Compressible Anelastic Dynamos

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Mike Proctor's 60th Birthday Celebration

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Motivation

Investigating compressible, rapidly rotating, dynamos in spherical shell geometry, driven by convection, using the anelastic approximation.

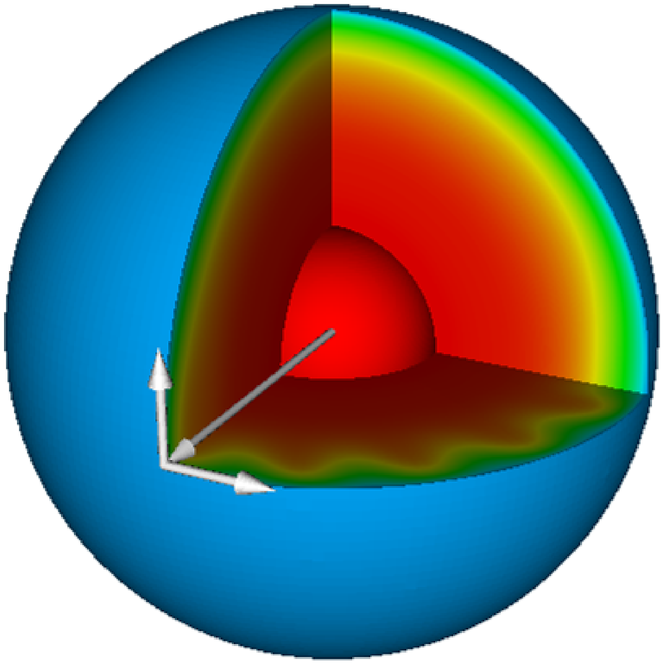
First aim is to establish simple solutions so the various codes (Leeds code, ASH code, Glatzmaier's original anelastic code, several others in progress) can be benchmarked against each other. Solutions steady in a drifting frame have well-defined energy, heat flux etc. which can be checked.

Codes use different methods: all have taken years to develop. Need large HPC machines to run, so relatively cheap benchmarks are at a premium.

Focussed mainly on modest Rayleigh numbers, and comparing with the very extensive simulations of the equivalent Boussinesq problem.

Second aim is to investigate what kind of dynamos are found in strongly compressible shells.

The configuration



Inner core: electrically insulating

Outer core: conducting fluid. Vacuum outside. Stress-free boundaries.

Gravity $\sim 1/r^2$: (massive inner core)

Anelastic approximation: no sound waves: $\nabla \cdot (\rho \mathbf{u}) = 0$. Toroidal-Poloidal expansion.

Viscous and Joule heating in entropy equation.

Rapid rotation, so low Ekman number $E = \nu/\Omega d^2$. Kinematic viscosity ν constant throughout shell, similarly κ and η.

Length scale d is the gap-width. Constant entropy on the boundaries, with entropy drop across the shell driving convection.

Polytropic Basic State

$$p = p_0 \zeta^{n+1}, \quad \rho = \rho_0 \zeta^n, \quad T = T_0 \zeta, \qquad \zeta = \frac{c_1}{r} + c_0$$

where c_1 and c_0 are constants.

Polytropic index $n = 1/(\gamma - 1) = 2$. Radius ratio $\frac{r_i}{r_o} = 0.35$.

Dimensionless parameters are:

$$Ra = \frac{GMd\Delta S}{\nu \kappa c_p}, \qquad Pr = \frac{\nu}{\kappa}, \qquad Pm = \frac{\nu}{\eta}, \qquad E = \frac{\nu}{\Omega d^2}$$

$$N_\rho = \ln\left(\frac{\rho_i}{\rho_o}\right), \qquad n: \text{ polytropic index}, \qquad \text{radius ratio} = \frac{r_i}{r_o}$$

ΔS is entropy drop across the shell. $N_\rho = 3$ has factor 20 density drop across shell, $N_\rho = 5$ factor 150. ν, κ, η all constant.

Anelastic equations

Use Lantz/Braginsky/Roberts formulation (basic state near adiabatic, $\Delta S/c_p$ small)

$$\frac{E}{Pm}\frac{D\mathbf{u}}{Dt} + 2\hat{\mathbf{z}} \times \mathbf{u} = -\nabla\left(\frac{p}{\rho}\right) + \frac{1}{\rho}(\nabla \times \mathbf{B}) \times \mathbf{B} + \mathbf{F}_\nu + \frac{Ra\,E\,Pm\,S\hat{\mathbf{r}}}{Pr\,r^2}$$

Buoyancy has form of entropy fluctuation plus a gradient.

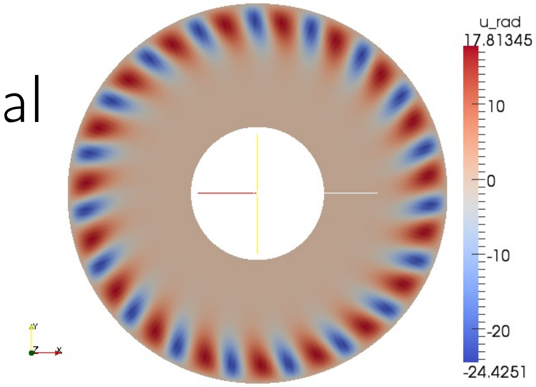
Entropy equation takes form

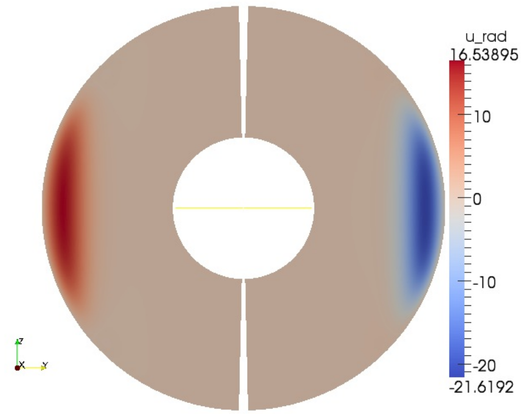
$$\frac{DS}{Dt} = \frac{Pm}{Pr}\zeta^{-n-1}\nabla \cdot \zeta^{n+1}\nabla S + \frac{Di}{\zeta}\left[E^{-1}\zeta^{-n}\left(\nabla \times \mathbf{B}\right)^2 + Q_\nu^*\right]$$

with $Di = \dfrac{c_1 Pr}{Pm\,Ra}$. Note that turbulent entropy diffusion assumed to dominate laminar temperature diffusion.

Hydrodynamic benchmark (no magnetic field)

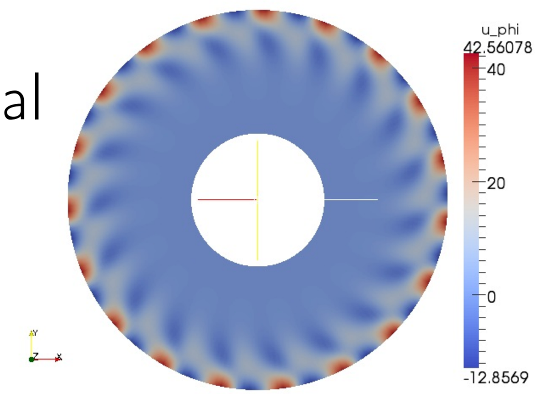
equatorial plane u_r

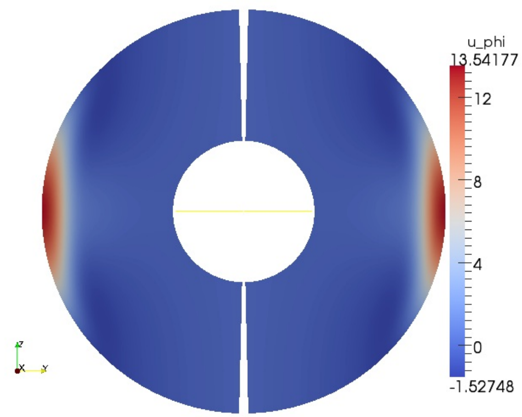




meridional slice u_r

equatorial plane u_ϕ





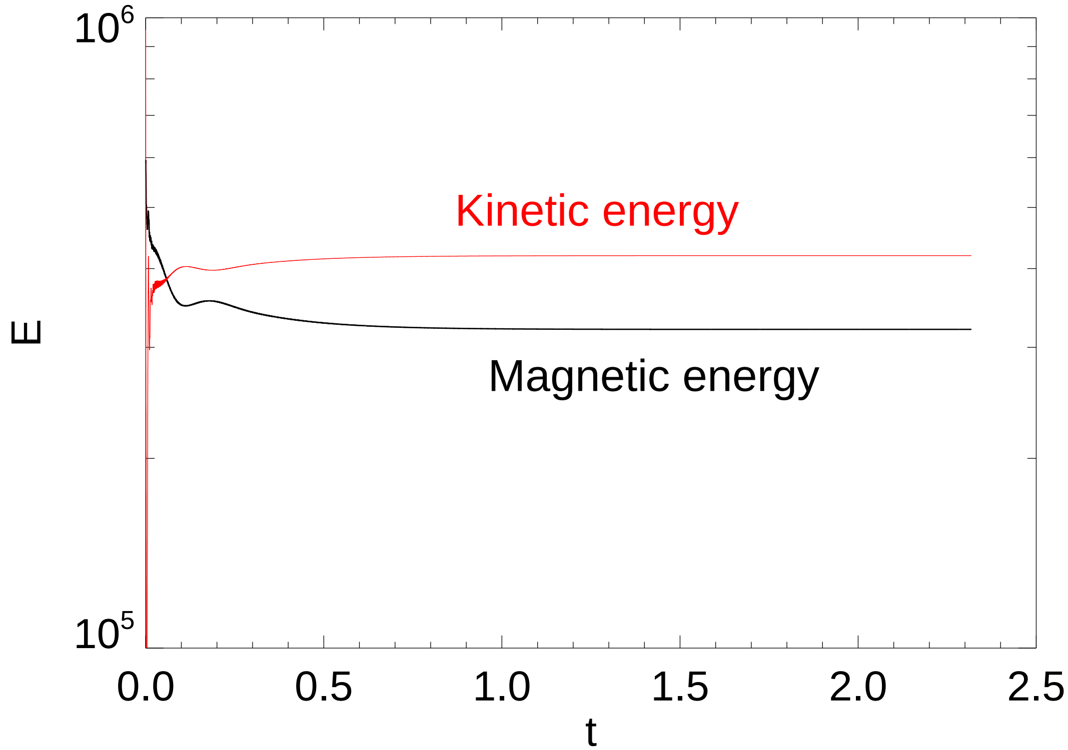
meridional slice zonal u_ϕ

$E = 1e - 03$, $Ra = 351,806$, $Pr = 1$, $N_\rho = 5$, $\eta = 0.35$, $n = 2$

Solution steady in a drifting frame, has $m = 19$ symmetry. Critical Ra for onset of convection $Ra = 283,175$, with $m = 20$.

Good agreement with other codes tested so far.

Saturation of the steady dipolar solution



$E = 2e-03$, $Ra = 80,000$, $Pr = 1$, $Pm = 50$, $N_\rho = 3$, $\eta = 0.35$, $n = 2$

Time here is measured in magnetic diffusion times, so t=2 corresponds to 100 viscous diffusion times. Timestep is 3e-06 (or less) on magnetic diffusion timescale. Fortunately, the solution does not require very high resolution: 64 radial points and 48 X 48 spherical harmonics gives well-resolved solutions.

Subcritical behaviour

The final solution is steady in a drifting frame, so the rolls propagate round, but the kinetic and magnetic energies don't change. Has exact $m = 7$ symmetry.

The critical Ra for onset of convection at these parameters is 81,327.75, with $m = 8$, so this solution at $Ra = 80,000$ is completely subcritical.

Not only would no dynamo grow from a small initial seed field, without magnetic field there would be no convection at all!

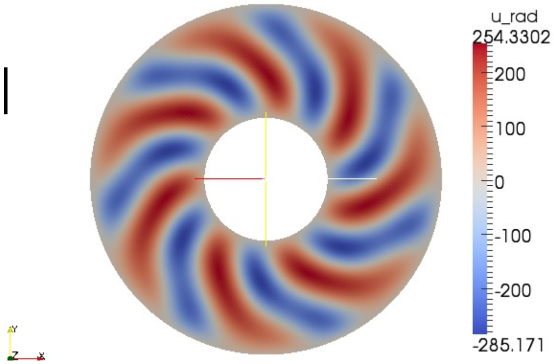
This behaviour has not (yet) been found in spherical Boussinesq dynamos. There the critical Ra for onset is usually much smaller than the Ra at which sustained magnetic fields are found.

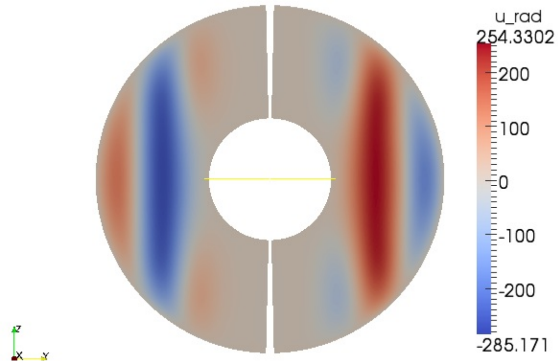
Suggests that magnetic field is having a bigger influence on compressible convection than in the Boussinesq case.

Also found at $Ra = 77,000$, $Pm = 100$.

Flow creating a steady dipolar solution

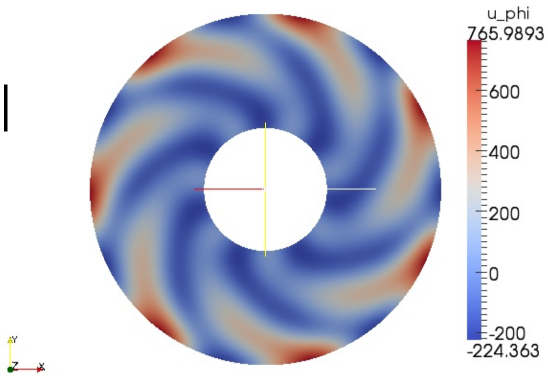
equatorial plane u_r

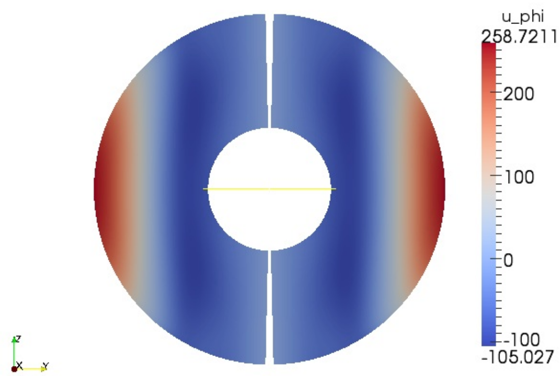




meridional slice u_r

equatorial plane u_ϕ



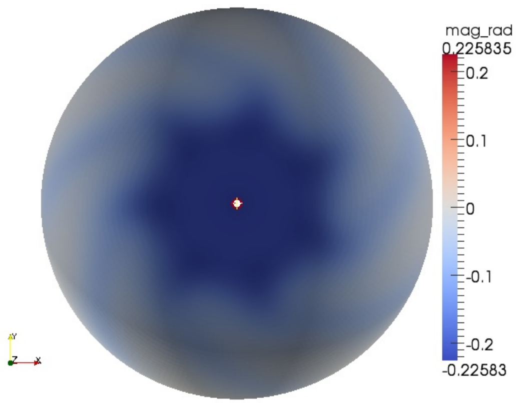


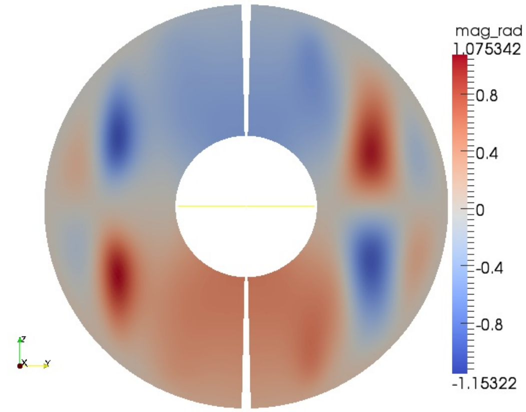
meridional slice zonal u_ϕ

$E = 2e-03,\ Ra = 80,000,\ Pr = 1,\ Pm = 50,\ N_\rho = 3,\ \eta = 0.35,\ n = 2$

Field for the dipole benchmark

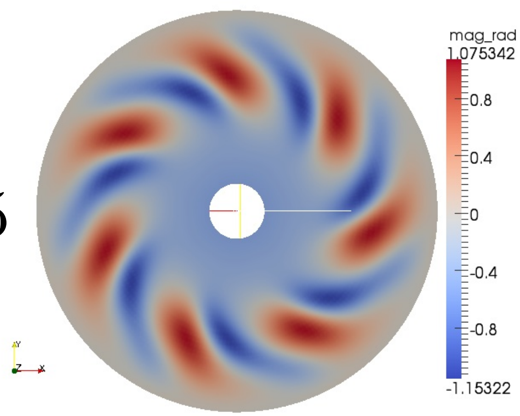
b_r viewed from N. Pole

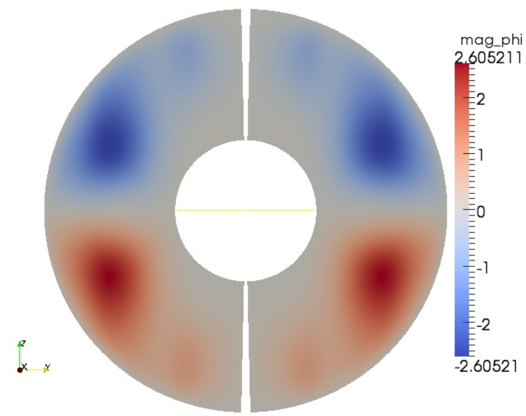




meridional slice b_r

z-slice at $z = 0.5$ b_r





meridional slice zonal b_ϕ

$E = 2e-03$, $Ra = 80,000$, $Pr = 1$, $Pm = 50$, $N_\rho = 3$, $\eta = 0.35$, $n = 2$

Steady solution at larger Pr

$E = 1e-03$, $Ra = 230,000$, $Pr = 6$, $Pm = 95$, $N_\rho = 3$, $\eta = 0.35$, $n = 2$. Resolution $64 \times 64 \times 64$.

Now looked at higher fluid Prandtl number cases. This reduces the importance of Reynolds stresses, and so reduces the zonal flow.

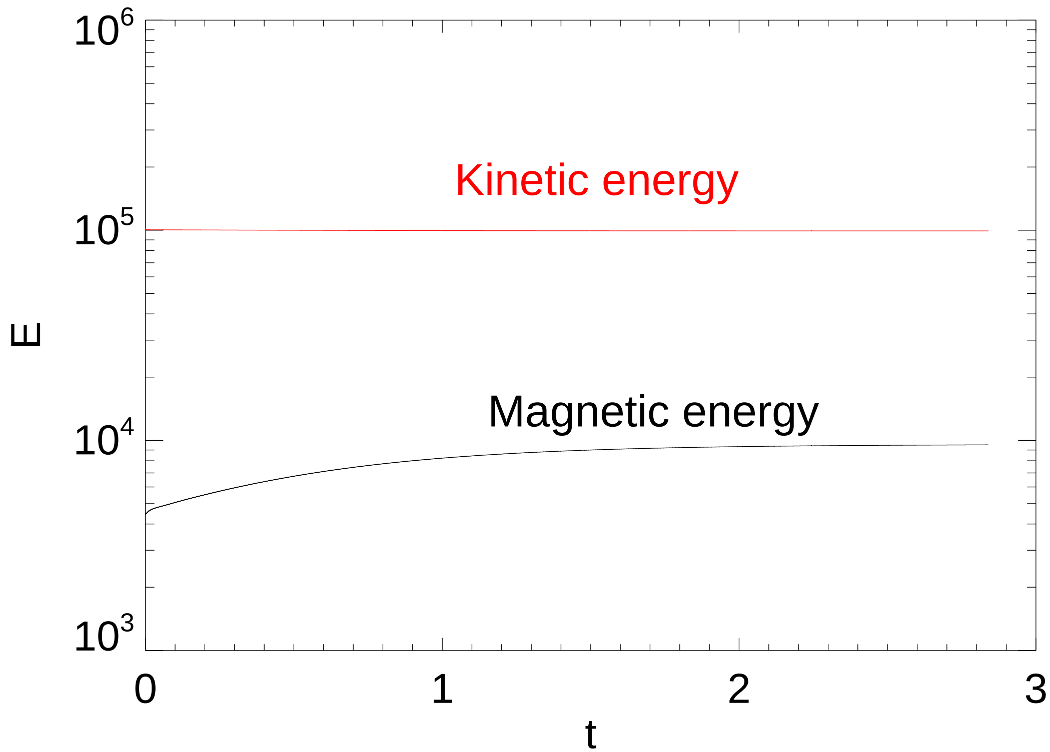
Found steady solutions at lower E (faster rotation). m increases, partly as a consequence of larger E and also because of larger Pr.

Critical Ra for onset of convection is $m = 14$, $Ra = 206,197.73$, $\omega = -25.288$, so this solution is not subcritical for convection.

Field configuration completely different: equatorial dipole, not an axial dipole.

These equatorial dipole dynamos take longer to saturate, and require slightly higher resolution. This solution has an exact $m = 2$ symmetry, though the convection is dominated by $m = 14$.

Saturation of the equatorial dipole solution

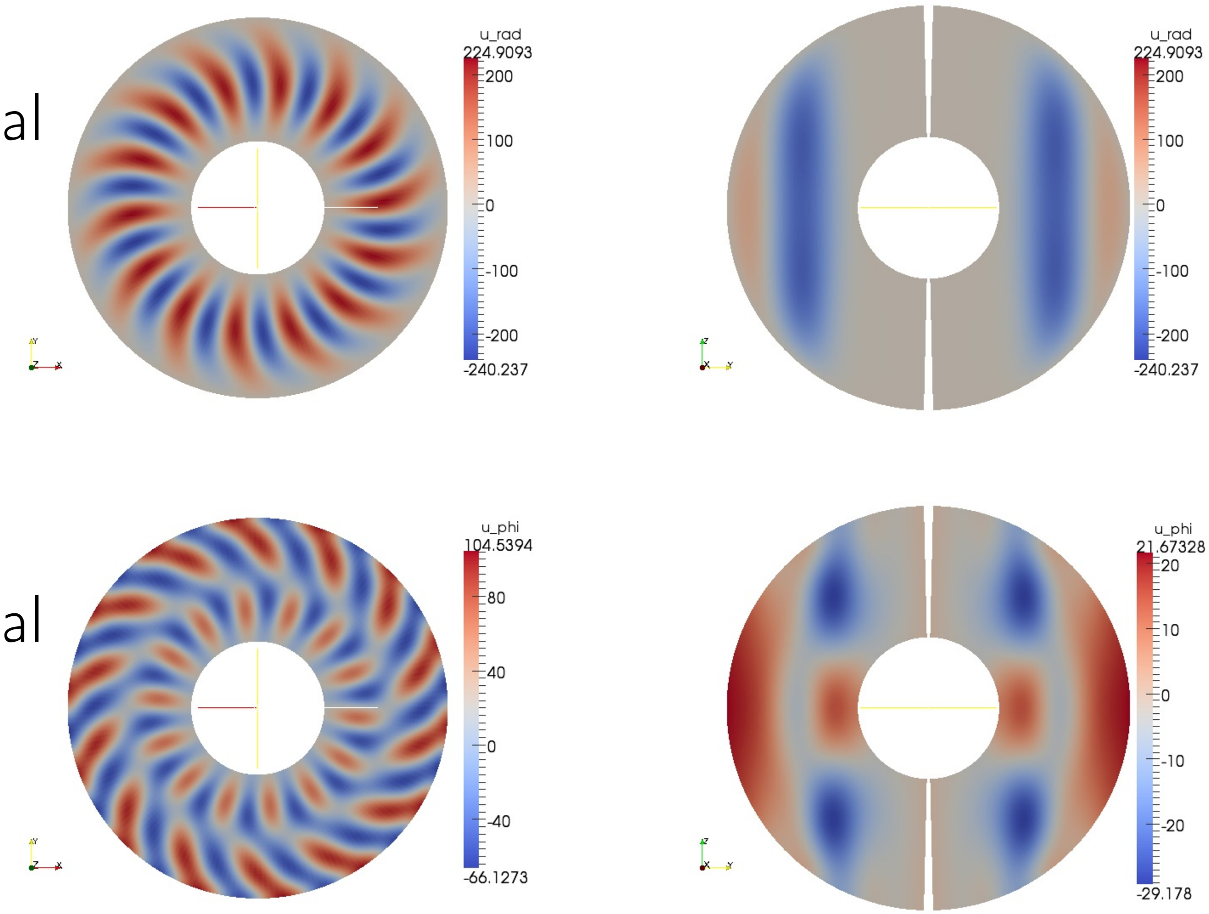


$E = 1e-03$, $Ra = 230,000$, $Pr = 6$, $Pm = 95$, $N_\rho = 3$, $\eta = 0.35$, $n = 2$.

Flow is not much affected by the field, and magnetic energy smaller than kinetic energy.

Flow creating the equatorial dipole solution

equatorial plane u_r



meridional slice u_r

equatorial plane u_ϕ

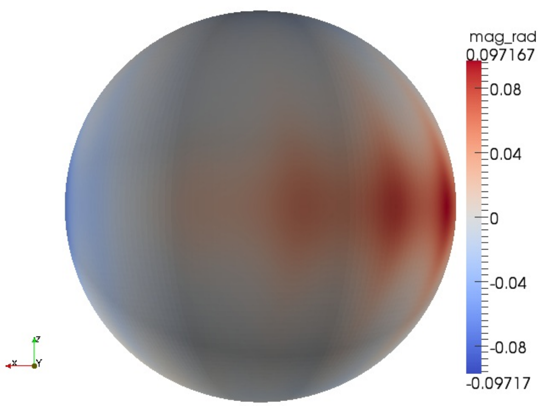
meridional slice zonal u_ϕ

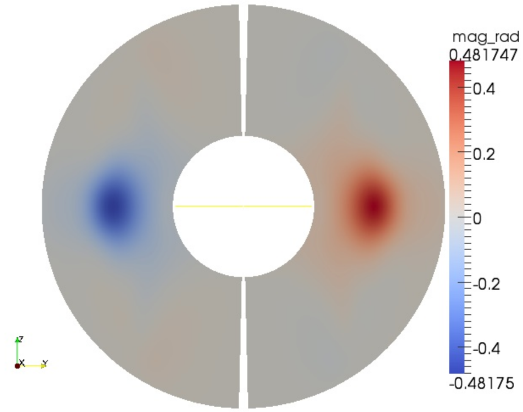
$E = 1e-03,\ Ra = 230,000,\ Pr = 6,\ Pm = 95,\ N_\rho = 3,\ \eta = 0.35,$ $n = 2.$

Note that critical Ra for onset of convection is 206,197.73, with $m = 14$

Field for the equatorial dipole

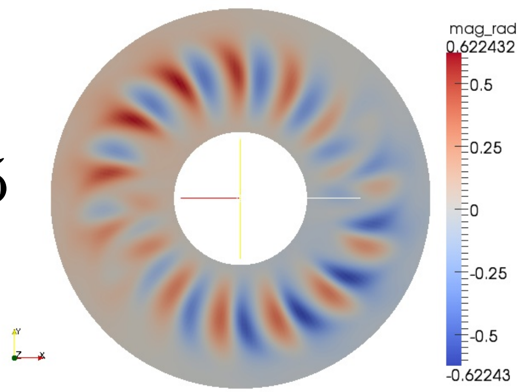
b_r viewed from equator

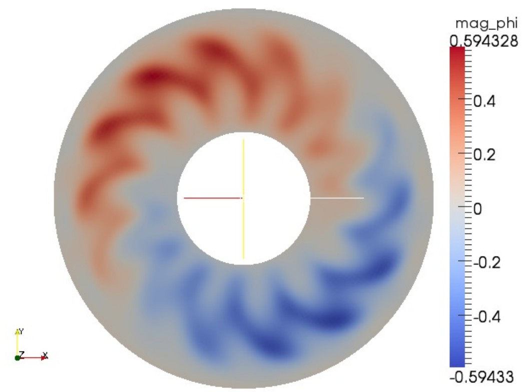




meridional slice b_r

z-slice at $z = 0.5$ b_r





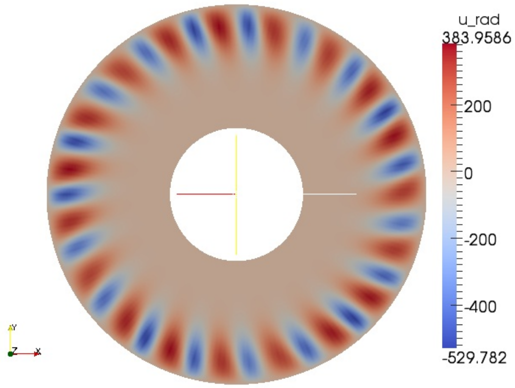
meridional slice zonal b_ϕ

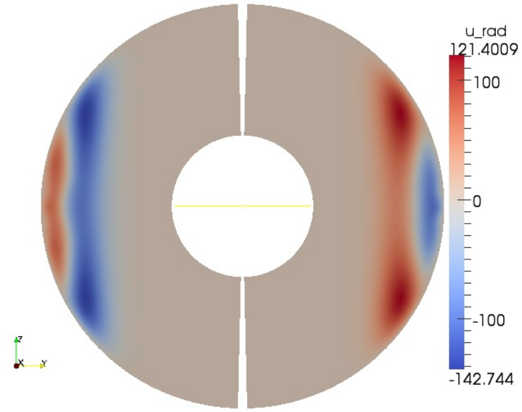
$E = 2e-03$, $Ra = 80,000$, $Pr = 1$, $Pm = 50$, $N_\rho = 3$, $\eta = 0.35$, $n = 2$

Field has quadrupolar parity about the equator.

Flow for an $N_\rho = 5$ steady solution

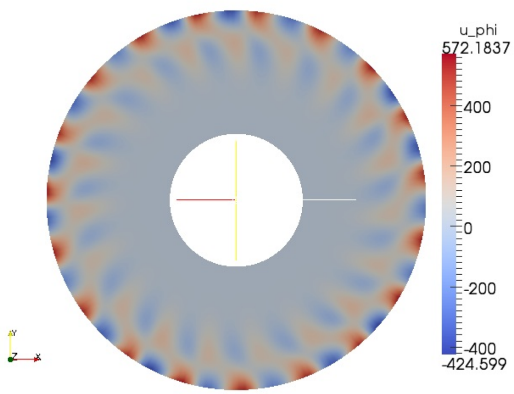
equatorial plane u_r

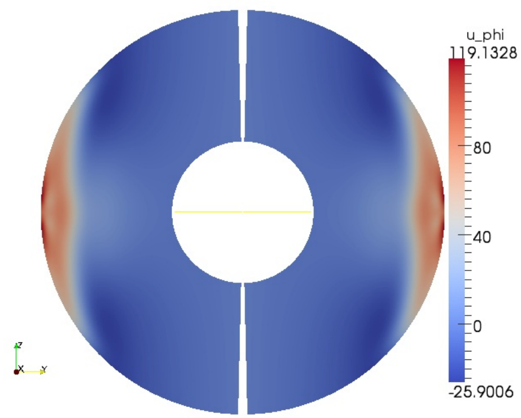




meridional slice u_r

equatorial plane u_ϕ





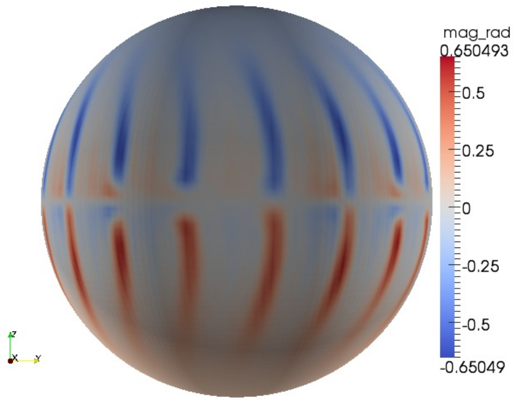
meridional slice zonal u_ϕ

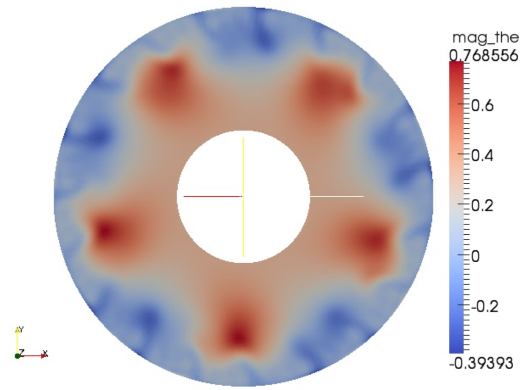
$E = 1e-03$, $Ra = 385,000$, $Pr = 1$, $Pm = 20$, $N_\rho = 5$, $\eta = 0.35$, $n = 2$.

Note that critical Ra for onset of convection is 283,175, with $m = 20$. $m = 19$, solution found here, is also close to marginal.

Field for an $N_\rho = 5$ steady solution

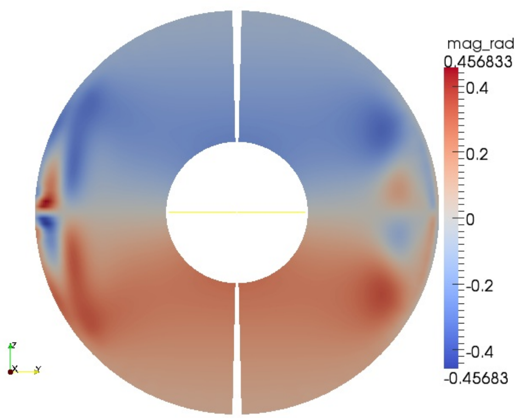
b_r viewed from equator

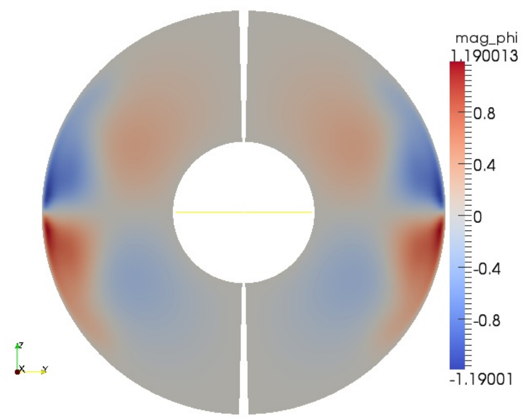




equatorial slice b_θ

meridional slice b_r

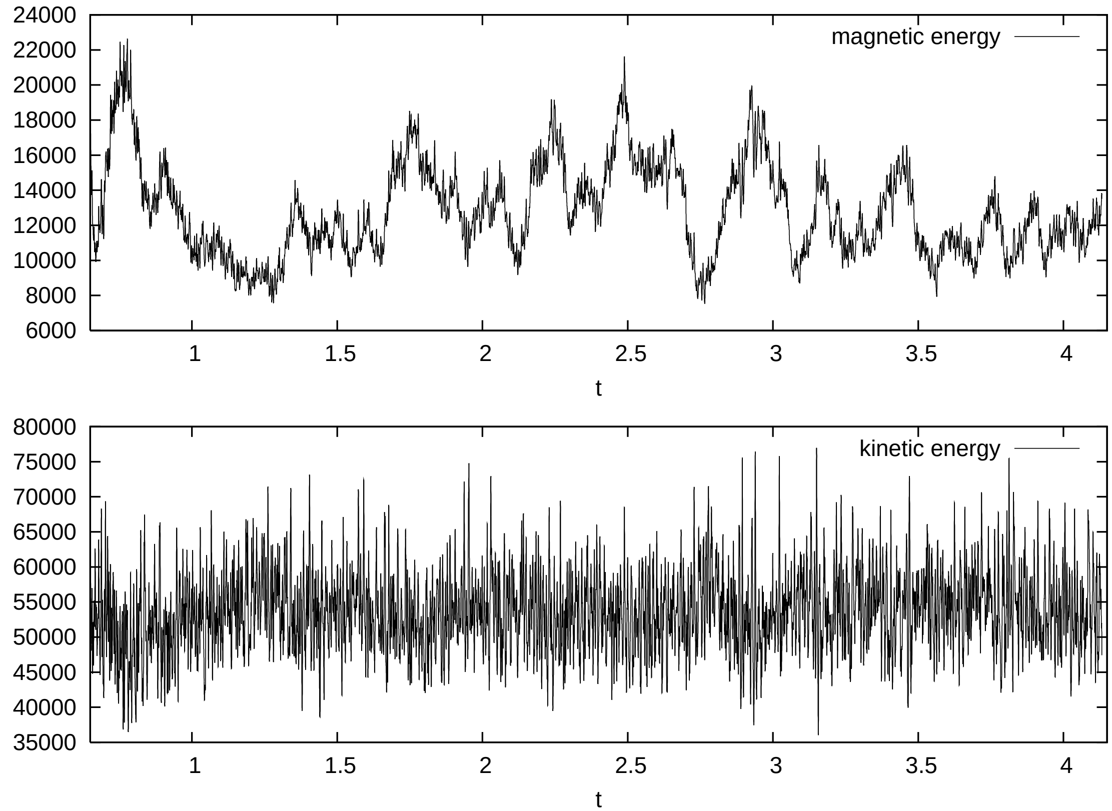




meridional slice zonal b_ϕ

Inside, the field has dipolar form, with an $m = 5$ perturbation form. Externally, the $m = 19$ structure is seen. This convectively imposed structure only occurs near outer boundary. Solution has small basin of attraction.

Tranverse dipole at high Ra

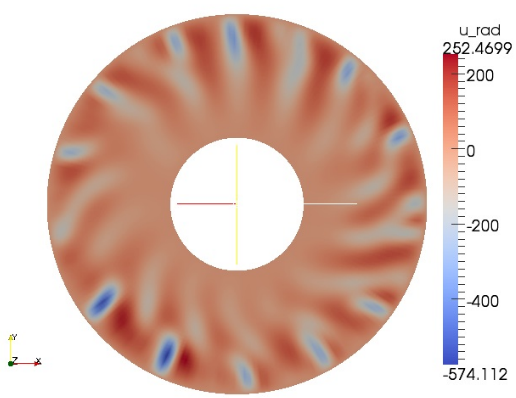


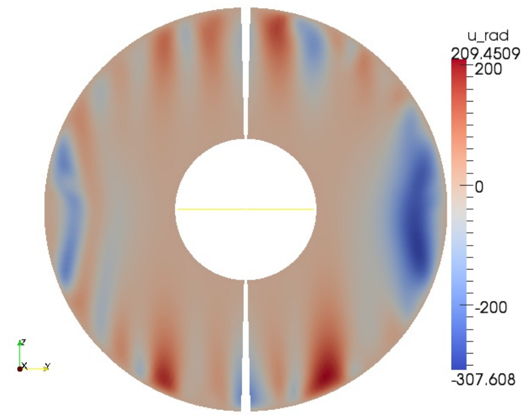
$E = 1e - 03$, $Ra = 764,000$, $Pr = 1$, $Pm = 5$, $N_\rho = 5$, $\eta = 0.35$, $n = 2$. Even at 3 times critical, flow and field fluctuate rapidly on the diffusion time-scale.

At Prandtl number $Pr = 1$ transverse dipoles, complicated versions of the symmetric equatorial dipole, are common.

Tranverse dipole at high Ra

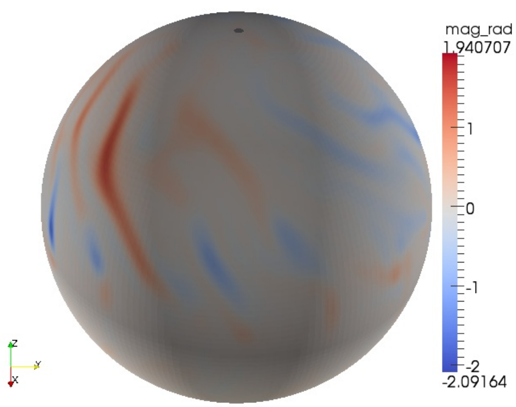
equatorial
slice
u_r

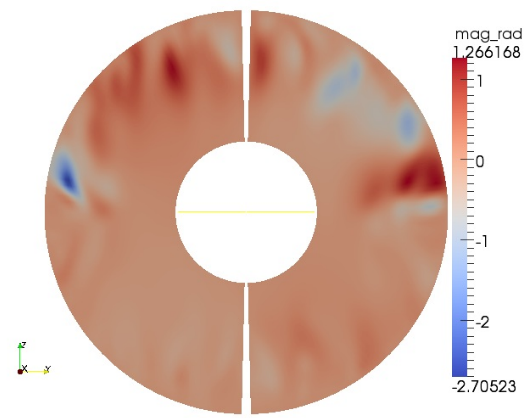




meridional
slice
u_r

b_r at
outer
surface



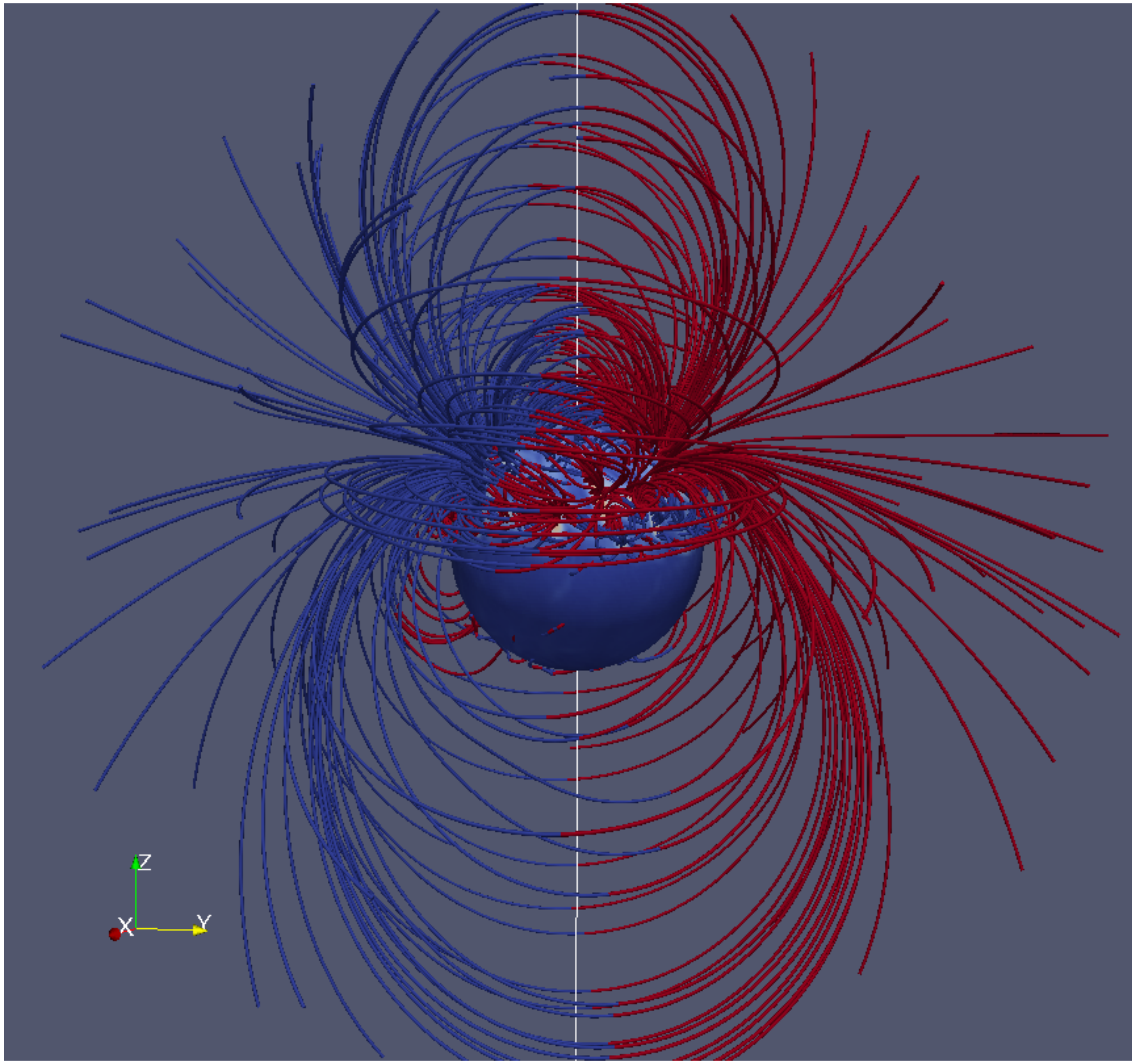


meridional
slice
b_r

Note the convection is now concentrated near the outer boundary. Resolution of $96 \times 192 \times 192$ required.

Field is generated mostly in Northern hemisphere.

Tranverse dipole: external field



The external fields associated with these transverse field solutions consist of starspots which can occur at any latitude.

Conclusions

(i) Nonmagnetic convection: compressibility increases critical azimuthal wavenumber. When N_ρ large, convection strongest near outer boundary.

(ii) Some solutions steady in a drifting frame have been found, suitable as benchmarks. These are only slightly supercritical, as secondary instabilities come in as Ra raised.

(iii) Surprisingly, magnetic solutions found at Ra below that required for onset of nonmagnetic convection. This has not yet been found for spherical Boussinesq models.

(iv) Compressible dynamos have more diverse field configurations than Boussinesq dynamos. In particular, transverse dipoles common as well as axial dipoles.

(v) Congratulations Mike !!