# The role of spectra in dynamo theory.

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Mathematical Aspects of Natural Dynamos: September 2003

# The Dynamo Problem

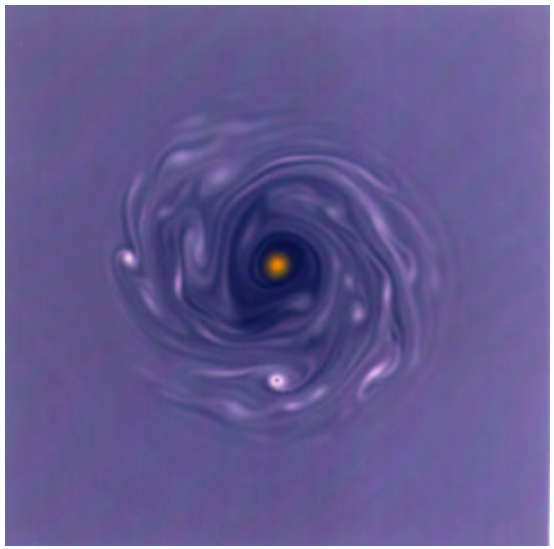
- In many astrophysical/geophysical/laboratory situations magnetic fields are generated by turbulent motions
  - High Reynolds number leads to turbulence.

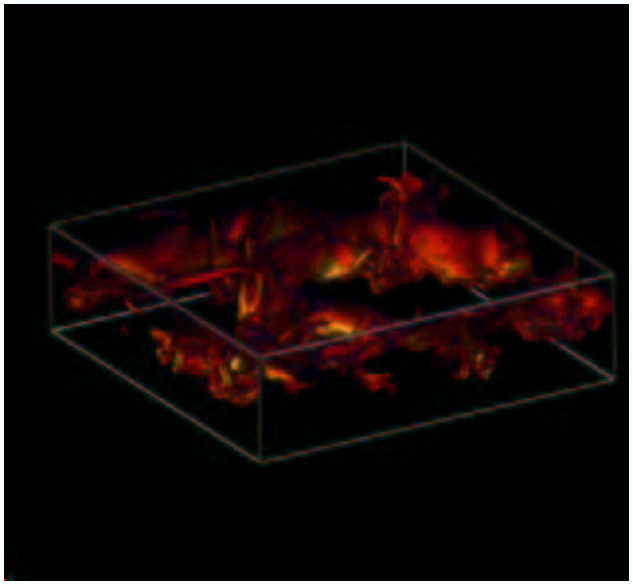
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{Rm} \nabla^2 \mathbf{B}$$

- How do properties of generated field depend on properties of the turbulence?
  - Reynolds numbers
  - Spectra KINEMATIC
  - Coherent Structures
- How do dynamos equilibrate
  - Role of magnetic field in modifying turbulence DYNAMIC

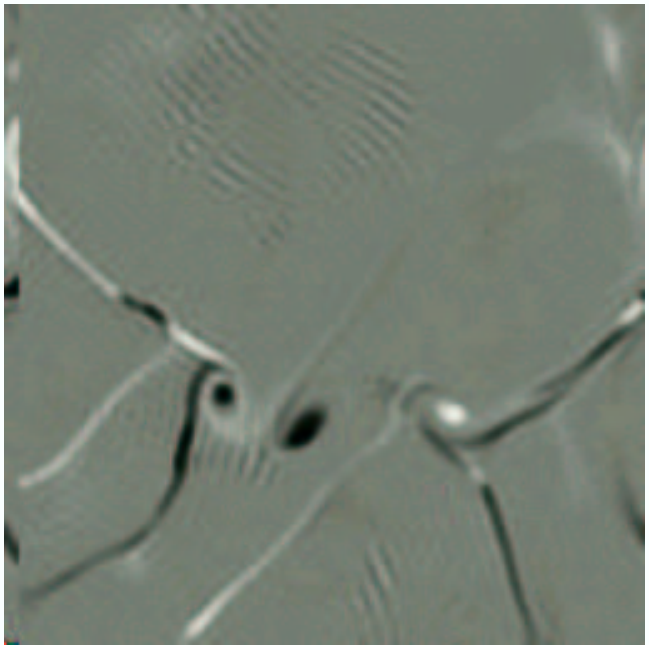
# Coherent Structures.

Bracco et al (1999)





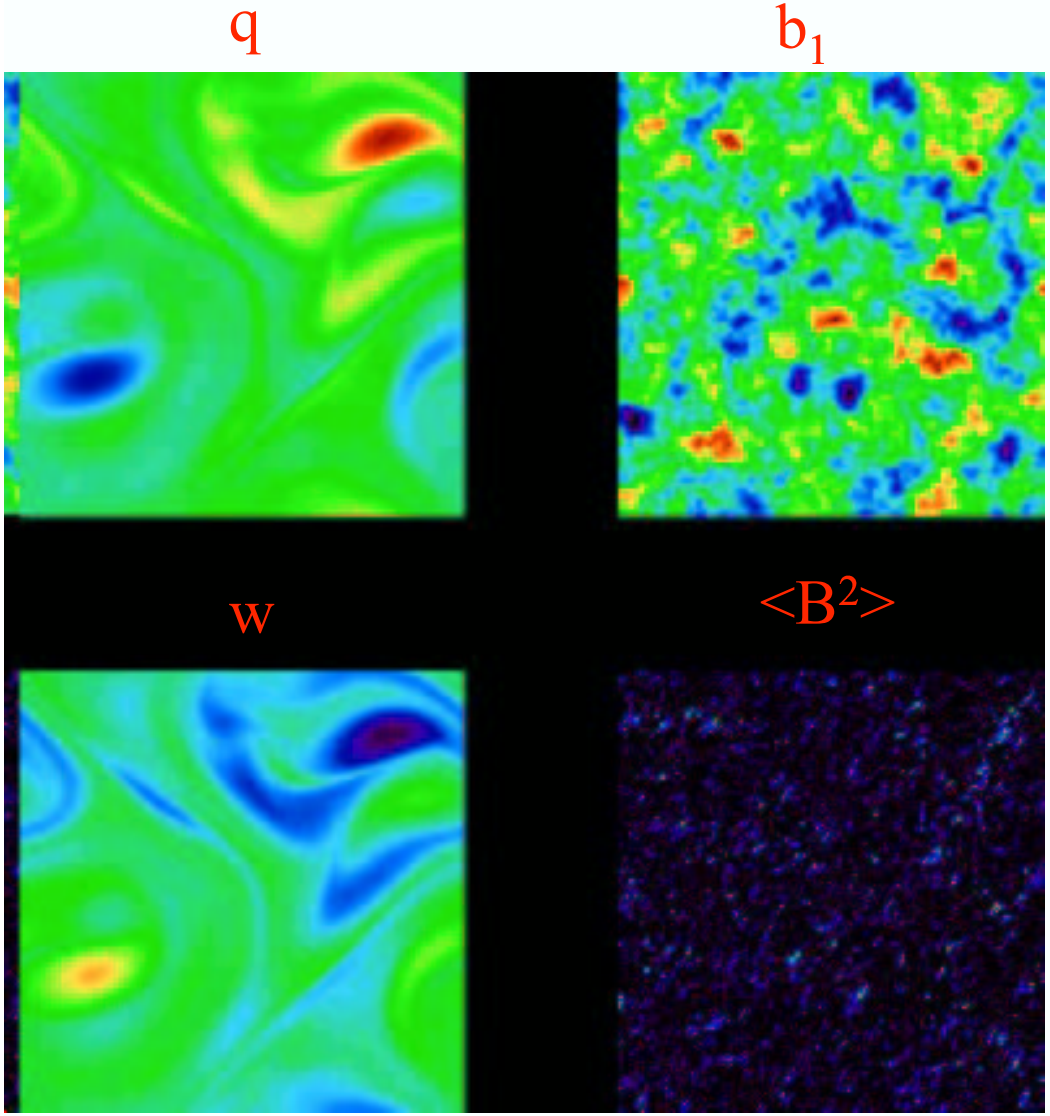
Brummell et al (2002)



Cattaneo (2001)

# Quasi-2d Dynamo solutions

(Llewellyn-Smith & Tobias JFM in press)

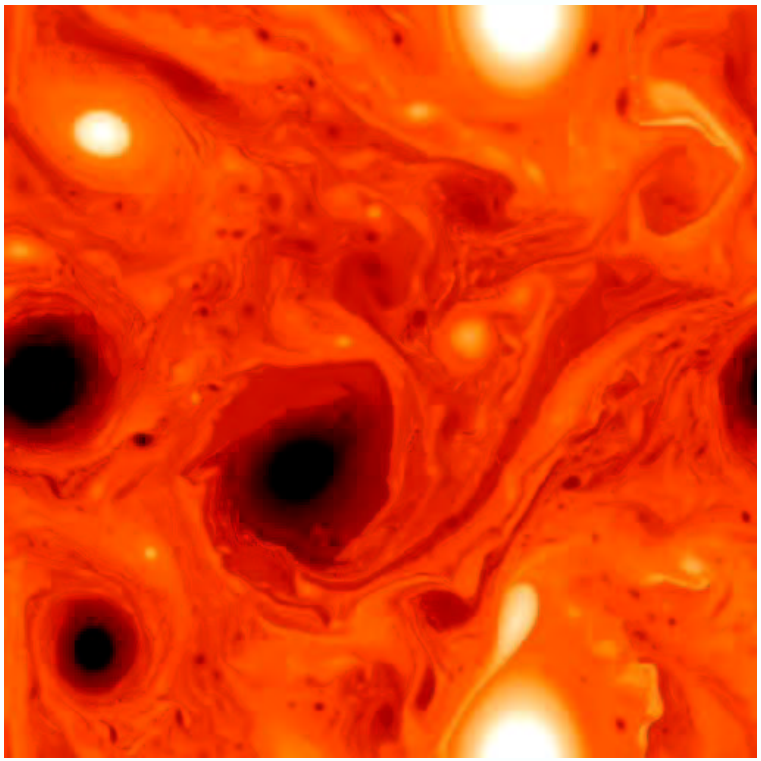



- **Solve 2D vorticity equation with steady forcing at high $R_e$.**

$$q = \nabla^2 \psi$$

- **Inverse cascade to large scale vortices**
- **Vortices advect and amplify magnetic field**
- **Small-scale magnetic fields are amplified**
- **Growth-rate is a function of k, $R_m$, $R_e$**

# Pseudo 3-d dynamos



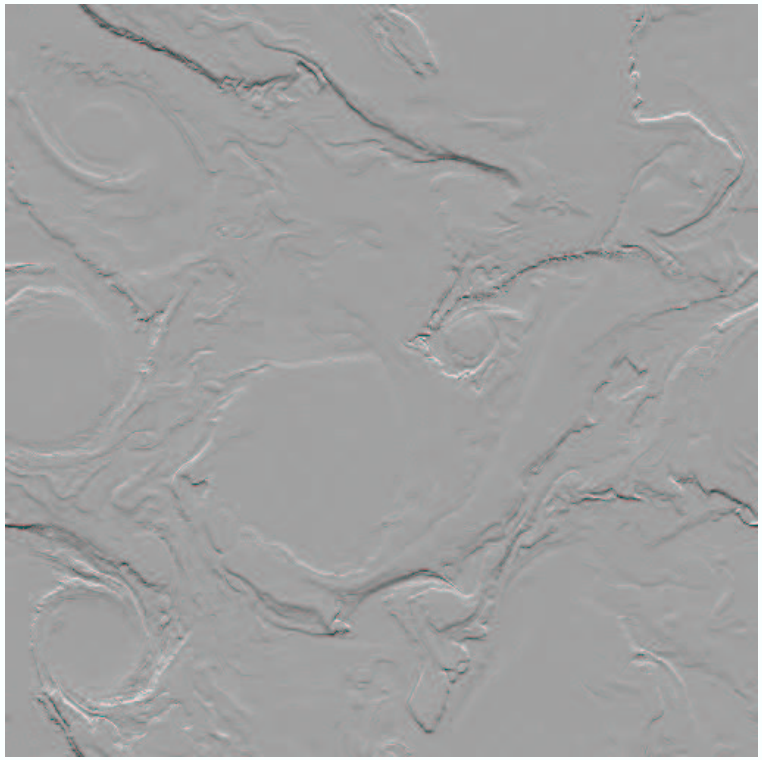
$$q = \nabla^{\lambda}\psi$$

$\lambda = 2 \rightarrow$ 2d Navier Stokes

$\lambda < 2 \rightarrow$ pseudo 3d dynamics LOCAL

Steepness of spectrum depends on $\lambda$

(e.g. Pierrehumbert et al 1992, Constantin 1999)



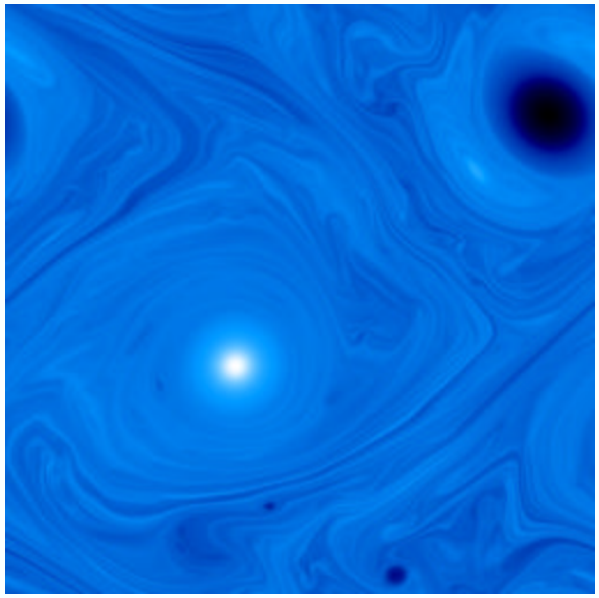
Very high Re, Rm calculations

$\nu = 0.00001$, $\eta = 0.001$, $\lambda = 1$, $k_f = 4$

(performed on Leeds 256 node Beowulf Cluster)

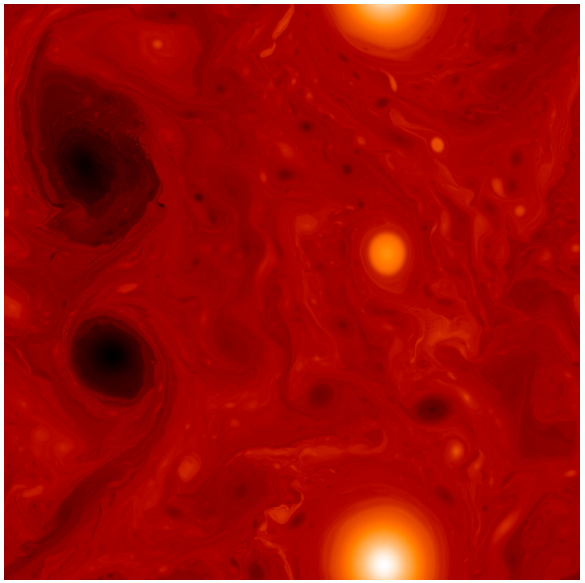
(Cattaneo & Tobias 2003)

# Dynamics varies depending on λ

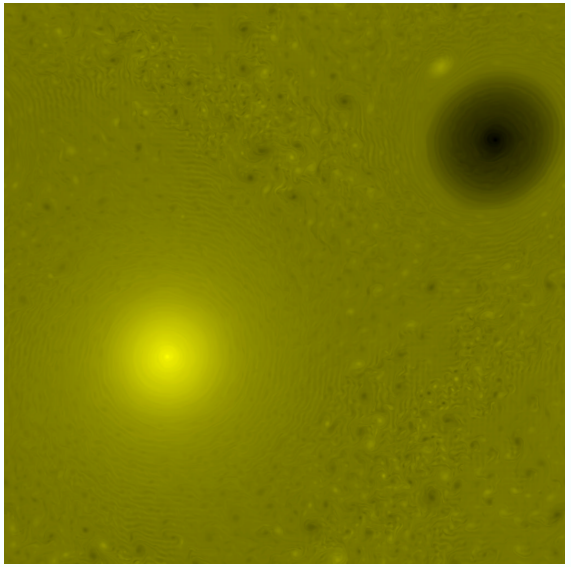


k=4
λ=2
2D NS

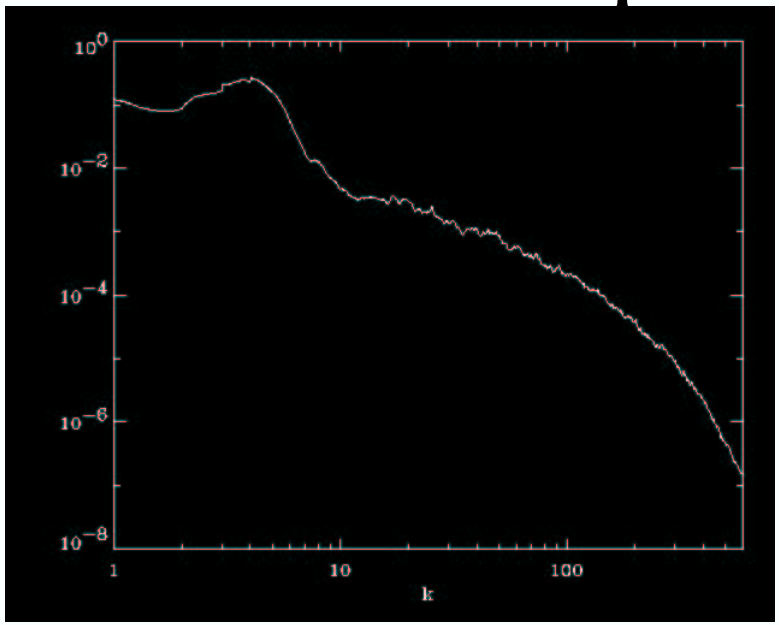
k=4
λ=1



k=100
λ=1

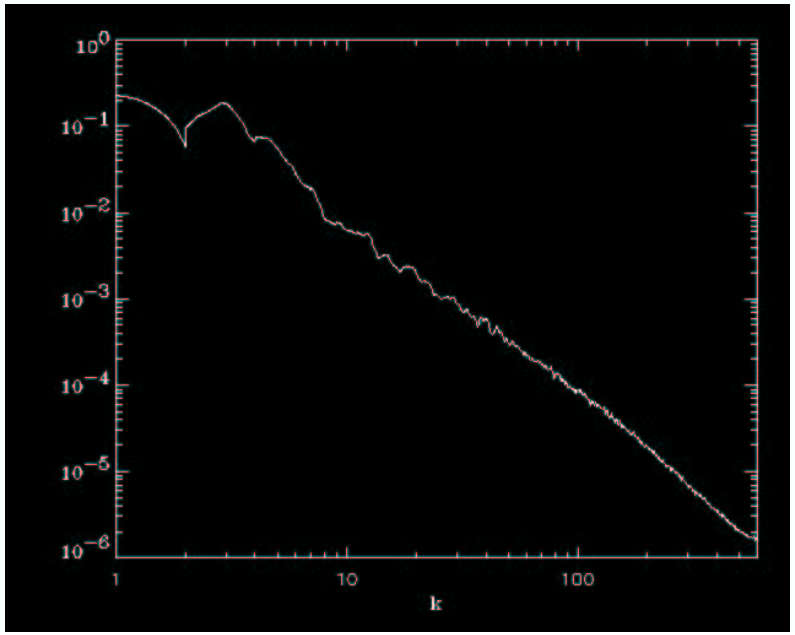


# Spectrum varies depending on λ

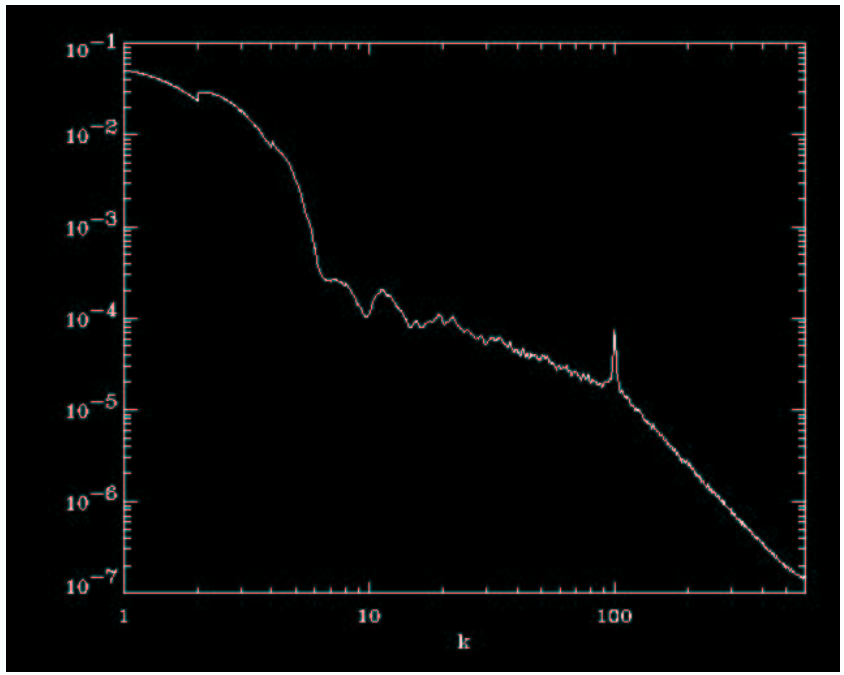



k=4
λ=2
2D NS
Complete inverse cascade

k=4
λ=1

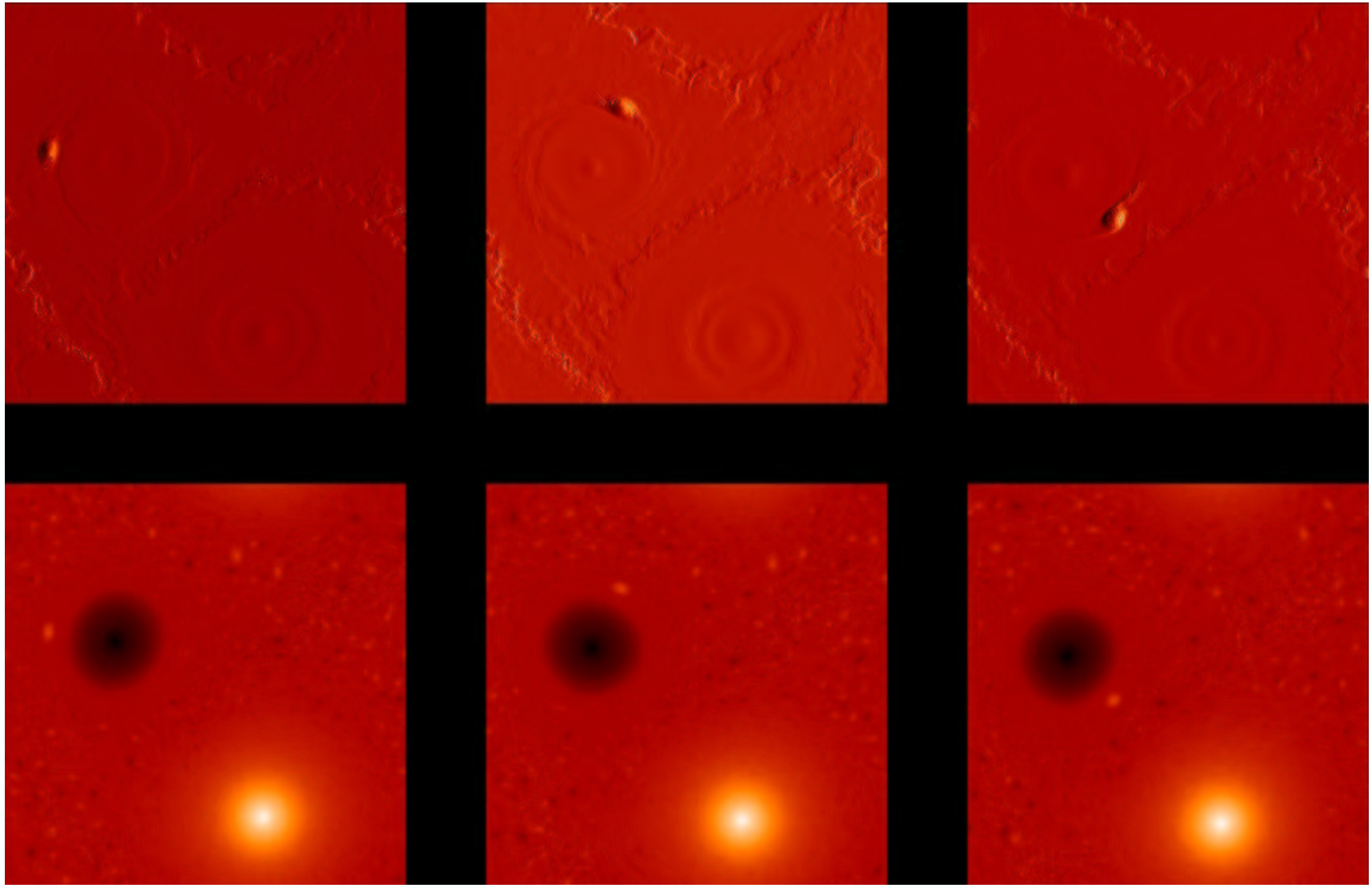



k=100
λ=1



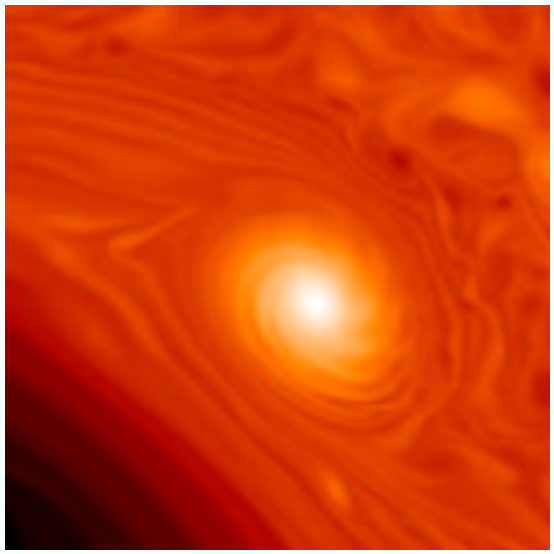
$B_x$

q/w

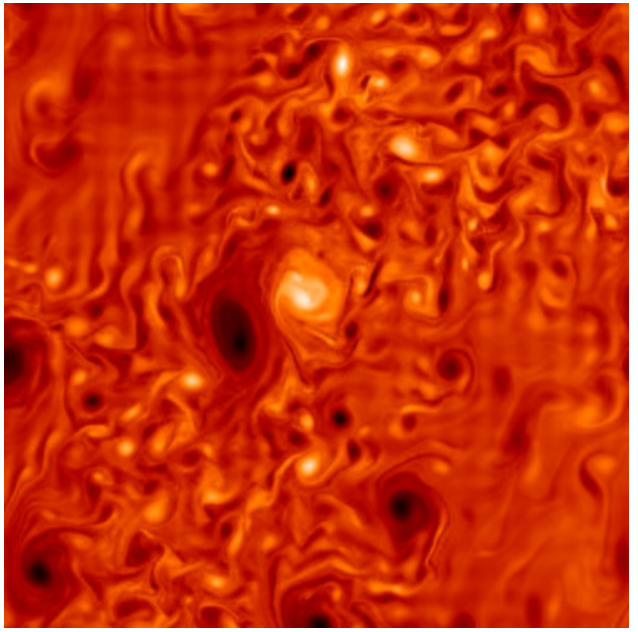


Field generation by small-scale vortices
Stays coherent for many turnover times
(Galloway-Proctor CP in action)

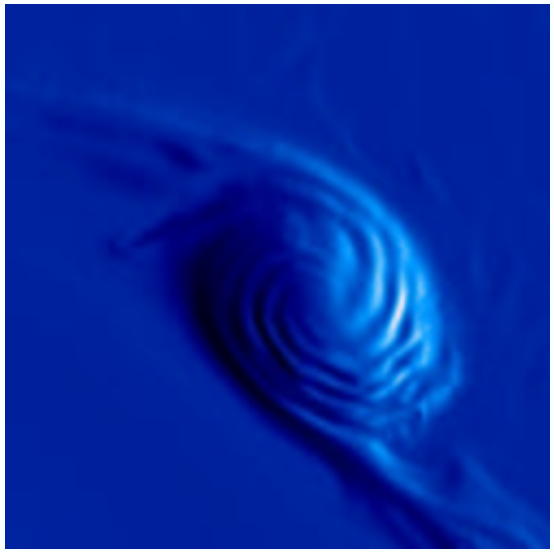
Close-ups



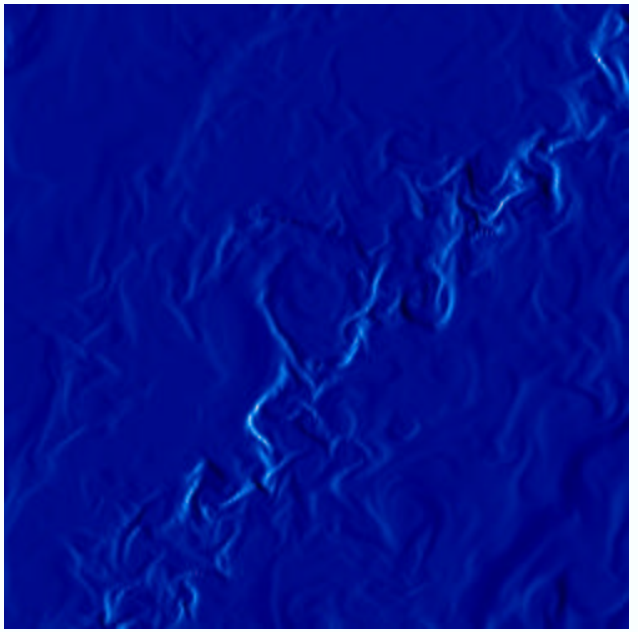
q



q



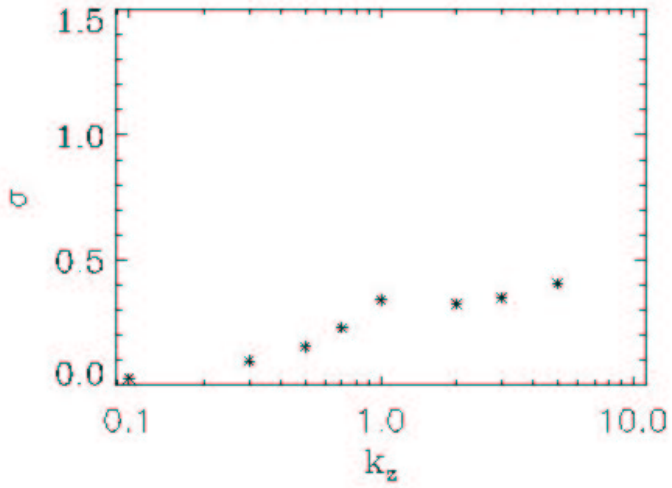
$B_x$



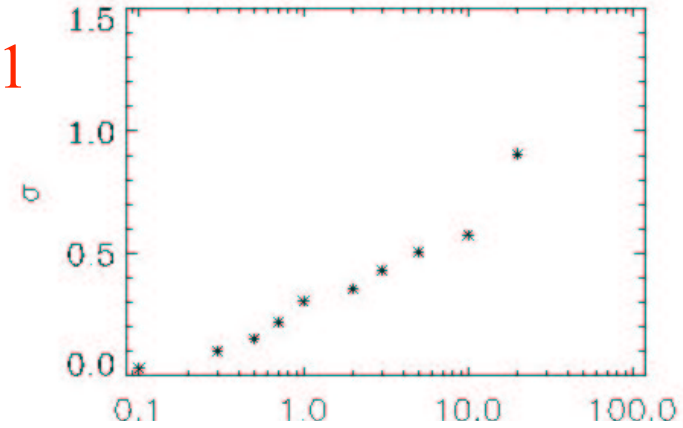
$B_x$

Vortices, vortices all the way down…

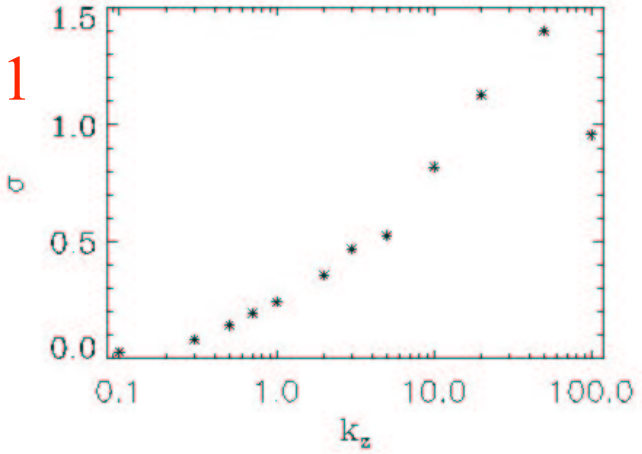
0.01




0.001



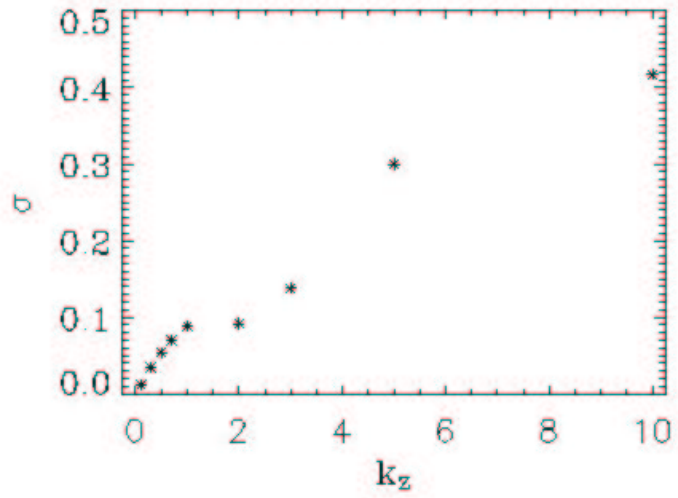

0.0001




$\Lambda=1$

Fast dynamo characteristics

Preferred $k_z$ is quite large

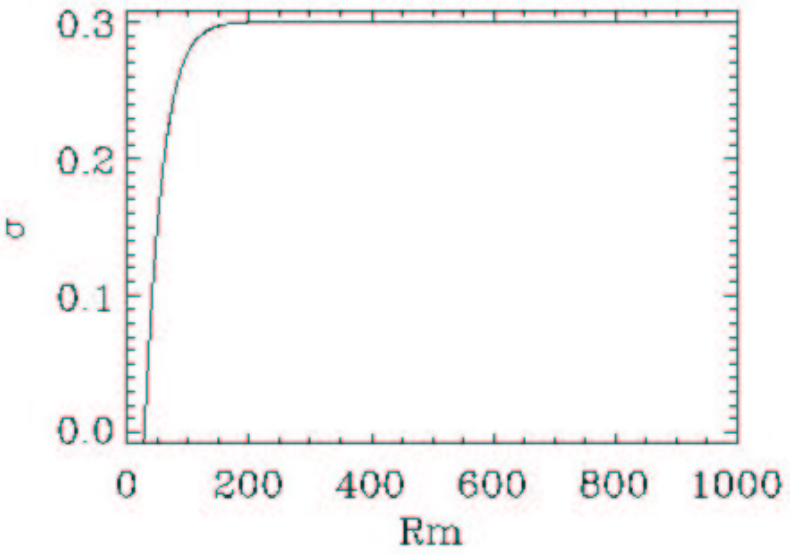



Even for flat slope ($k_f$=100) (cf short correlation time results)

# "Quick" dynamos

- **Can we understand what is going on in turbulent cascades dominated by coherent structures?**
- **Simplest assumption is that most field generation is done by the smallest/fastest eddy for which Rm~1.**
  - **All other eddies don't really contribute.**
- **Slight improvement:**
  - **Each eddy can act as a fast dynamo on its own turnover time.**
  - **Calculate**
    - **Local Rm**
    - **Local turnover time**
    - **Assume each scale acts a quick dynamo independently**
  - **"local growth rate" is then a function of spectral slope**

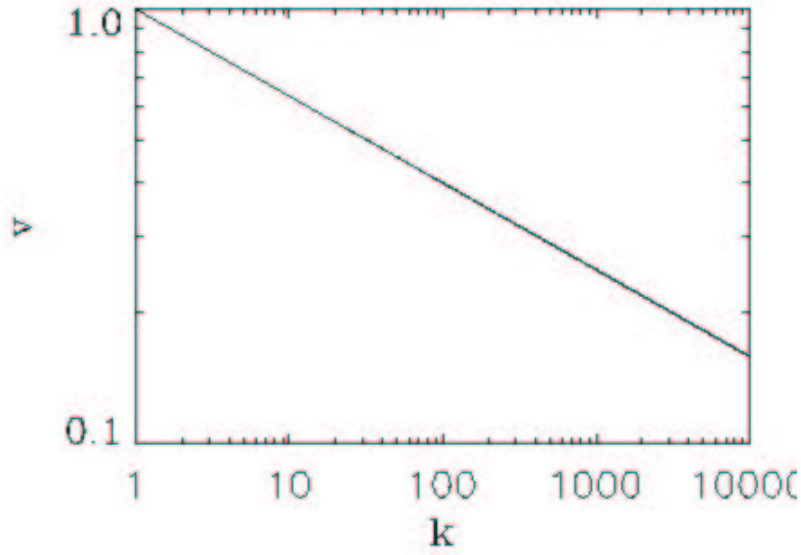
$$v \propto k^{-\alpha}$$



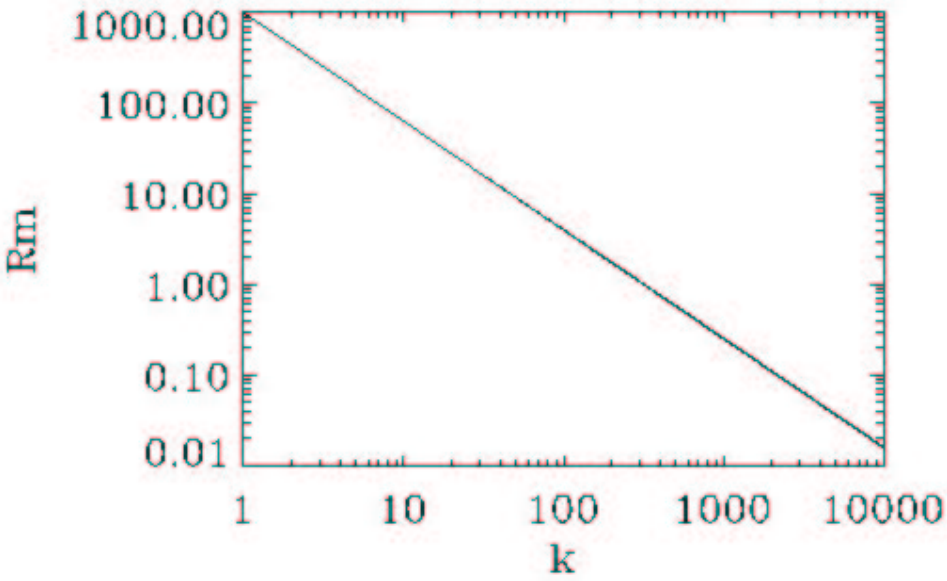

A "quick" dynamo is one where the growth-rate reaches a large fraction of its maximum when Rm~O(1)-O(100)
Quick/fast or quick/slow

p=0.2

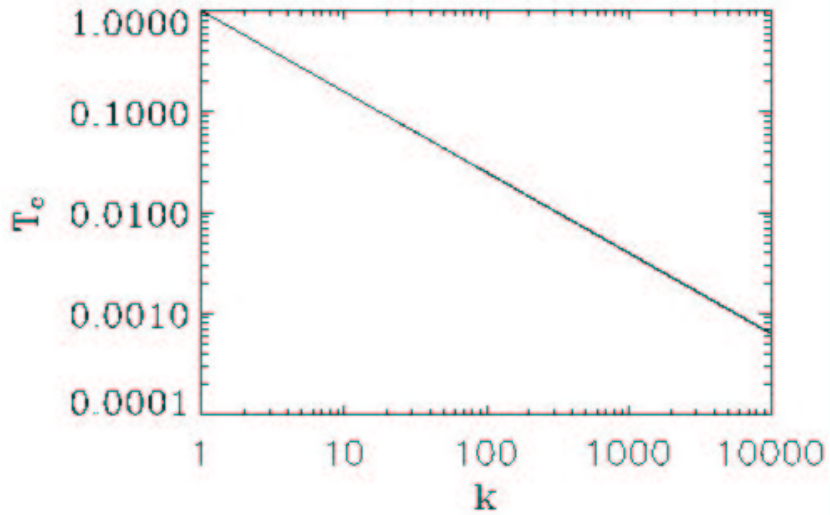
$v \propto k^{-\alpha}$



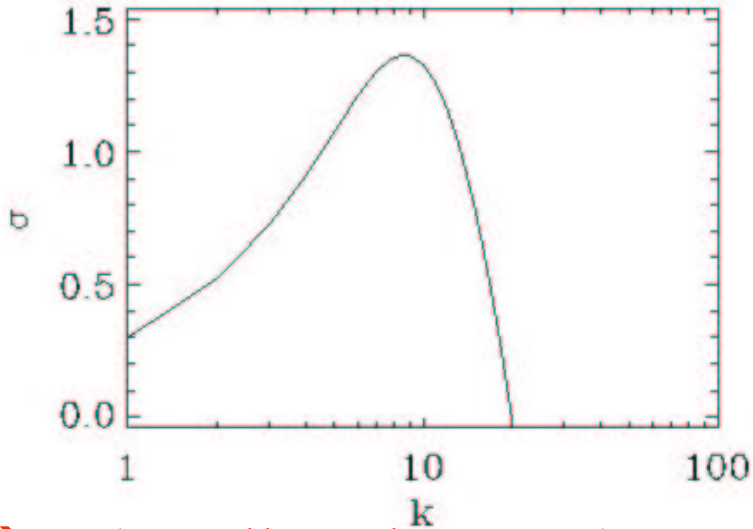

$Rm \propto k^{-(\alpha+1)}$




$T_{turn} \propto k^{\alpha-1}$




$\sigma$



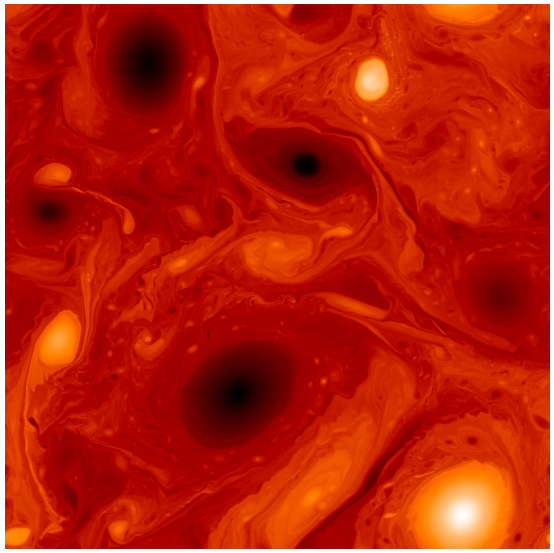

max(σ) can be as k→0 (usually at k~10-20)

# Filtered Dynamos

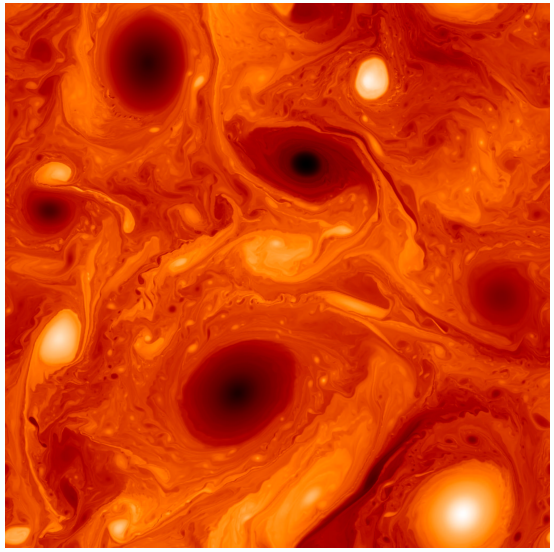
- Solve full 2d or pseudo-3d NS equations
  - Get coherent structures with eddies on various scales and appropriate phase relations
- Filter velocity field
  - Cut off large scales
  - Cut off small scales (to be done)
  - Or just keep “important scales” (to be done)
- Solve induction equation with filtered velocity field
  - Growth rates
  - Form of generated field.

# Filtered Dynamos (preliminary results)

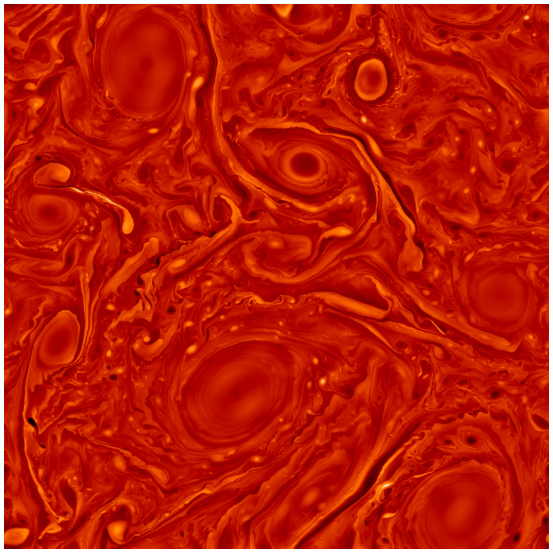
k=0



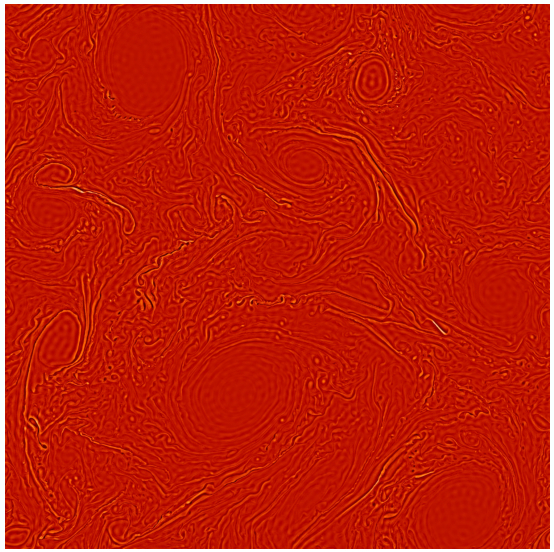
k=1



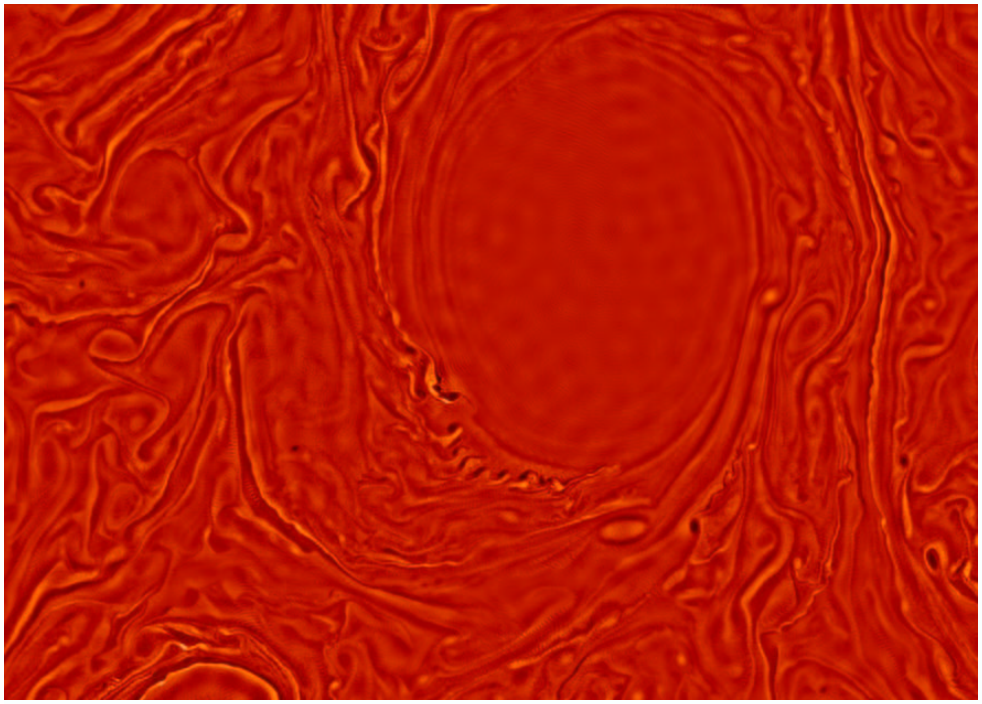
k=10



k=200

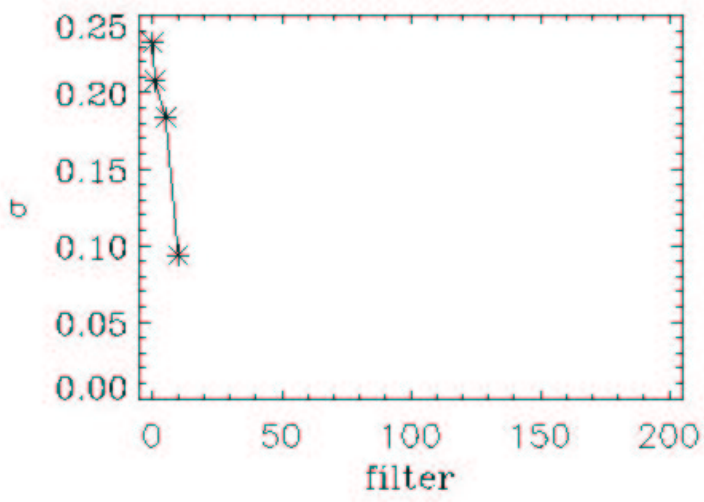


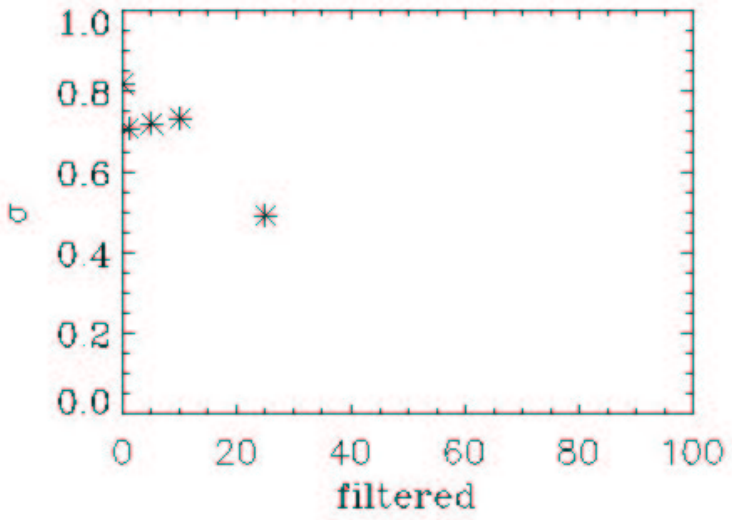
# Filtered Dynamos



k=200

Growth rate begins to fall
When filter reaches ~10-20



# Conclusions

- **Dynamos generate magnetic field by eddies in real space**
- **Astrophysical/Geophysical Turbulence may be neither homogeneous nor isotropic nor have a short correlation time (on any scale of interest!).**
- **Coherent structures within the turbulence may play an important role.**
- **Can get some idea of the behaviour of coherent structures from**
  - **"Quick dynamo" theory.**
  - **Filtered spectra**