INFLUENCE OF BUOYANCY PROFILE ON THE GEOMAGNETIC FIELD

David Gubbins[1,2,3]

Chris Davies[1,3]

1. School of Earth & Environment, University of Leeds, UK
2. Miller Institute & Dept Earth & Planetary Science, University of California, Berkeley
3. IGPP, Scripps Institution of Oceanography, University of California at San Diego

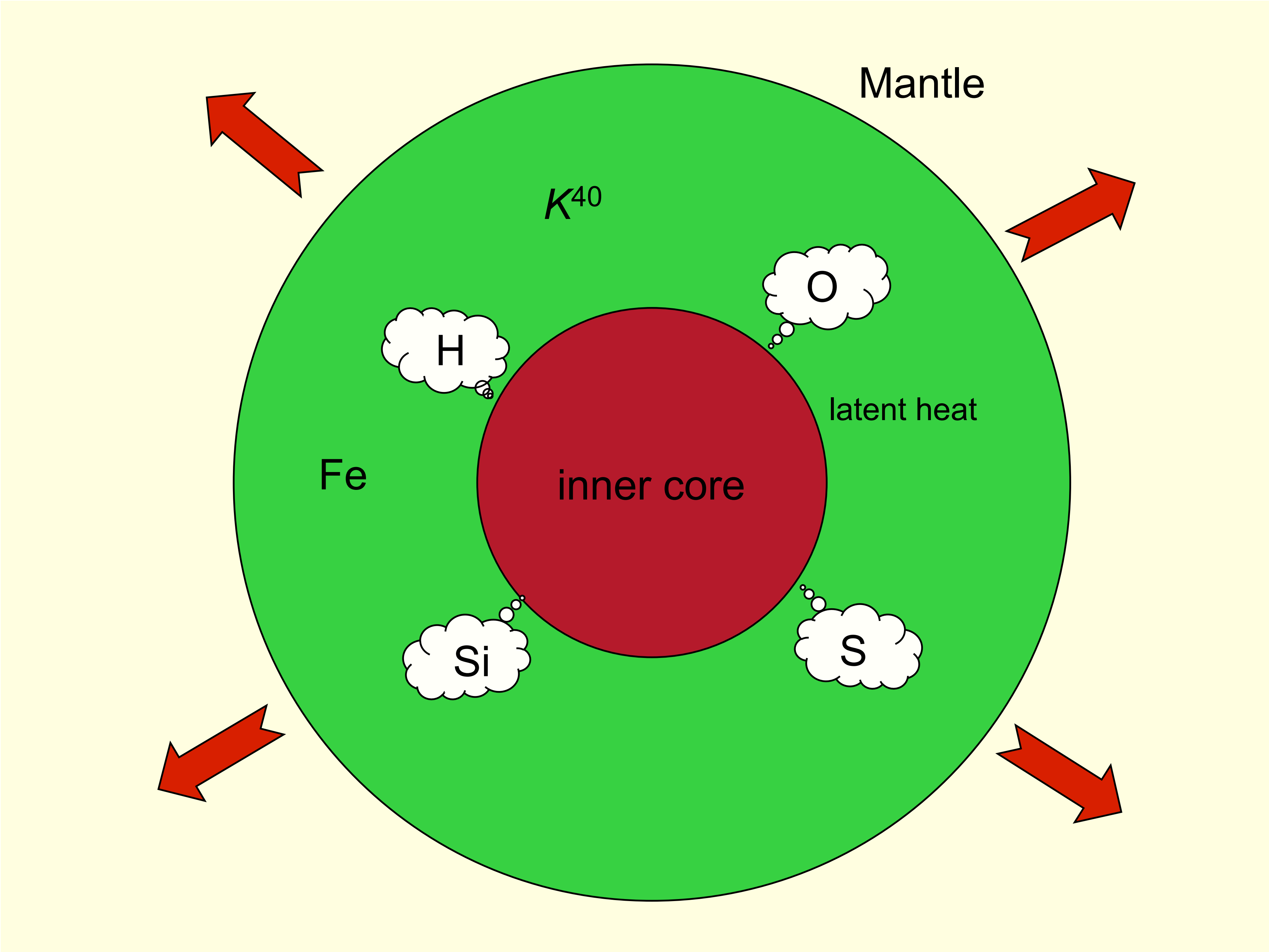
UNIVERSITY OF LEEDS

Cecil H. and Ida M. Green
INSTITUTE OF GEOPHYSICS AND PLANETARY PHYSICS

The Adolph C. and Mary Sprague
Miller Institute for Basic Research in Science
University of California, Berkeley

PLAN

- Geodynamo is driven by thermochemical convection that depends on the thermal evolution of the core
- Make a good but simple approximation to the buoyancy profile from specific core evolution models
- Explore numerical geodynamo solutions to see which ones (if any) look like the Earth


Mantle
K^{40}
O
H
latent heat
Fe
inner core
Si
S

SOURCES OF BUOYANCY

- Latent heat – released at bottom & removed at top
- Specific heat – released internally (non-uniform)
- Radiogenic heating – uniform internal
- Compositional – released at bottom and mixed internally (uniform)
- Adiabatic – heat lost to convection by conduction down the adiabat
- Barodiffusion – light material lost by diffusion down the pressure gradient

THE REFERENCE STATE

- Pressure is nearly hydrostatic: $\dfrac{dP}{dr} = -\rho g$
- Convective velocity >> diffusion =>
- core is well mixed in composition and entropy
- temperature is adiabatic

Remove this and use the Boussinesq equations

Temperature in the core is found by integrating up from the inner core boundary, where T is the known melting temperature

$$T(r) = T_{\mathrm{i}} \exp\left(\int_{r_{\mathrm{i}}}^{r} -\frac{g\gamma}{\phi} dr \right)$$

Time evolution of the (logarithm) of temperature is then the same everywhere:

$$\frac{1}{T_o}\frac{dT_o}{dt} = \frac{1}{T}\frac{dT}{dt}$$

Evolution related to rate of drop of temperature at the core-mantle boundary AND uniform radiogenic heating

APPROXIMATE ADIABATIC GRADIENT

Ignoring variation of thermal expansion with radius gives an adiabatic temperature that is quadratic in *r:* this is accurate to about 8 K.

$$T_a(r) = T_\mathrm{i} - \frac{(T_\mathrm{i} - T_\mathrm{o})}{(r_\mathrm{o}^2 - r_\mathrm{i}^2)}\left(r^2 - r_\mathrm{i}^2\right) = \frac{1}{(r_\mathrm{o}^2 - r_\mathrm{i}^2)}[(T_\mathrm{i} r_\mathrm{o}^2 - T_\mathrm{o} r_\mathrm{i}^2) - (T_\mathrm{i} - T_\mathrm{o}) r^2]$$

Subtracting this from the heat diffusion equation is equivalent to introducing a uniform heat sink

$$q_\mathrm{a} = k \nabla^2 T_\mathrm{a}$$

Specific Heat varies as the adiabatic temperature:

$$q_\mathrm{s} = -\rho C_p \frac{dT_\mathrm{a}(r)}{dt} = -\rho C_p \frac{T_\mathrm{a}(r)}{T_\mathrm{o}} \frac{dT_\mathrm{o}}{dt}$$

COMBINING SOURCES OF BUOYANCY

The gravitational force is proportional to $\alpha_{\mathrm{T}} T + \alpha_c c$ so define a cotemperature

$$T_{\mathrm{co}} = T + \frac{\alpha_c}{\alpha_T} c$$

and combine the 2 diffusion equations into 1 by setting the diffusion constants equal to each other $D = \kappa$ (they are turbulent values anyway).

Compromise on boundary conditions:

At $r = r_{\mathrm{o}}$: constant heat flux (specified), zero mass flux, dT_{co}/dr

At $r = r_{\mathrm{i}}$: constant T, c, T_{co}

NON-DIMENSIONAL EQUATIONS

$$
\begin{aligned}
Pr^{-1}E\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right] + 2\hat{\boldsymbol{\Omega}}\times\mathbf{u} &= -\nabla P + \vartheta\hat{\boldsymbol{r}} + (\nabla\times\mathbf{B})\times\mathbf{B} + E\nabla^2\mathbf{u} \\
\frac{\partial \mathbf{B}}{\partial t} &= \nabla\times(\mathbf{u}\times\mathbf{B}) + q^{-1}\nabla^2\mathbf{B} \\
\frac{\partial \vartheta}{\partial t} + \mathbf{u}\cdot\nabla\vartheta &= \nabla^2\vartheta - ERa\, u_r T'
\end{aligned}
$$

$$
Pr = \frac{\nu_\mathrm{T}}{\kappa_\mathrm{T}}; \quad q = \frac{\kappa_\mathrm{T}}{\eta}; \quad E = \frac{\nu_\mathrm{T}}{\Omega d^2}; \quad ERa = \frac{2\alpha_T \kappa g_0 d^3\,(T_\mathrm{i} - T_\mathrm{o})}{\Omega\kappa_\mathrm{T}^2\,(r_\mathrm{o} + r_\mathrm{i})}
$$

Ra has the form of a conventional Rayleigh number:

$$
Ra = 2\frac{\alpha_T g_0 \kappa d^5\,(T_\mathrm{i} - T_\mathrm{o})}{\kappa_\mathrm{T}^2 \nu_\mathrm{T}\,(r_\mathrm{o} + r_\mathrm{i})},
$$

EQUIVALENT BASIC STATE CONDUCTION TEMPERATURES

$$
\begin{aligned}
T'_{\mathrm{a}} &= -2\frac{k}{k_{\mathrm{T}}}\frac{(T_{\mathrm{i}}-T_{\mathrm{o}})}{(r_{\mathrm{o}}^2-r_{\mathrm{i}}^2)}r \\
T'_{\mathrm{r}} &= \frac{q_{\mathrm{r}}}{3k_{\mathrm{T}}}r \\
T'_{c} &= -\frac{}{3\kappa_{\mathrm{T}}}\frac{\alpha_c}{\alpha_T}\frac{4\pi r_{\mathrm{i}}^2\rho_{\mathrm{i}}c_0}{\tau_r M_{\mathrm{oc}}}\left(r-\frac{r_{\mathrm{o}}^3}{r^2}\right)\frac{T_{\mathrm{i}}}{T_{\mathrm{o}}}\frac{dT_{\mathrm{o}}}{dt} \\
T'_{\mathrm{s}} &= \frac{1}{\kappa_{\mathrm{T}}\left(r_{\mathrm{o}}^2-r_{\mathrm{i}}^2\right)}\left[\frac{1}{3}(T_{\mathrm{i}}r_{\mathrm{o}}^2-T_{\mathrm{o}}r_{\mathrm{i}}^2)r-\frac{1}{5}(T_{\mathrm{i}}-T_{\mathrm{o}})r^3\right]\frac{1}{T_{\mathrm{o}}}\frac{dT_{\mathrm{o}}}{dt} \\
T'_{\mathrm{L}} &= \frac{\rho_{\mathrm{i}}L}{\tau_r k_{\mathrm{T}}}\frac{T_{\mathrm{i}}}{T_{\mathrm{o}}}\frac{dT_{\mathrm{o}}}{dt}\frac{r_{\mathrm{i}}^2}{r^2} \\
T'_{\mathrm{b}} &= \frac{\alpha_c^2\alpha_D g_0}{\alpha_T\bar{\rho}\kappa_{\mathrm{T}}}r.
\end{aligned}
$$

General form:

$$T'(r) = \frac{\beta^{(b)}}{r^2} + \beta^{(i)}r + \beta^{(s)}r^3$$

RAYLEIGH NUMBERS

For the Earth:

$$Ra = \frac{2\alpha_{\mathrm{T}} g_0 \kappa d^5 \left(T_{\mathrm{i}} - T_{\mathrm{o}}\right)}{\kappa_{\mathrm{T}}^2 \nu_{\mathrm{T}} \left(r_{\mathrm{o}} + r_{\mathrm{i}}\right)} \approx 2.16 \times 10^{12}$$

with Ekman number and asymptotic critical Rayleigh numbers:

$$E = 10^{-9}; \quad E^{-1} = 10^{9}; \quad E^{-4/3} = 10^{12}$$

Ra in the Earth is close to the critical Rayleigh number for non-magnetic rotating convection with turbulent diffusivities.

For this calculation:

$$E = 10^{-4}; \quad E^{-1} = 10^{4}; \quad E^{-4/3} = 2.15 \times 10^{5}$$

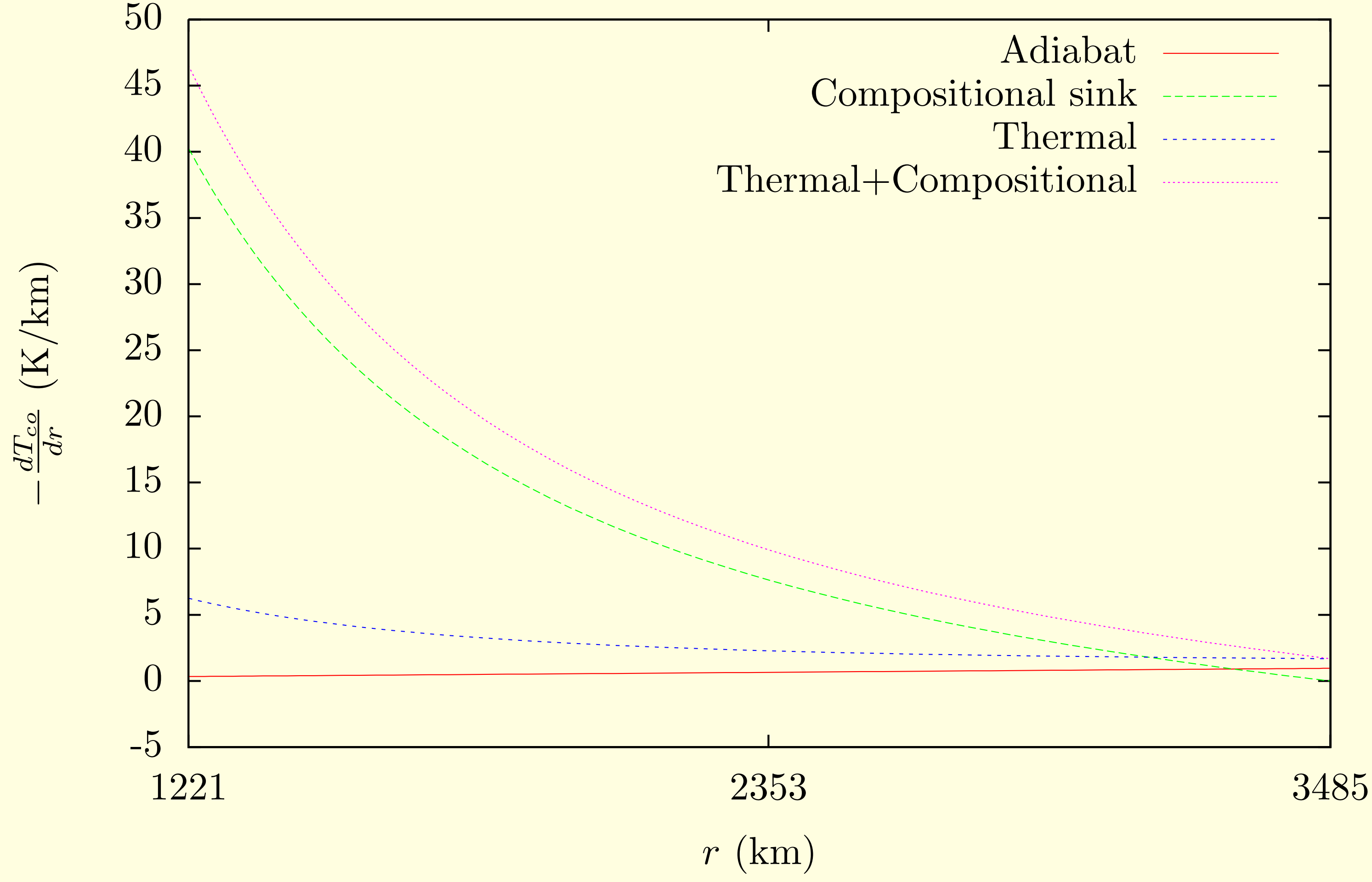
3 END-MEMBER CORE EVOLUTION MODELS

$$T'(r) = \frac{\beta^{(b)}}{r^2} + \beta^{(i)} r + \beta^{(s)} r^3$$

	Case 1			Case 2			Case 3		
	$dT_o/dt = 123$ K/Gyr $h = 0$			$dT_o/dt = 69$ K/Gyr $h = 0$			$dT_o/dt = 12$ K/Gyr $h = 4$ pW/kg		
a	0	-1	0	0	-1	0	0	-1	0
r	0	0	0	0	0	0	0	1.06	0
c	6.59	-1.90	0	3.70	-1.10	0	0.64	-0.18	0
s	0	1.23	-0.09	0	0.69	-0.05	0	0.12	-0.01
L	2.74	0	0	1.54	0	0	0.27	0	0
Total	9.33	-1.67	-0.09	5.24	-1.4	-0.05	0.91	0.00	-0.01
	(b)	*(i)*	*(s)*	*(b)*	*(i)*	*(s)*	*(b)*	*(i)*	*(s)*

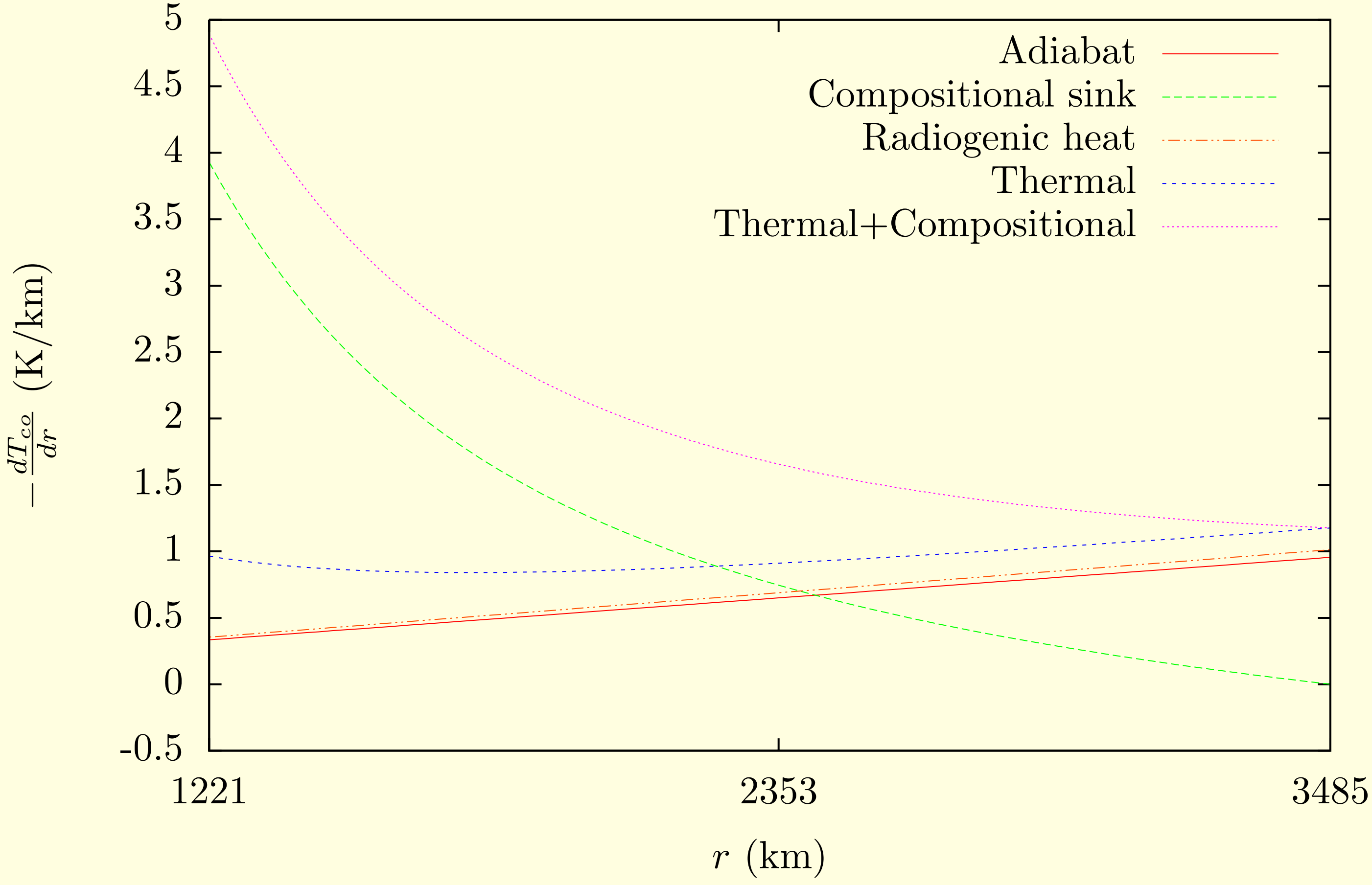
APPLIED CONDUCTION COTEMPERATURES

Radial variation of cotemperature gradient with radius for $dT_o/dt = 123$ K/Gyr and $h = 0$

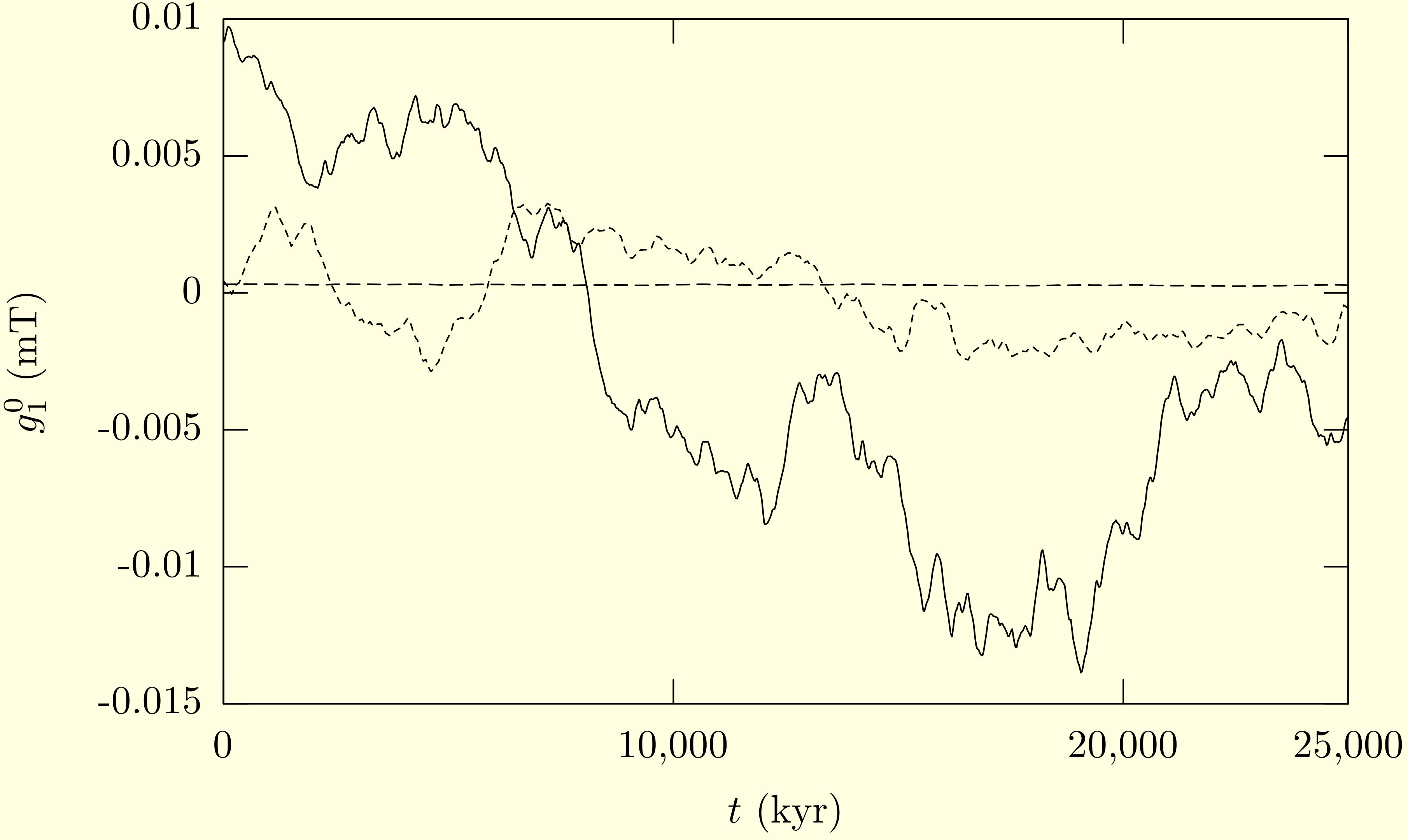


APPLIED CONDUCTION COTEMPERATURES

Radial variation of cotemperature gradient with radius for $dT_o/dt = 12$ K/Gyr and $h = 4 \times 10^{-12}$ W/kg

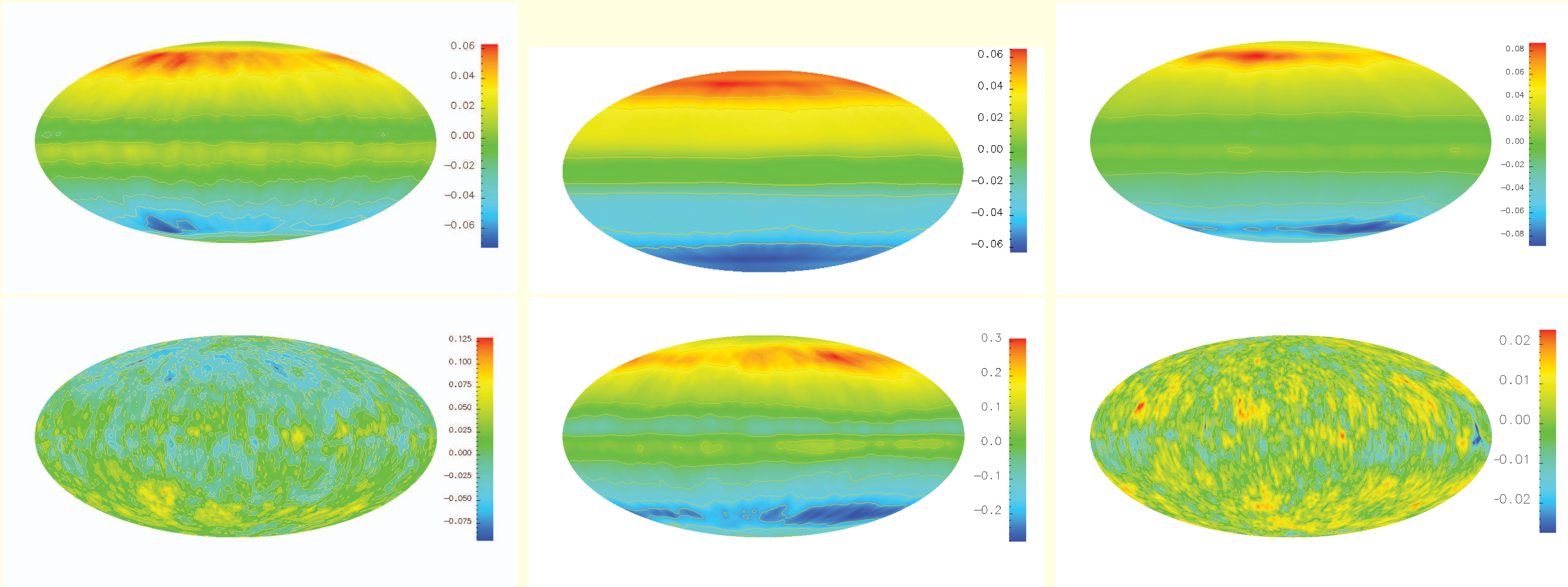


DIPOLE MOMENT

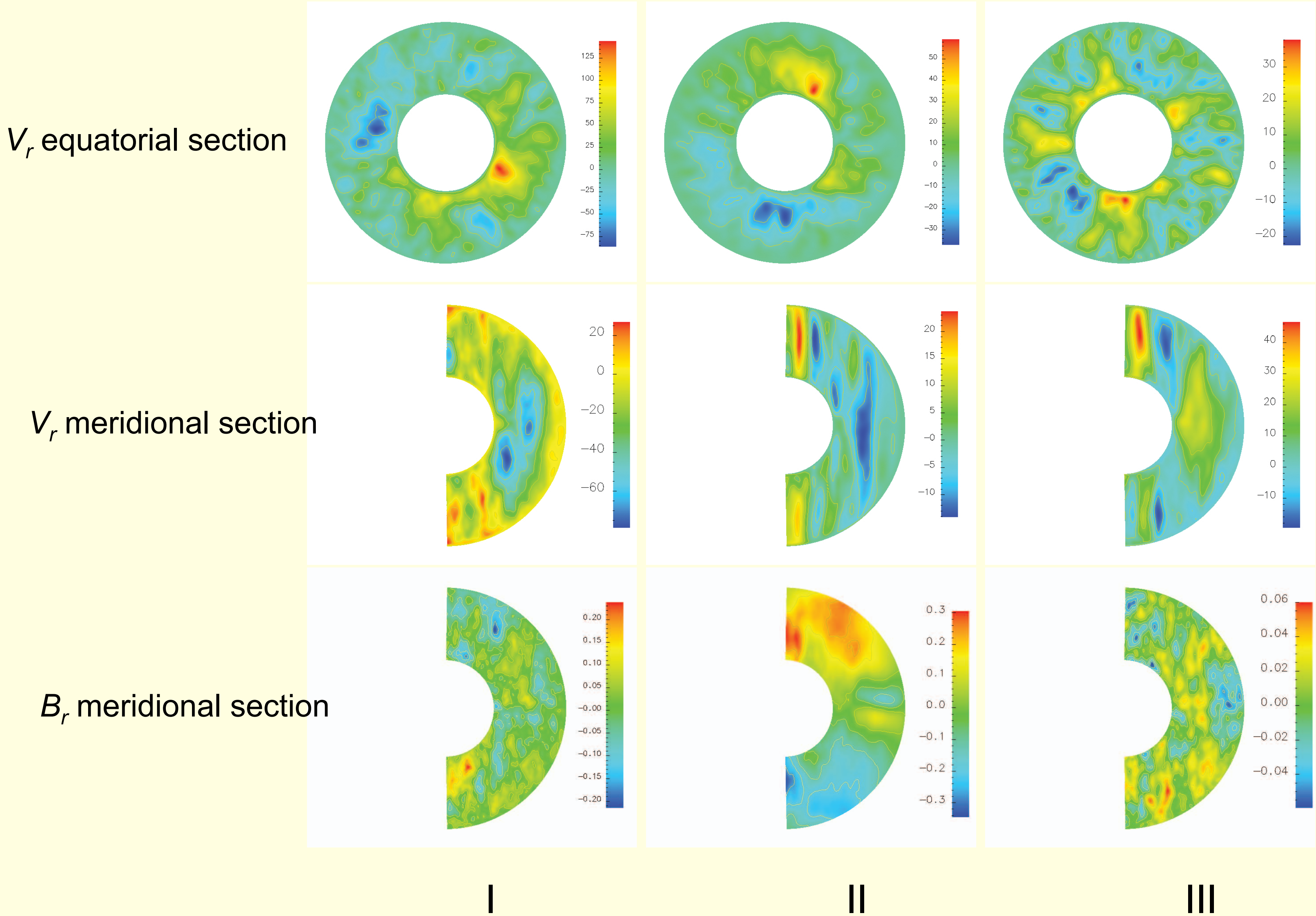


TIME-AVERAGED (100 kyr) SURFACE B_r

ERa = 20 (top), 200 (bottom)



CASE I CASE II CASE III


V_r equatorial section
V_r meridional section
B_r meridional section
125
100
75
50
25
0
−25
−50
−75
50
40
30
20
10
0
−10
−20
−30
30
20
10
0
−10
−20
20
0
−20
−40
−60
20
15
10
5
−0
−5
−10
40
30
20
10
0
−10
0.20
0.15
0.10
0.05
−0.00
−0.05
−0.10
−0.15
−0.20
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
0.06
0.04
0.02
0.00
−0.02
−0.04
I
II
III

CONCLUSIONS

- Geodynamo runs at somewhere near the critical Rayleigh number for non-magnetic convection, 1000 times that for magnetoconvection
- Compositional buoyancy dominates
- Convection is driven most strongly at the bottom of the core, hardly at all at the top
- Internal heating does not produce a sufficiently large dipole
- Inadequate cooling does not produce a strong enough dipole, nor sufficient time variations.