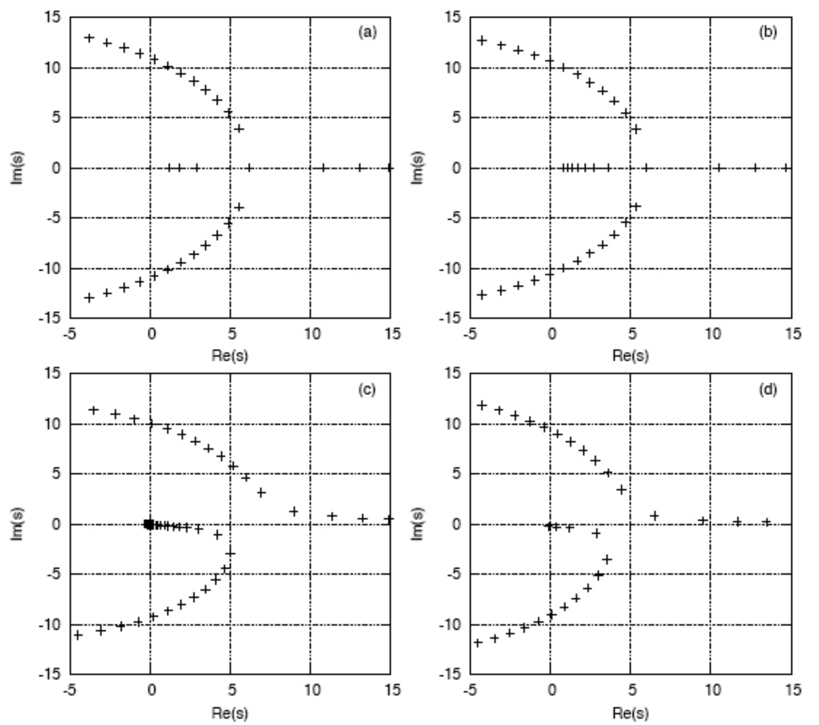


Figure 2.6: *Plots of eigenvalues s in the complex plane as R is varied. The crosses are plotted at intervals of 200 in R. In each case $\mathbf{k}$ is fixed and equal to $(k_c, 0)$, where k_c is the critical wavenumber (i.e. the one that minimizes R_c). (a) Vertical field ($\phi = 0°$), with $\hat{\kappa} = 1$. (b) $\phi = 15°$ and $\hat{\kappa} = 1$. (c) As (b), but with a fixed angle condition at the bottom of the layer (the other three cases use a tied field condition at the bottom.) (d) $\phi = 15°$ and $\hat{\kappa} = (z + 1/2)^{-3}$. Note: in case (c) the locus of eigenvalues passes through the origin, whereas in case (d) it merely passes very close to the origin – it in fact crosses the imaginary axis slightly below the origin.*

Answer lies in an interesting mutual adjointness property of the equations.

Geophys. Astrophys. Fluid Dyn. 2010 (in press)

Effects of boundary conditions on the onset of convection with tilted magnetic fields and rotation vectors

M. R. E. PROCTOR*, N. O. WEISS, S. D. THOMPSON and N. T. ROXBURGH
Department of Applied Mathematics and Theoretical Physics,
Centre for Mathematical Sciences, University of Cambridge, Cambridge CB3 0WA , UK

Consider 2D solutions with e.g. $\mathbf{B}= \nabla\times(A\mathbf{e}_y)$,

$$A=\mathrm{Re}(\hat{A}(z)e^{ikx})$$

For tilted field condition: $\hat{A}'=-ik\tan\phi\,\hat{A}$

Line tied: $\hat{A}=0$

Potential: $\hat{A}'=-|k|\hat{A}$

The system can be written in the matrix form

$$s\mathsf{C}\boldsymbol{X} = \mathsf{F}\boldsymbol{X}$$

where $\boldsymbol{X} = \begin{bmatrix} A \\ \psi \\ \theta \end{bmatrix}$. We also define the related vector $\boldsymbol{Y} = \begin{bmatrix} -(D^2 - k^2)A \\ \psi \\ \theta \end{bmatrix}$.

The differential operators C, F take the form

$$\mathsf{C} \quad = \quad \begin{bmatrix} Q\zeta & 0 & 0 \\ 0 & -\frac{1}{\sigma}(D^2 - k^2) & 0 \\ 0 & 0 & R \end{bmatrix} \quad \text{and}$$

$$\mathsf{F} \quad = \quad \begin{bmatrix} Q\zeta^2(D^2 - k^2) & Q\zeta(\cos\phi D + ik\sin\phi) & 0 \\ -Q\zeta(D^2 - k^2)(\cos\phi D + ik\sin\phi) & -(D^2 - k^2)^2 & -iRk \\ 0 & iRk & R(D^2 - k^2) \end{bmatrix}.$$

Introduce the (generalised) inner product

$$\langle \boldsymbol{X}_1, \boldsymbol{X}_2 \rangle \equiv \int_0^1 \mathbf{Y}_1^{*\mathsf{T}} \boldsymbol{X}_2 \, dz = \int_0^1 \boldsymbol{X}_1^{*\mathsf{T}} \mathbf{Y}_2 \, dz = \langle \boldsymbol{X}_2, \boldsymbol{X}_1 \rangle^*,$$

as long as A_1, A_2^* obey the same boundary conditions.

Define $\tilde{\boldsymbol{X}}^{\mathsf{T}} = [\tilde{A}, \tilde{\psi}, \tilde{\theta}]$, where $\tilde{A}, \tilde{\psi}, \tilde{\theta}$ obey the same boundary conditions as A, ψ, θ respectively. Then

$$\langle \tilde{\boldsymbol{X}}, (s\mathbf{C}\boldsymbol{X} - \mathbf{F}\boldsymbol{X}) \rangle = \int_0^1 \tilde{\boldsymbol{Y}}^{*\mathsf{T}}(s\mathbf{C}\boldsymbol{X} - \mathbf{F}\boldsymbol{X})dz = \int_0^1 \boldsymbol{Y}^{\mathsf{T}}(s^*\mathbf{C}\tilde{\boldsymbol{X}} - \mathbf{F}^{\dagger}\tilde{\boldsymbol{X}})^* = \langle (s^*\mathbf{C}\tilde{\boldsymbol{X}} - \mathbf{F}^{\dagger}\tilde{\boldsymbol{X}}), \boldsymbol{X} \rangle,$$

$$\mathbf{F}^{\dagger} = \begin{bmatrix} Q\zeta^2(D^2 - k^2) & -Q\zeta(\cos\phi D + ik\sin\phi) & 0 \\ Q\zeta(D^2 - k^2)(\cos\phi D + ik\sin\phi) & -(D^2 - k^2)^2 & -iRk \\ 0 & iRk & R(D^2 - k^2) \end{bmatrix}.$$

Thus $\mathbf{F}^{\dagger}$ is the formal adjoint of $\mathbf{F}$, and therefore its spectrum is the complex conjugate of that of $\mathbf{F}$. Now replace $\tilde{\boldsymbol{X}}$ by $\boldsymbol{Z} = \begin{bmatrix} -\tilde{A} \\ \tilde{\psi} \\ \tilde{\theta} \end{bmatrix}$, then $s^*\mathbf{C}\tilde{\boldsymbol{X}} = \mathbf{F}^{\dagger}\tilde{\boldsymbol{X}}$ becomes

$$s^*\mathbf{C}\boldsymbol{Z} = \mathbf{F}\boldsymbol{Z}.$$

So get identical problem, with s being replaced by s^*!

Does not work for tilted field b.c.'s but does work for others, even with asymmetrical mechanical and thermal b.c.'s that are self adjoint.

Mean field theory for MHD states

Is traditional mean field theory relevant to long wavelength instabilities of fully evolved small-scale dynamos?

New work with David Hughes and Alice Courvoisier suggests that in many cases traditional ideas should be modified.

PROCEEDINGS OF THE ROYAL SOCIETY A

Proc. R. Soc. A (2010) **466**, 583–601
doi:10.1098/rspa.2009.0384
Published online 29 October 2009

Self-consistent mean-field magnetohydrodynamics

By A. Courvoisier[1,*], D. W. Hughes[1] and M. R. E. Proctor[2]

[1]*Department of Applied Mathematics, University of Leeds, Leeds LS2 9JT, UK*
[2]*DAMTP, Centre for Mathematical Sciences, University of Cambridge, Cambridge CB3 0WA, UK*

General linear stability of an MHD state

Consider basic state ***U(x,t)***, ***B(x,t)***. Equations for small perturbations ***u(x,t)***, ***b(x,t)***:

$$\begin{aligned} \frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{U}\cdot\nabla\boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{U} &= -\nabla p + \boldsymbol{B}\cdot\nabla\boldsymbol{b} + \boldsymbol{b}\cdot\nabla\boldsymbol{B} + Pm\nabla^2\boldsymbol{u}, \\ \frac{\partial \boldsymbol{b}}{\partial t} + \boldsymbol{U}\cdot\nabla\boldsymbol{b} + \boldsymbol{u}\cdot\nabla\boldsymbol{B} &= \boldsymbol{B}\cdot\nabla\boldsymbol{u} + \boldsymbol{b}\cdot\nabla\boldsymbol{U} + \nabla^2\boldsymbol{b}, \end{aligned}$$

plainly the growth of disturbances is not decided by the induction equation alone!

$$\boldsymbol{u} = \overline{\boldsymbol{u}}(\boldsymbol{X},T) + \boldsymbol{u}'(\boldsymbol{x},\boldsymbol{X},t,T)$$

Consider now situation when perturbation fields have mean and fluctuating parts, e.g.

Then we can formally write down equations for the mean field and flow:

$$\frac{\partial \overline{\boldsymbol{u}}}{\partial T} + \frac{\partial}{\partial X_j}(\overline{U_j \boldsymbol{u}' + u_j' \boldsymbol{U}}) = -\nabla_X \overline{p} + \frac{\partial}{\partial X_j}(\overline{B_j \boldsymbol{b}' + b_j' \boldsymbol{B}}) + Pm \nabla_X^2 \overline{\boldsymbol{u}},$$

$$\frac{\partial \overline{\boldsymbol{b}}}{\partial T} = \nabla_X \times (\overline{\boldsymbol{U} \times \boldsymbol{b}' + \boldsymbol{u}' \times \boldsymbol{B}}) + \nabla_X^2 \overline{\boldsymbol{b}},$$

Since fluctuation equations are linear, ignoring diffusion these can be written in the form

$$\frac{\partial \overline{u}_i}{\partial T} + \frac{\partial}{\partial X_j}(\Gamma_{ijl}^{(1)} \overline{u}_l + \Gamma_{ijl}^{(2)} \overline{b}_l) = -\frac{\partial \overline{p}}{\partial X_i},$$

$$\frac{\partial \overline{b}_i}{\partial T} = \epsilon_{ijk} \frac{\partial}{\partial X_j}(\alpha_{kl}^{(1)} \overline{b}_l + \alpha_{kl}^{(2)} \overline{u}_l).$$

$\boldsymbol{\alpha}^{(1)}$ is the usual α-effect. $\boldsymbol{\Gamma}^{(1)}$ is the AKA ("anisotropic kinematic α effect")
Use 'short-sudden' approximation to get explicit formulae

$$\boldsymbol{u}' = \tau(\overline{\boldsymbol{b}} \cdot \nabla \boldsymbol{B} - \overline{\boldsymbol{u}} \cdot \nabla \boldsymbol{U}), \quad \boldsymbol{b}' = \tau(\overline{\boldsymbol{b}} \cdot \nabla \boldsymbol{U} - \overline{\boldsymbol{u}} \cdot \nabla \boldsymbol{B}).$$

$$\Gamma_{ijl}^{(2)} = 2\tau(\gamma_{ijl} + \gamma_{jil}), \quad \alpha_{il}^{(2)} = -2\tau \epsilon_{imn} \gamma_{mnl}, \quad \text{where}$$

$$\gamma_{ijl} = \overline{U_i \frac{\partial B_j}{\partial X_l}}.$$

These equations can be validated for ong-wavelength instabilities of simple 2D state. The basic state is an MHD state as we impose a uniform magnetic field **B**0 . Flow driven by 2D forcing.

Choose forcing that drives simple flows when $B_0 = 0$

$$\boldsymbol{U}_0 = \nabla \times (\psi \hat{\boldsymbol{z}}) + w\hat{\boldsymbol{z}} \equiv \boldsymbol{U}_H + w\hat{\boldsymbol{z}}.$$

(a) CP flow of Galloway & Proctor

$$\psi = w = \sqrt{3/2}\,(\cos(x + \cos t) + \sin(y + \sin t))\,.$$

(b) AKA flow

$$\boldsymbol{U}_0 = \frac{1}{\sqrt{2}}\,(\cos{(y + Re^{-1}t)} + \sin{(y + Re^{-1}t)},$$
$$\cos{(x - Re^{-1}t)} - \sin{(x - Re^{-1}t)},$$
$$2(\cos{(y + Re^{-1}t)} + \cos{(x - Re^{-1}t)})\,.$$

(c)MW+ flow of Otani

$$\psi = -w = (\cos x \cos^2 t - \cos y \sin^2 t)$$

where in each case, (since $\nabla \times (\boldsymbol{U} \cdot \nabla \boldsymbol{U}) = 0$)

$$\boldsymbol{F} \equiv \boldsymbol{F}_H + F\hat{\boldsymbol{z}} = (\partial_t - Re^{-1}\nabla^2)\boldsymbol{U}_0.$$

Start with perturbation equations:

$$\begin{aligned} \dot{\boldsymbol{u}} - Re^{-1}\nabla^2\boldsymbol{u} &= -\nabla\pi + \nabla\cdot(\boldsymbol{Bb} + \boldsymbol{bB} - \boldsymbol{Uu} - \boldsymbol{uU}) + \boldsymbol{B}_0\cdot\nabla\boldsymbol{b}, \\ \dot{\boldsymbol{b}} - Rm^{-1}\nabla^2\boldsymbol{b} &= \nabla\times(\boldsymbol{B}\times\boldsymbol{u} + \boldsymbol{b}\times\boldsymbol{U}) + \boldsymbol{B}_0\cdot\nabla\boldsymbol{u}, \\ \nabla\cdot\boldsymbol{u} &= \nabla\cdot\boldsymbol{b} = 0. \end{aligned}$$

Separate variables in z, t so that

$$(\boldsymbol{b}(\boldsymbol{x},t), \boldsymbol{u}(\boldsymbol{x},t), \pi(\boldsymbol{x},t)) = (\boldsymbol{H}(x,y,t), \boldsymbol{V}(x,y,t), \Pi(x,y,t))\; e^{ikz+p(k)t},$$

Write variables as sum of mean and fluctuating parts:

$$\big(\overline{\boldsymbol{b}}(z,t), \overline{\boldsymbol{u}}(z,t), \overline{p}(z,t)\big) = (\langle\boldsymbol{H}\rangle, \langle\boldsymbol{V}\rangle, \langle\Pi\rangle)\; e^{ikz+p(k)t}, \quad \boldsymbol{H} = \langle\boldsymbol{H}\rangle + \boldsymbol{h}, \text{ etc.}$$

Expand in powers of k: $\boldsymbol{H} = \sum_n \boldsymbol{H}_n k^n$, $p(k) = \sum_{n>0} p_n k^n$.

Obtain sequence of problems for mean and fluctuating parts

Can calculate growth rates correct to leading order in wavenumber k in terms of basic state.

Leading order average takes the form

$$p_1\langle \boldsymbol{H}_0\rangle = i\hat{\boldsymbol{z}} \times \langle \boldsymbol{v}_0 \times \boldsymbol{B} + \boldsymbol{U} \times \boldsymbol{h}_0\rangle,$$
$$p_1\langle \boldsymbol{V}_0\rangle = -i\hat{\boldsymbol{z}}\Pi_0 + i\hat{\boldsymbol{z}} \cdot \langle \boldsymbol{h}_0\boldsymbol{B} + \boldsymbol{B}\boldsymbol{h}_0 - \boldsymbol{v}_0\boldsymbol{U} - \boldsymbol{U}\boldsymbol{v}_0\rangle,$$
$$\hat{\boldsymbol{z}} \cdot \langle \boldsymbol{H}_0\rangle = 0, \quad \hat{\boldsymbol{z}} \cdot \langle \boldsymbol{V}_0\rangle = 0, \quad \text{or}$$

$$p_1\langle \boldsymbol{H}_0\rangle = i\hat{\boldsymbol{z}} \times \left(\boldsymbol{\alpha}^B \cdot \langle \boldsymbol{H}_0\rangle + \boldsymbol{\alpha}^U \cdot \langle \boldsymbol{V}_0\rangle\right),$$
$$p_1\langle \boldsymbol{V}_0\rangle = -i\hat{\boldsymbol{z}}\Pi_0 + i\hat{\boldsymbol{z}} \cdot \left(\boldsymbol{\Gamma}^B \cdot \langle \boldsymbol{H}_0\rangle + \boldsymbol{\Gamma}^U \cdot \langle \boldsymbol{V}_0\rangle\right).$$

In component form we have

$$p_1 \begin{bmatrix} H_{0x} \\ H_{0y} \\ V_{0x} \\ V_{0y} \end{bmatrix} = i \begin{bmatrix} -\alpha^B_{21} & -\alpha^B_{22} & -\alpha^U_{21} & -\alpha^U_{22} \\ \alpha^B_{11} & \alpha^B_{12} & \alpha^U_{11} & \alpha^U_{12} \\ \Gamma^B_{131} & \Gamma^B_{132} & \Gamma^U_{131} & \Gamma^U_{132} \\ \Gamma^B_{231} & \Gamma^B_{232} & \Gamma^U_{231} & \Gamma^U_{232} \end{bmatrix} \begin{bmatrix} H_{0x} \\ H_{0y} \\ V_{0x} \\ V_{0y} \end{bmatrix} = \mathcal{M} \begin{bmatrix} H_{0x} \\ H_{0y} \\ V_{0x} \\ V_{0y} \end{bmatrix}.$$

The first order growth rate is then an eigenvalue of the matrix $\mathcal{M}\,k$.

Example: MW+ flow

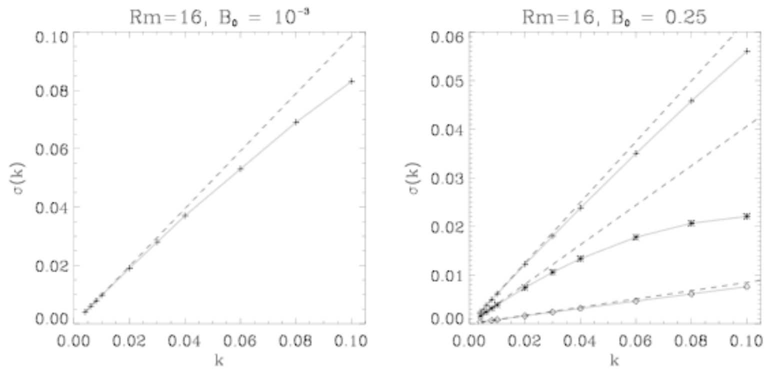


Figure 3. Left: $\sigma(k)$ for $Rm = 16$, $\mathcal{B} = 0.001$. The dashed line corresponds to $\lambda\, k$, where λ is the positive eigenvalue of $\boldsymbol{\mathcal{A}}$. Right: $\sigma(k)$ for the magnetic problem (+) and the full problem ($\star$, velocity mode, and $\diamond$, magnetic mode) for $Rm = 16$ and $\mathcal{B} = 0.25$. The dashed lines corresponds to $\lambda\, k$, where λ stands for the positive eigenvalue(s) of $\boldsymbol{\mathcal{A}}(\mathcal{B})$ (for the magnetic problem) and of $\boldsymbol{\mathcal{M}}$ (for the full problem).

Example: AKA

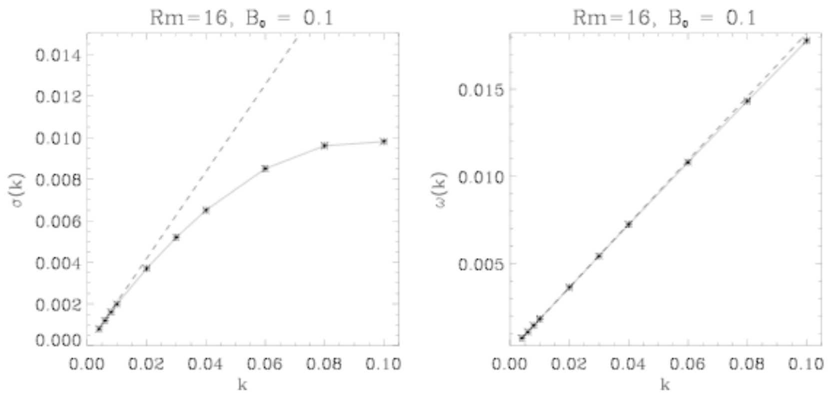
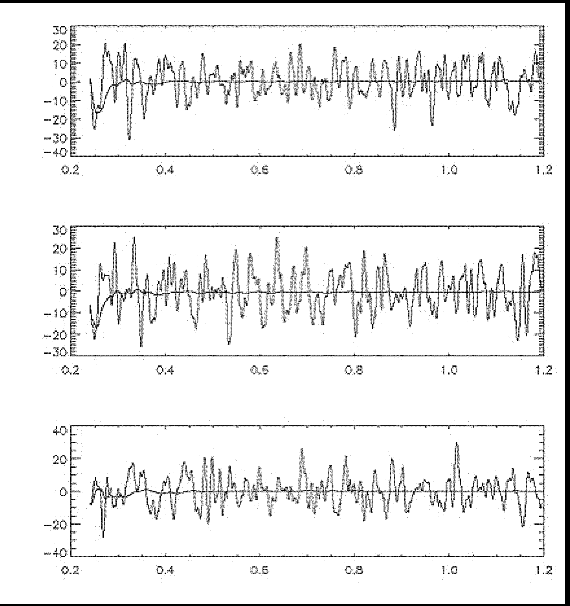


Figure 2. $\sigma(k)$ (left) and $\omega(k)$ (right) for the AKA forcing; $Rm = 16$, $\mathcal{B} = 0.1$. The dashed lines corresponds to $Re[\lambda]\, k$ (left) and $Im[\lambda]\, k$ (right), where λ is the eigenvalue of $\boldsymbol{\mathcal{M}}$ with the largest real part.

What about aperiodic MHD states?

Direct calculation now impossible. But can in principle measure the mean coefficients by evaluating EMF and Reynolds stress by imposing uniform magnetic fields and flows on an MHD state with no mean.

Problem: getting adequate signal/noise



With grateful thanks to all my collaborators!

Malkus	Straughan	Postlethwaite	Brummell
Richardson	Lega	Rincon	Roxburgh
Schekochihin	Rucklidge	Julien	Cattaneo
Moffatt	Galloway	Jones	Rempel
Mann	Jouve	Houghton	Knobloch
Meunier	Hollerbach	Matthews	Soward
Weiss	Lesur	Bushby	Brownjohn
Jenkins	Sellar	Hurlburt	Melbourne
Pipin	Dawes	Gubbins	Chapman
Tobias	Ogilvie	Fearn	Holyer
Belvedere	Childress	Vasil	Metzener
Hughes	Lythe	Thompson	Wissink