Convectively-driven dynamos in a compressible layer

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“Convection, magnetoconvection and dynamo theory” (MREP60)
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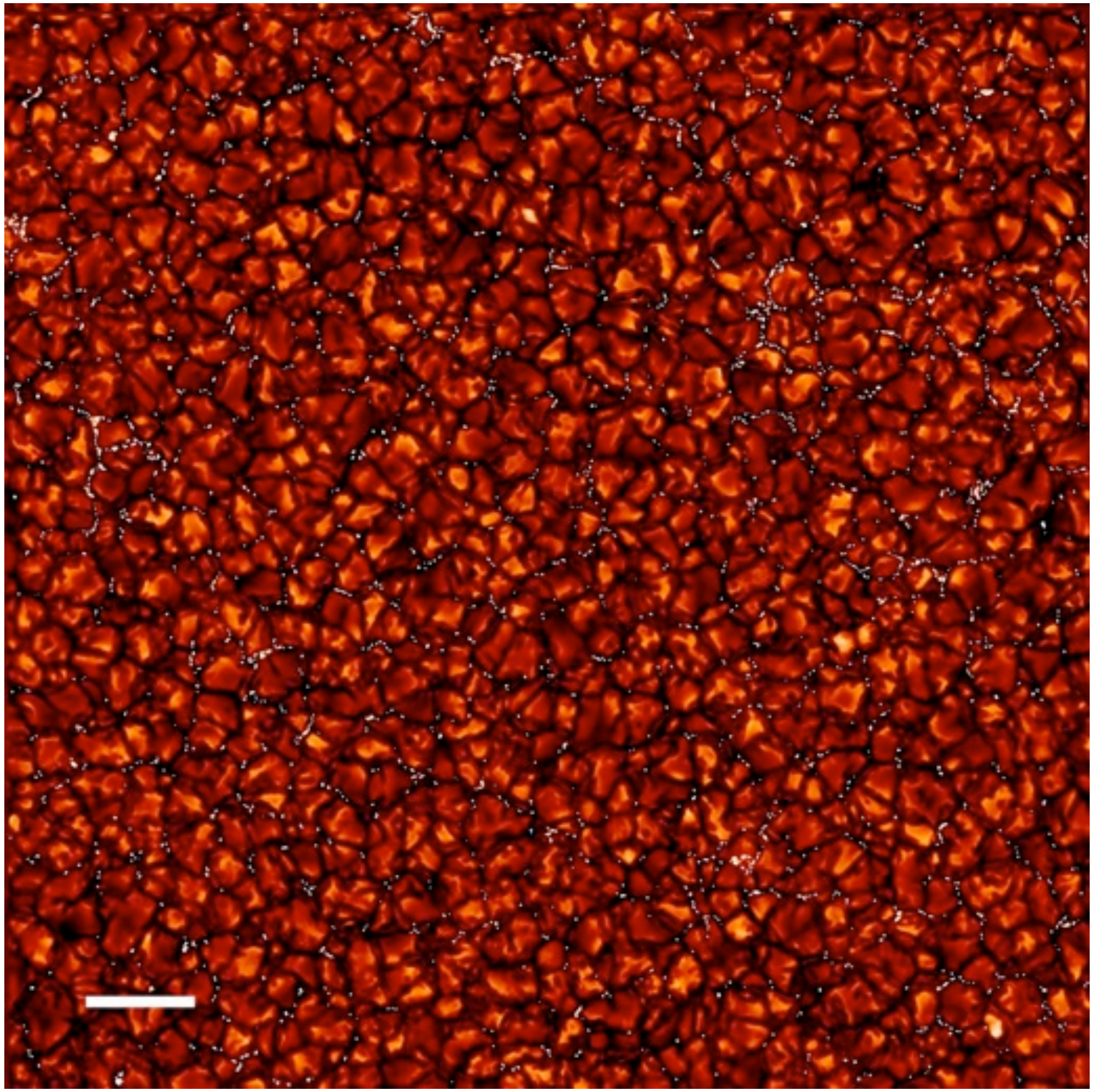
1. Motivation: Solar observations

The quiet Sun:

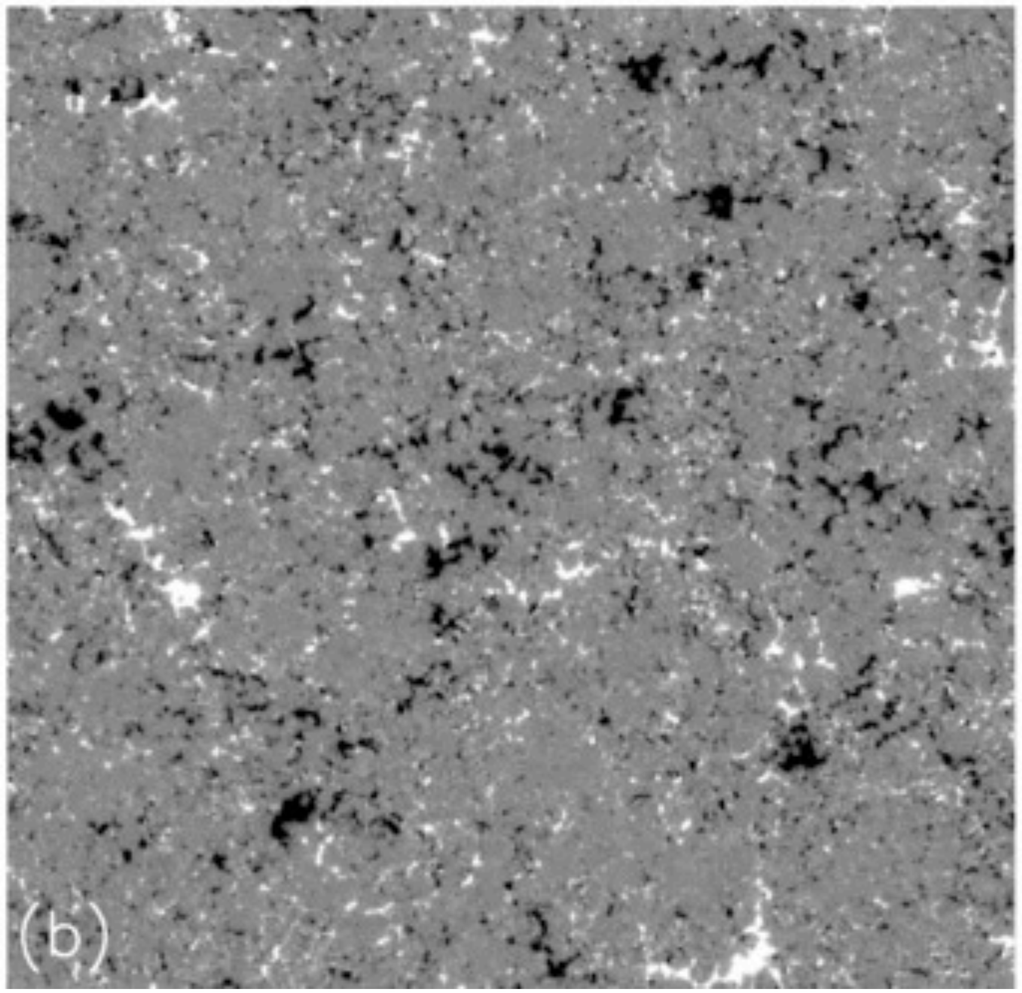
- Magnetic flux concentrations accumulate in the intergranular lanes

- Strong magnetic fields appear as bright points in G-band images of the Sun (Right)



(Sanchez Almeida et al., 2010, ApJL)

1. Motivation: Solar observations (cont.)



100Mm

Main features:

- The magnetic flux regions in the quiet Sun are of mixed polarity
- Field strengths in excess of a Kilogauss

Super-equipartition fields:

$$B_z > B_{eq} \approx 400G$$

Above: The line-of-sight component of the magnetic field in the quiet Sun (Parnell et al., 2009, ApJ)

1. Motivation: Previous models

It is likely that (some fraction of) the quiet Sun magnetic fields are generated locally by small-scale convective motions

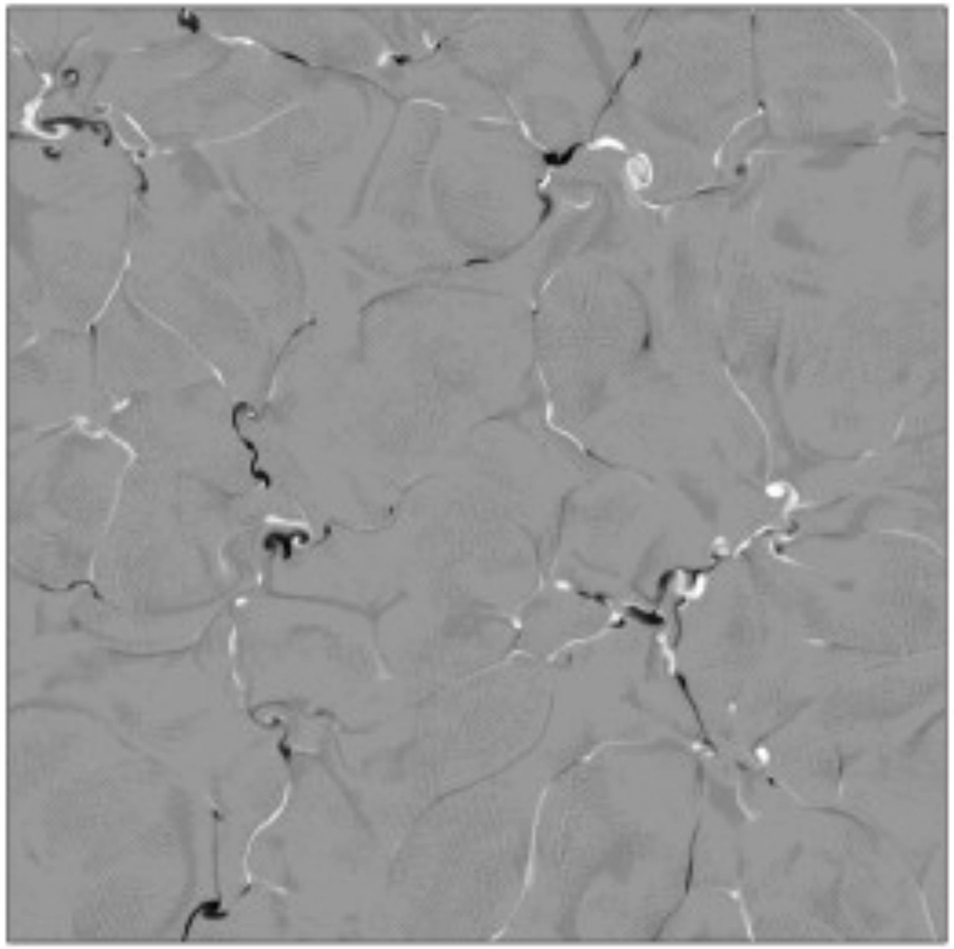
Right: Dynamo action in Boussinesq convection (Cattaneo, 1999, ApJ)

• An efficient dynamo:

$$\frac{\text{Magnetic Energy}}{\text{Kinetic Energy}} \approx 0.2$$

• As in the quiet Sun, mixed polarity magnetic flux concentrations form in the convective downflows

B_z

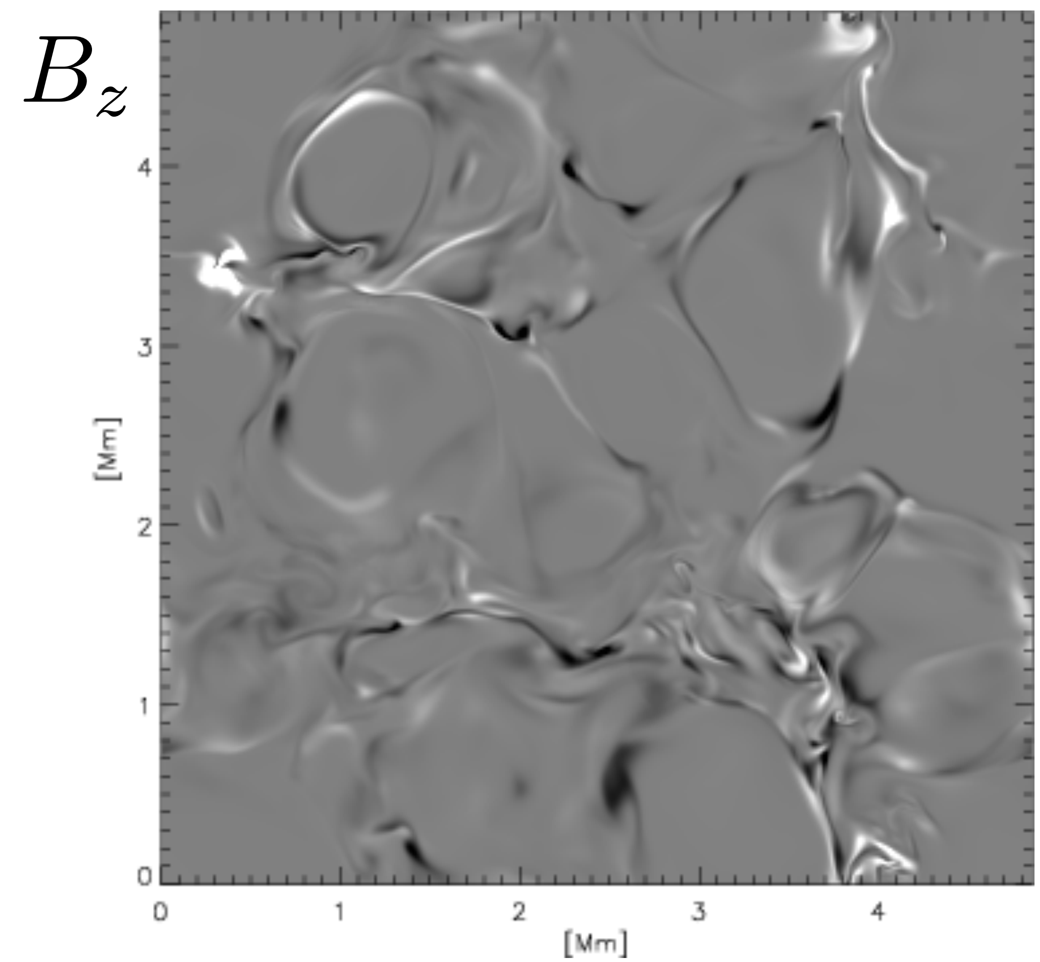


1. Motivation: Previous models (cont.)

Dynamo action in "radiative" compressible convection
(Vögler & Schüssler, 2007, A&A)

Similar field structures to Boussinesq case, but even best case is less efficient:

$$\frac{\text{Magnetic Energy}}{\text{Kinetic Energy}} \approx 0.025$$



Other compressible models:

- Abbett, 2007, ApJ
- Käpylä et al., 2008, ApJ
- Brummell et al., 2010, GAFD
- .....

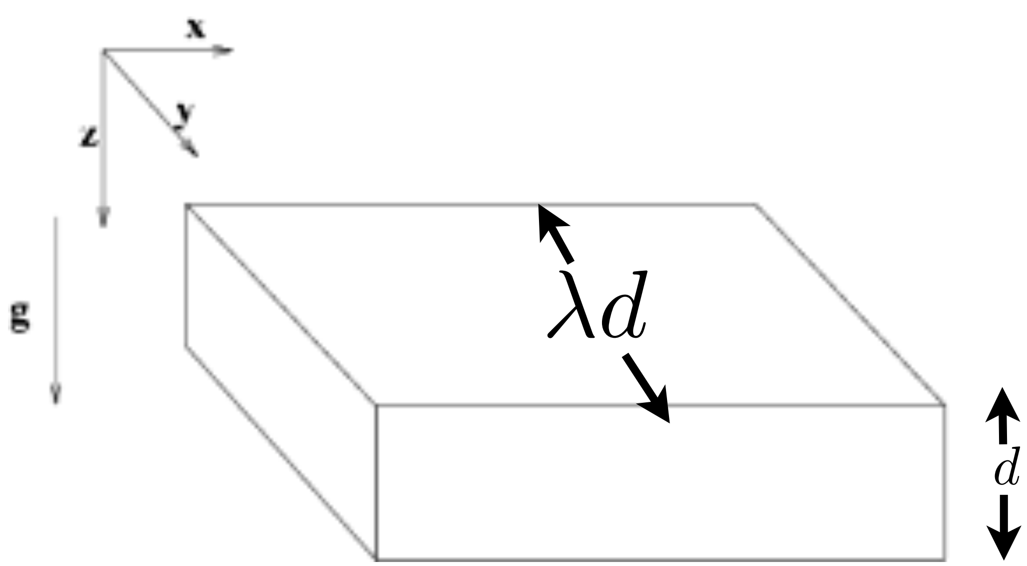
2. Model Setup

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \qquad P = \mathcal{R}\rho T$$

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla P + \rho g \hat{\mathbf{z}} + \frac{1}{\mu_0}(\nabla \times \mathbf{B}) \times \mathbf{B} + \mu \left[\nabla^2 \mathbf{u} + \frac{1}{3}\nabla(\nabla \cdot \mathbf{u})\right]$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \qquad \nabla \cdot \mathbf{B} = 0$$

$$\rho c_V \left[\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T\right] = -P\nabla \cdot \mathbf{u} + K\nabla^2 T + Q_\nu + Q_\eta$$



$(\lambda = 4 \text{ or } 8)$

Idealised boundary conditions:

Horizontal: All variables periodic

Vertical: Impermeable, stress-free, fixed temperature, vertical magnetic field

3. Numerical results: λ=4

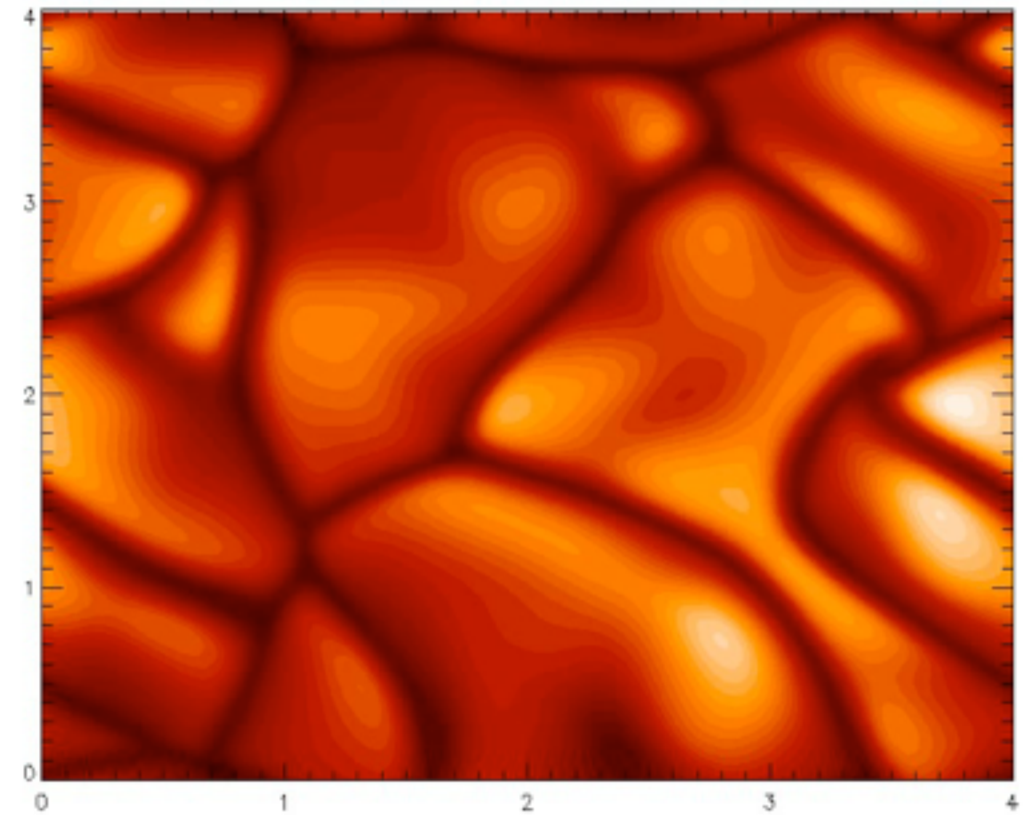
Initial condition: Fully-developed hydrodynamic convection. Insert a seed (vertical) magnetic field with no net flux

Moderately stratified layer:

$$T_{base}/T_{top} = 4$$

$$\overline{\rho}_{base}/\overline{\rho}_{top} \approx 4$$

Right: Temperature contours in a horizontal plane just below the upper surface



Reynolds number (fixed): $\mathcal{R}e = \frac{\rho_{mid} U_{rms} d}{\mu} \approx 150$

Magnetic Reynolds number (variable): $\mathcal{R}m = \frac{U_{rms} d}{\eta}$

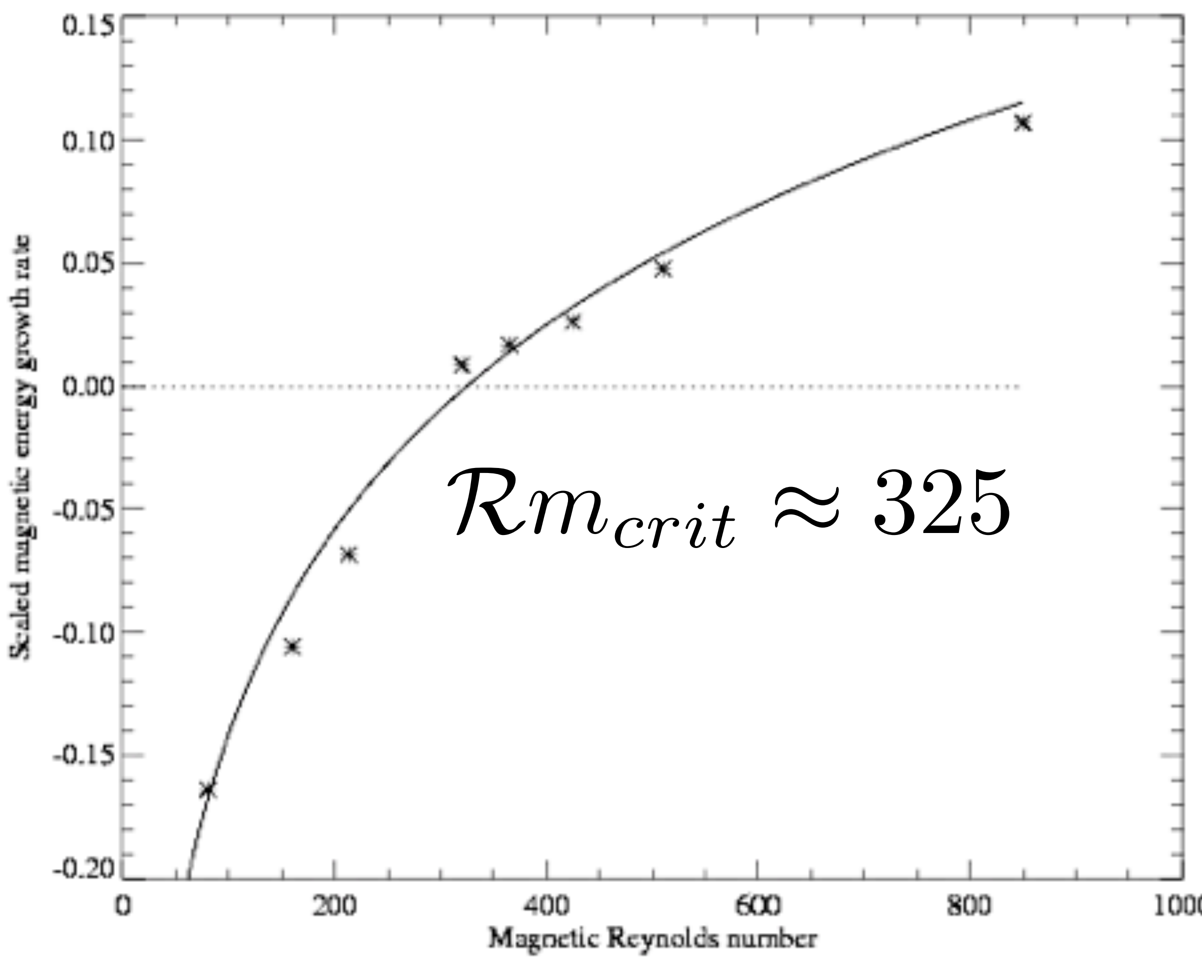
Prandtl number (fixed): $\sigma = \frac{\mu c_P}{K} = 1$

3. Numerical results: λ=4 (cont.)

Kinematic Phase: Whilst the field is weak, the magnetic energy either grows or decays exponentially. This plot shows the growth rate as a function of the magnetic Reynolds number

Key Points:

- A logarithmic "best fit" curve
- As in Boussinesq convection (Cattaneo, 1999, ApJ) and previous compressible calculations (e.g. Vögler & Schüssler, 2007, A&A), the peak growth rate is comparable to the convective turnover time



Magnetic Prandtl number:

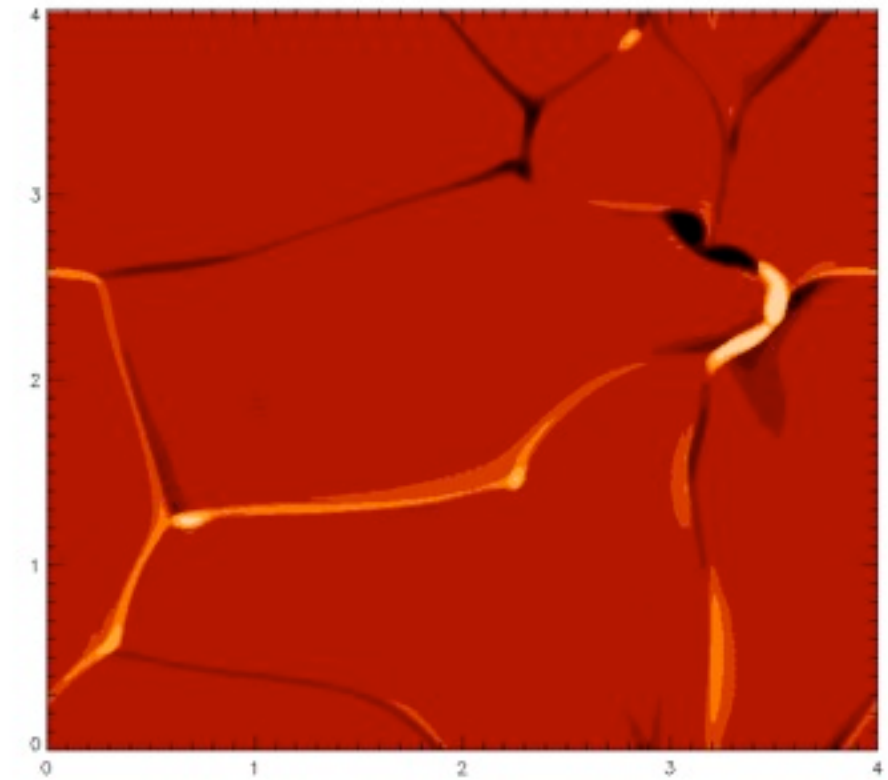
If $\mathcal{R}m > \mathcal{R}m_{crit}$ then $Pm = \mathcal{R}m/\mathcal{R}e > 2.17$

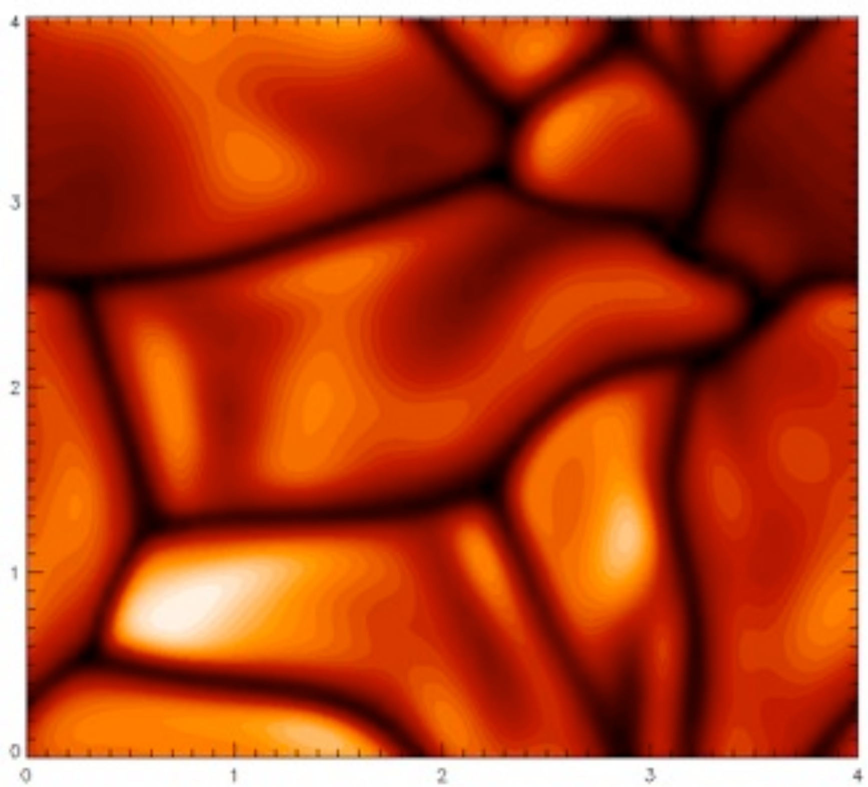
3. Numerical results: λ=4 (cont.)

Nonlinear results: $\mathcal{R}m \approx 480$

Mixed polarity magnetic flux accumulates in the convective downflows, where high magnetic pressure leads to partial evacuation (Note: logarithmically-spaced contours used for Bz)

B_z





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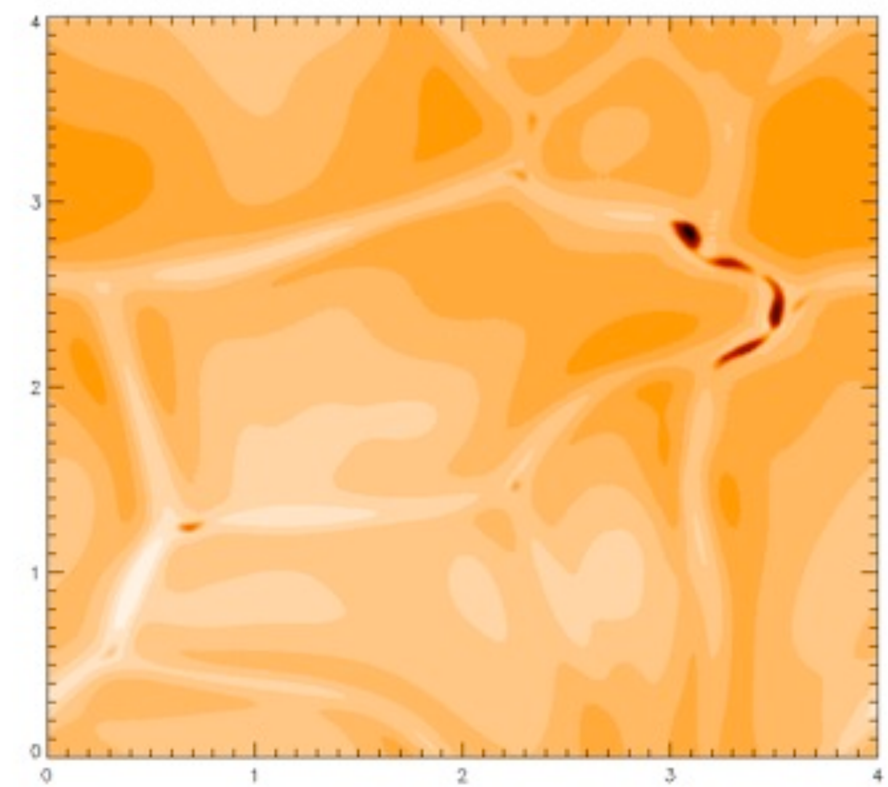
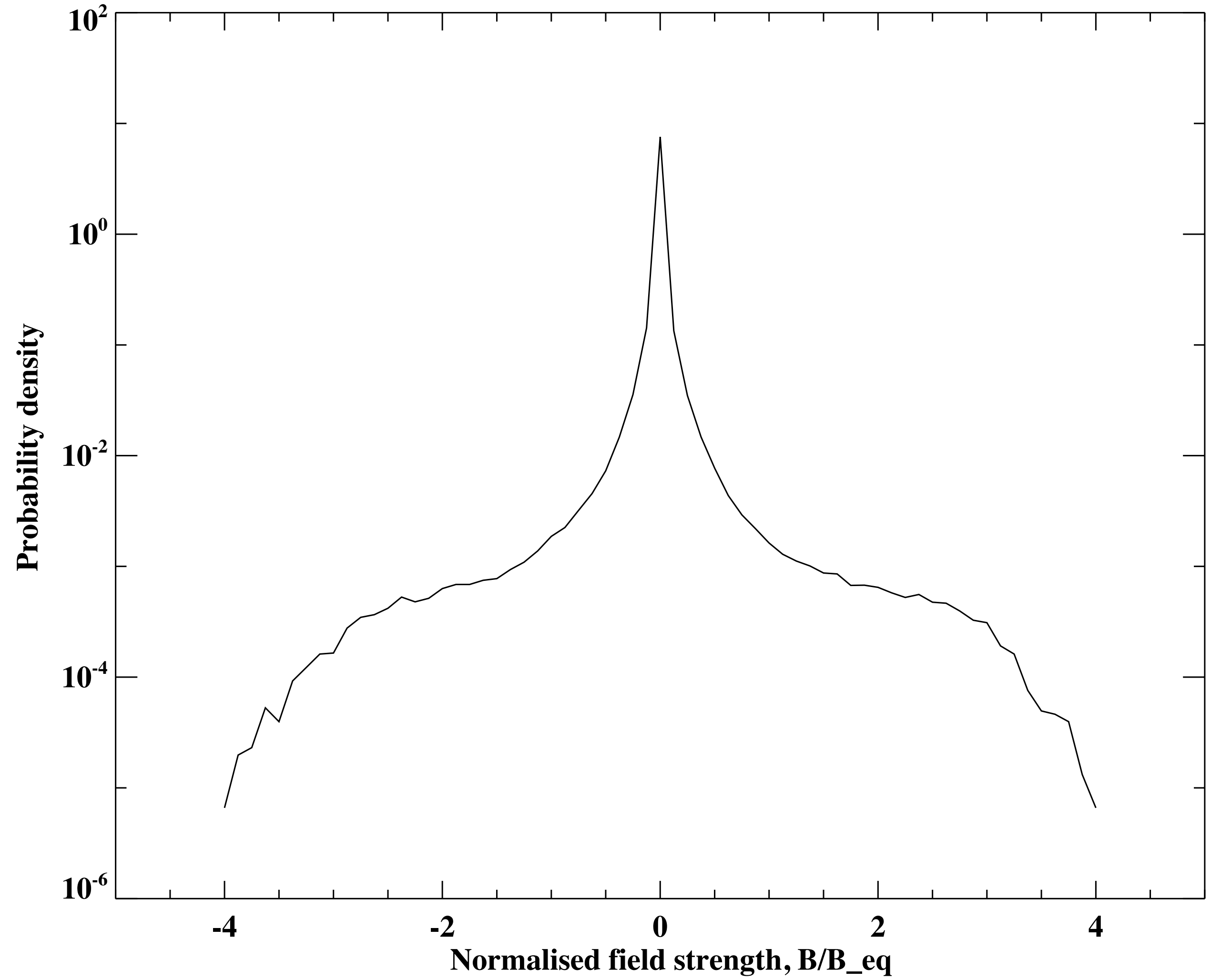


3. Numerical results: λ=4 (cont.)

Right: $\mathcal{R}m \approx 480$
Time-averaged probability density function for B_z/B_{eq} at the upper surface



"Equipartition" field strength:

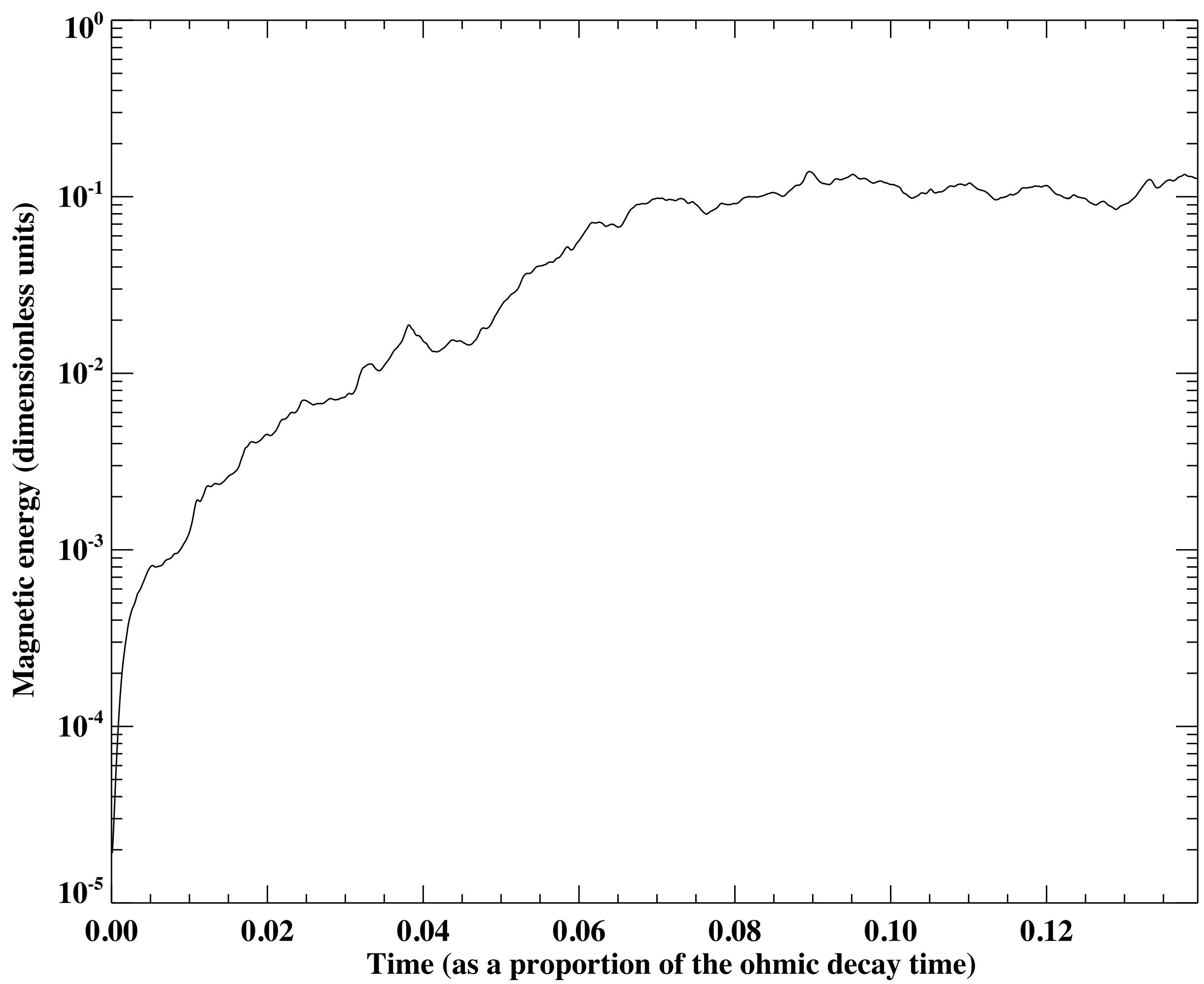
$$B_{eq} = \sqrt{\mu_0 \overline{\rho U_{surf}^2}}$$

- Super-equipartition field strengths qualitatively similar to those observed in the quiet Sun
- Partial evacuation plays an important role in the field intensification process. Related to convective collapse models (e.g. Spruit, 1979, SoPh), although more of an "adjustment" than a well-defined instability.

3. Numerical results: λ=4 (cont.)

The global saturation level of the dynamo:

Right: The magnetic energy as a function of time for $\mathcal{R}m \approx 480$



$$\frac{\text{Magnetic Energy}}{\text{Kinetic Energy}} \approx 0.05$$

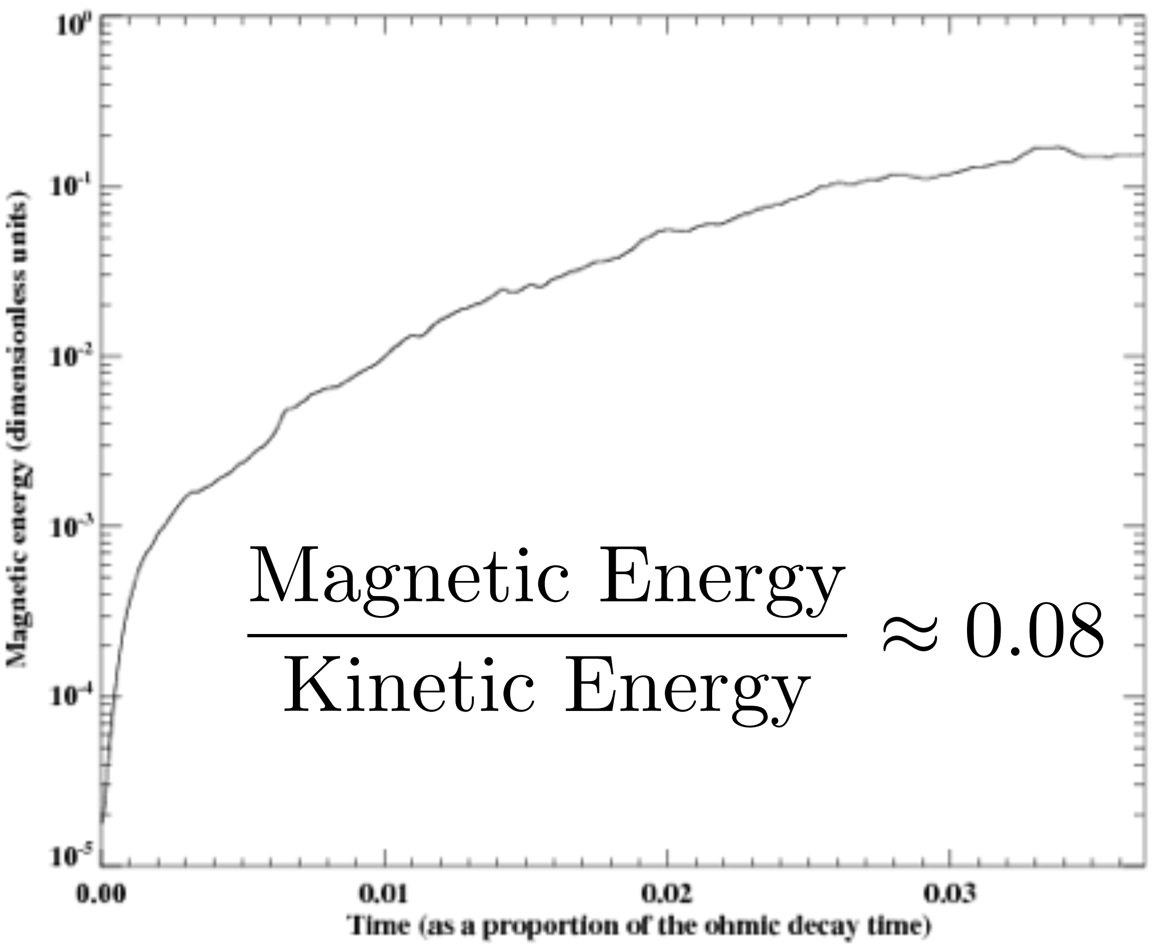
Comparison with the Boussinesq dynamo of Cattaneo (1999):
Larger domain, but comparable $\mathcal{R}e$ and σ. In that case:

$$\mathcal{R}m \approx 1000 \Longrightarrow \frac{\text{Magnetic Energy}}{\text{Kinetic Energy}} \approx 0.2$$

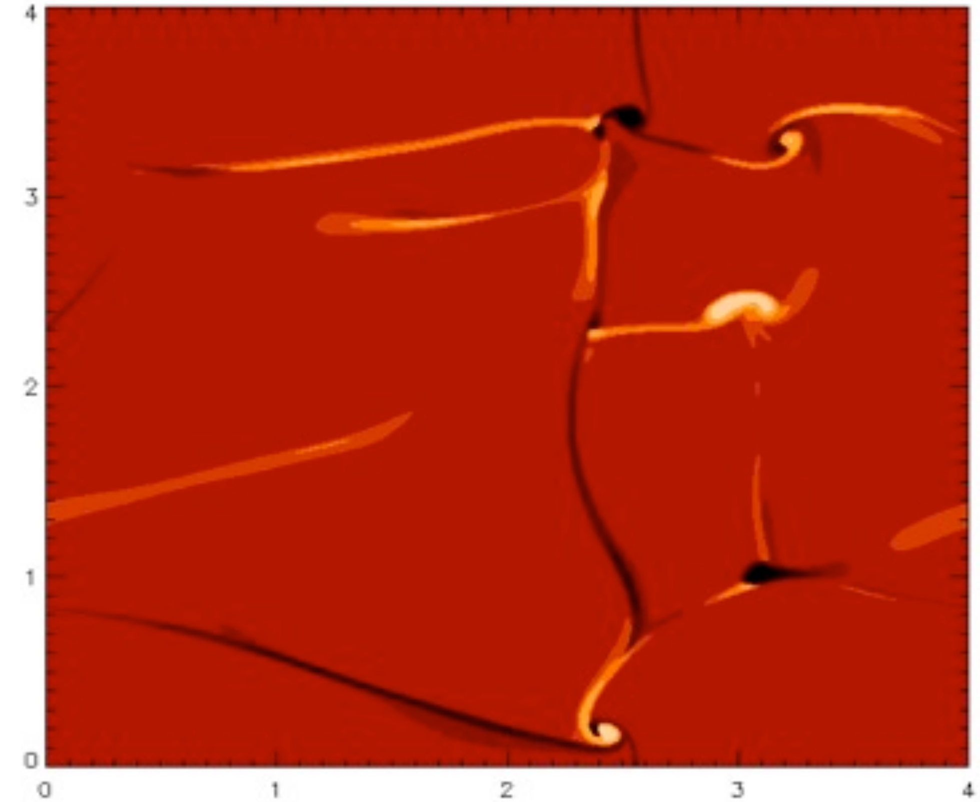
3. Numerical results: λ=4 (cont.)

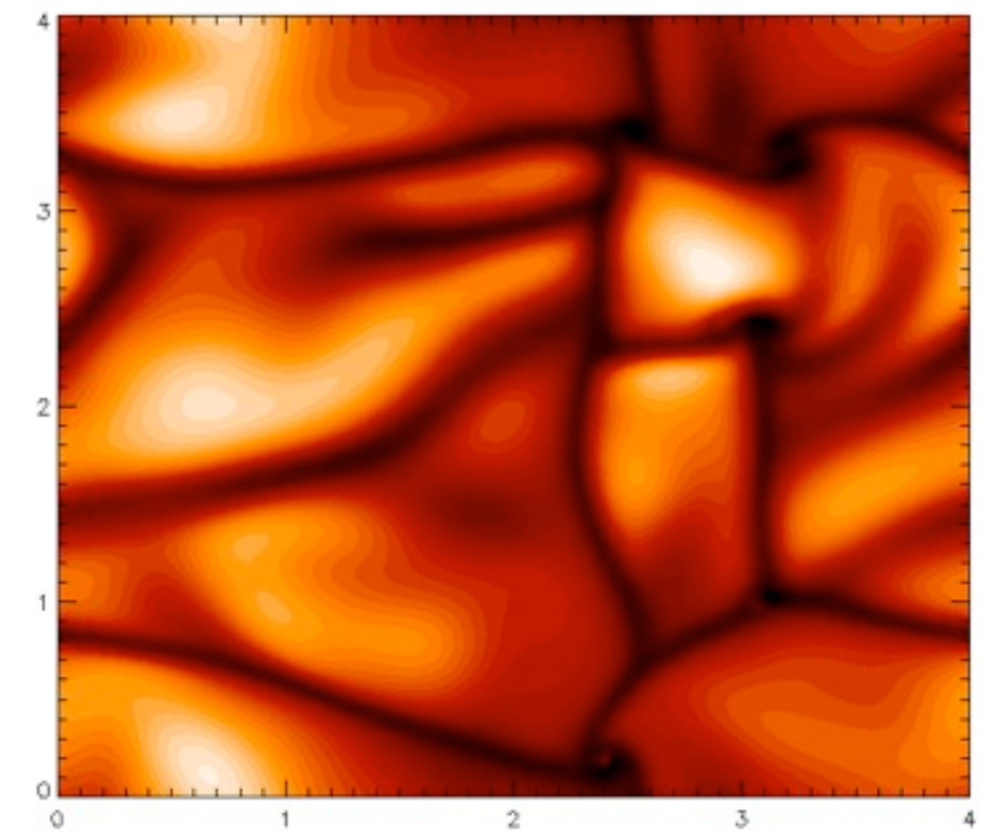
Nonlinear results: $\mathcal{R}m \approx 800$

Higher magnetic Reynolds number leads to a higher saturation level for the dynamo (still not close to Boussinesq levels but still growing...)



$$\frac{\text{Magnetic Energy}}{\text{Kinetic Energy}} \approx 0.08$$

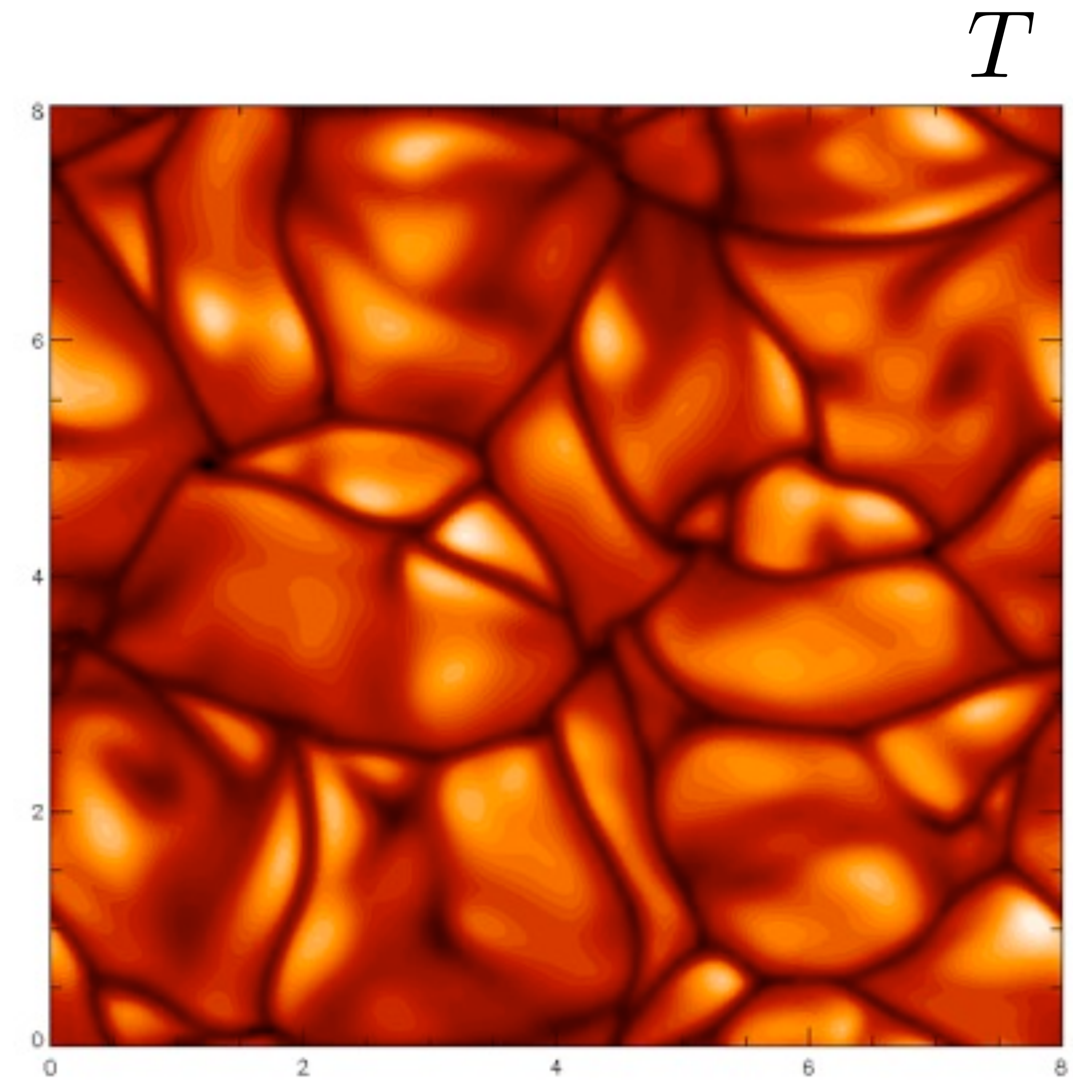


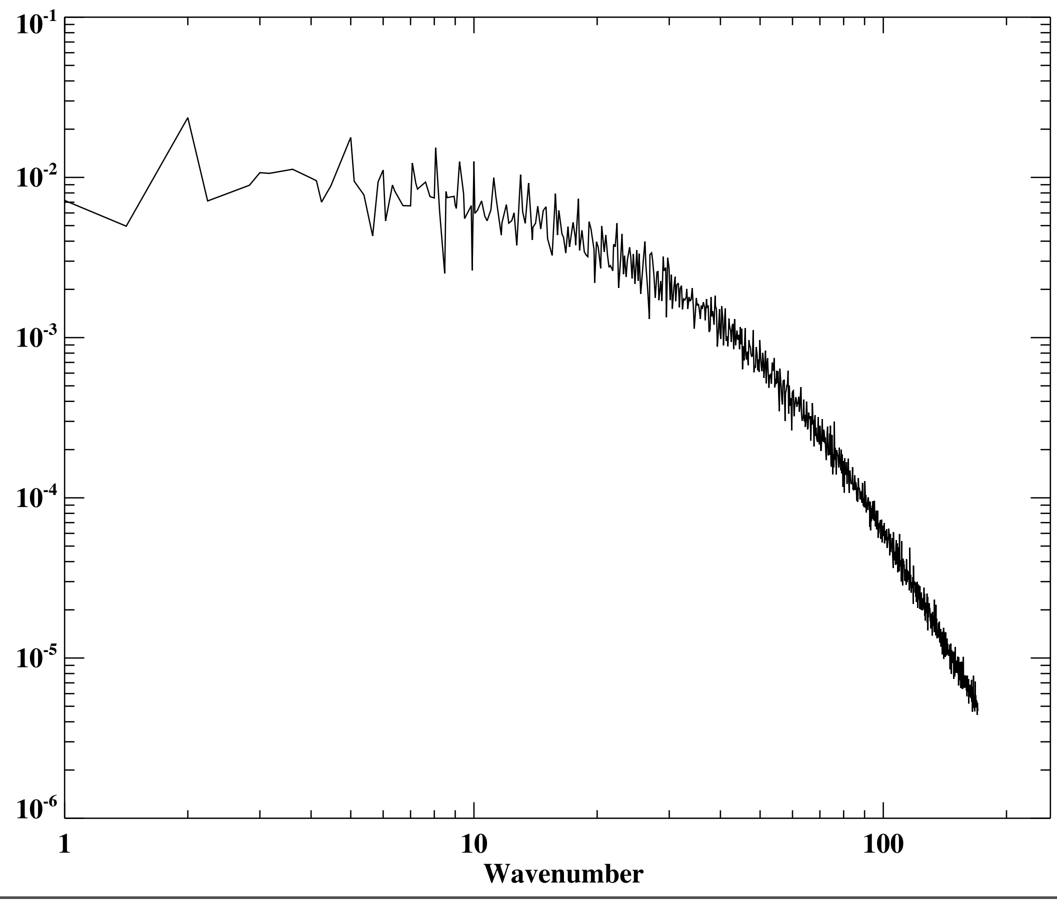


4. Numerical results: λ=8

Does the box size matter?

Preliminary study: Combine 4 copies of a nonlinear λ=4 dynamo calculation into an 8x8x1 domain. Add a random (thermal) perturbation then evolve until the initial imposed symmetry is no longer present....





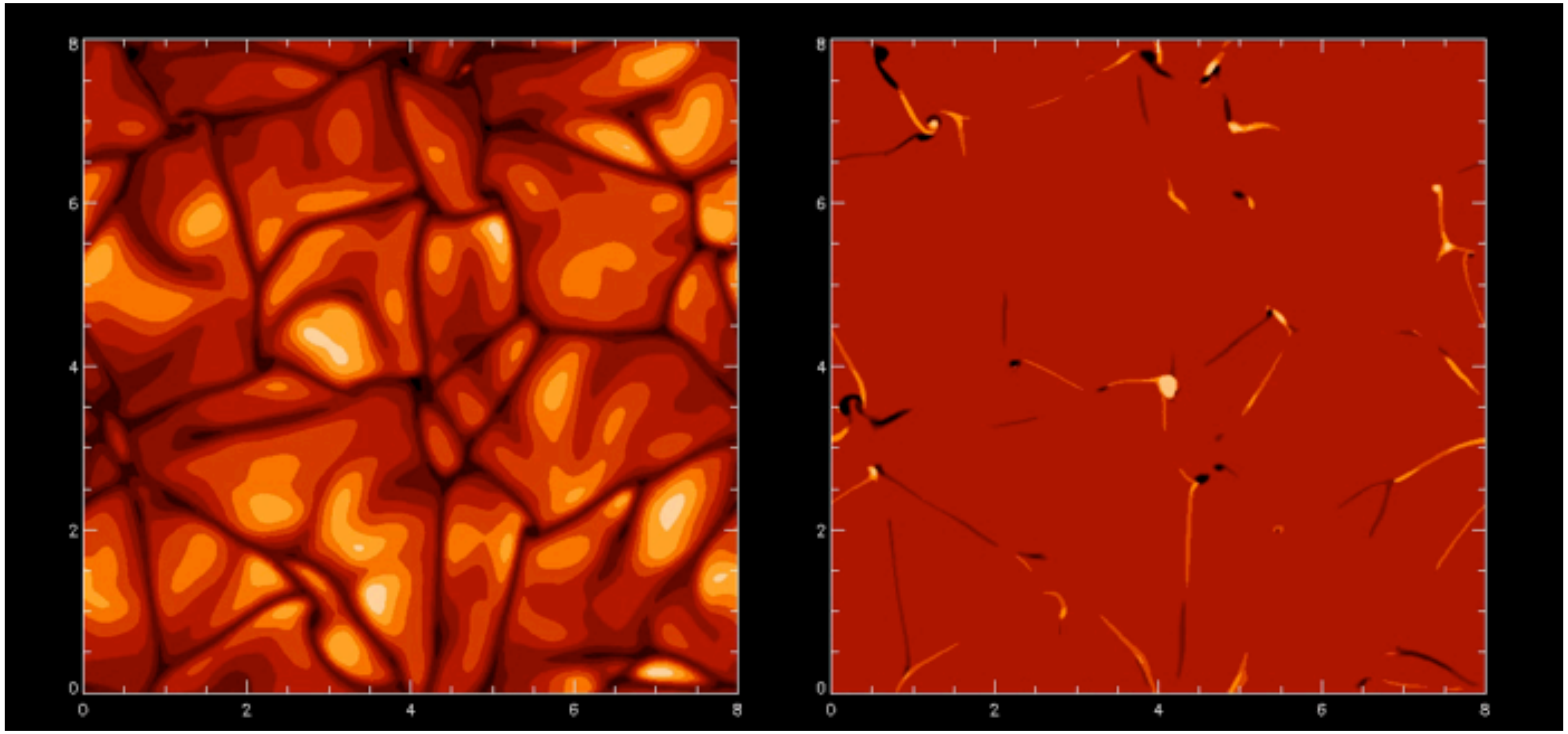
Left: (Horizontal) kinetic energy spectrum. The peak at k=2 almost certainly corresponds to a **mesogranular** scale (e.g. Rincon et al., 2005, A&A) rather than an artefact of the imposed symmetry...

4. Numerical results: λ=8 (cont.)

Dynamo action in the larger domain for $\mathcal{R}m \approx 480$:

$$\frac{\text{Magnetic Energy}}{\text{Kinetic Energy}} \approx 0.065$$

T



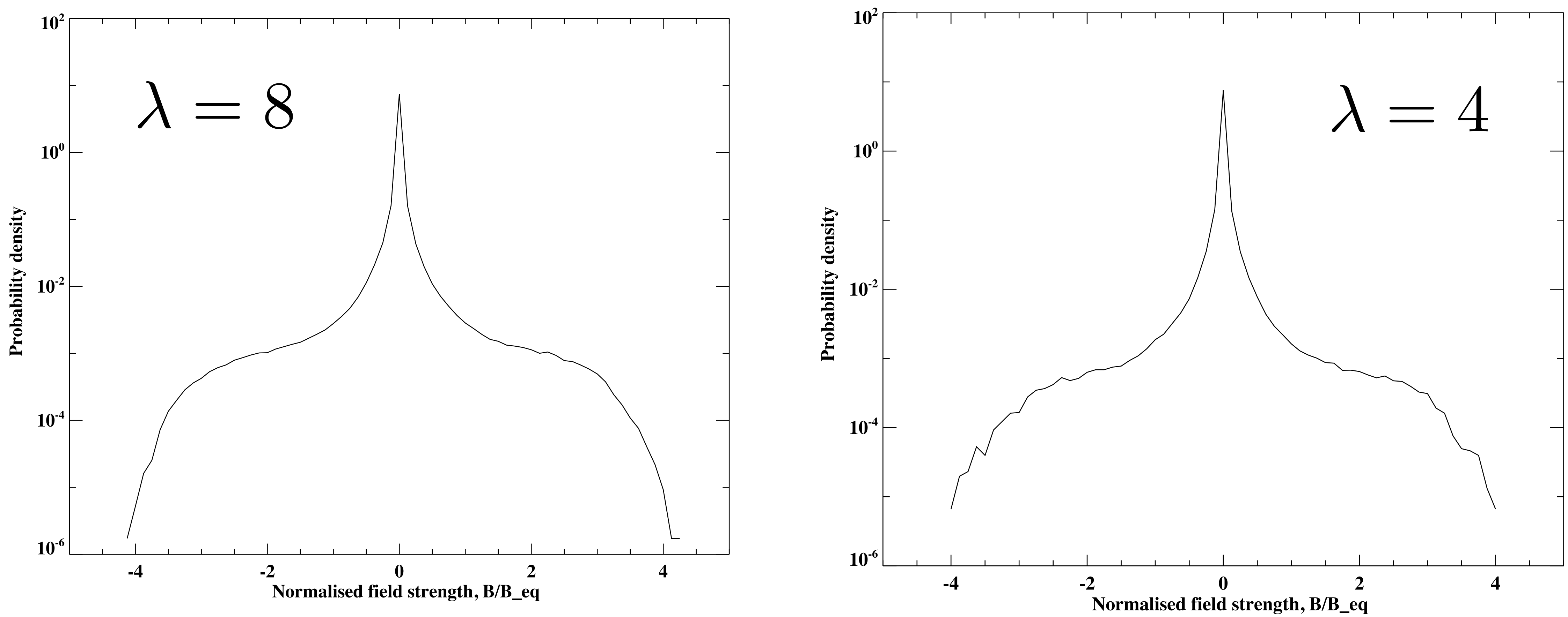
B_z

• Persistent magnetic field concentrations associated with mesogranules(?). Certainly evolve over a longer timescale than granules

• Slightly higher saturation level than equivalent λ=4 case

4. Numerical results: λ=8 (cont.)

A comparison of Probability Density Functions (PDFs) for Bz:



- The PDFs are very similar, although possibly slightly more stretched in the larger box PDF
- Suggests that the peak field strength is only weakly dependent upon the domain size

4. Summary

- Compressible convection can drive a small-scale dynamo at relatively modest magnetic Reynolds numbers

- In the parameter regime under consideration, the growth rate of magnetic energy appears to have a logarithmic dependence upon Rm (probably depends crucially upon the range of values of Pm)

- Comparisons with Boussinesq studies **suggest** that compressible dynamos tend to saturate at a lower level than similar Boussinesq calculations (although higher values of Rm may be able to produce dynamos of comparable efficiency?)

- Preliminary calculations in larger domains suggest that the presence of mesoscale structures may have a weak positive influence upon the saturation level of the dynamo